<div align="center">

Creative Computing II
Complex Numbers
19th January 2011

</div>

This lab sheet covers the manipulation and use of complex numbers with pen and paper and with *Octave*, and the construction of the Mandelbrot set.

1. Using pencil and paper, calculate the values of the following expressions:

   (a) $(1 + 2i) + (3 - 4i)$

   (b) $(3 + \frac{9}{4}i) - (4 - \frac{5}{8}i)$

   (c) $2 \times (\frac{8}{3} + 2i)$

   (d) $i \times (7 - 9i)$

   (e) $(2 + 3i) \times (1 - 4i)$

   (f) $\frac{1}{2 - 4i}$

   (g) $\frac{15 - 2i}{2 - 4i}$

   (h) $|(3 + 4i)(5 - 12i)|$

   (i) $|\sqrt{(15 + 20i)}|$

   (j) $\arg\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

   (k) $\arg\left(\sqrt{i}\right)$

2. For each of the expressions in part 1, check your answer using *Octave*, using the built-in support for complex numbers.

3. There are many interesting applications of complex numbers; one is in the construction of *fractals*. You have already met fractal objects in CC112: the L-system objects such as the Koch curve, and the biomorph tree-like objects constructed by Richard Dawkins. This part of the lab is about another fractal object, the *Mandelbrot set*.

   (a) We will be applying a transformation repeatedly to complex numbers within the range $-2 \leq \Re z, \Im z \leq 2$. The first step is to construct a matrix representing that region of the complex plane. Construct in *Octave* a 5×5 complex matrix so that the real part of the (y, x)th entry is $3 - y$ and the imaginary part is $x - 3$. Your matrix should be printed by *Octave* like

$$
\begin{matrix}
-2 + 2i & -1 + 2i & 0 + 2i & 1 + 2i & 2 + 2i \\
-2 + 1i & -1 + 1i & 0 + 1i & 1 + 1i & 2 + 1i \\
-2 + 0i & -1 + 0i & 0 + 0i & 1 + 0i & 2 + 0i \\
-2 - 1i & -1 - 1i & 0 - 1i & 1 - 1i & 2 - 1i \\
-2 - 2i & -1 - 2i & 0 - 2i & 1 - 2i & 2 - 2i
\end{matrix}
$$

   (hint: you may find the *Octave* function `repmat` useful in your answer – though there are many ways to achieve this)

(b) The transformation we will be applying is the mapping of a point on the complex plane to itself squared, plus its original value: $z \to z^2 + c$. In order to do this, store the value of your array from part 3a in the *Octave* variable `c`, and use `c` to initialize the matrix `z`. Then to perform one iteration, use the *Octave* expression `z = z.^2 + c;`, which will elementwise square each entry in `z` and add the corresponding entry in `c`.

(c) Repeat this mapping a few times (say four), then inspect the contents of the matrix `z`. You should find that some of the values have grown very large, while others are 0 or 1.

(d) The *Mandelbrot set* is the set of numbers z which do not grow without bound under the iteration $z \to z^2 + c$. From your examination of the `z` matrix in part 3c, suggest numbers which are probably in the *Mandelbrot set*. (If you can, *prove* that those numbers do not diverge under the mapping).

(e) If the modulus of a complex number under the mapping exceeds 2, then the mapping will diverge. This fact enables us to visualise the results of your iteration, by treating all the numbers whose modulus exceeds 2 equally. Use the *Octave* expression `imagesc(min(abs(z),2))` to view the contents of your matrix. (You may wish to read the online help for `imagesc`).

(f) The matrix we started with is insufficiently detailed to see the structure of the Mandelbrot set. Repeat this question with a 513×513 matrix for the numbers in the range $-2 \leq \Re z, \Im z \leq 2$; you may need to do more iterations in part 3c to see the fine detail of the structure emerge.

Other resources:

- Penrose, R., *The Road to Reality*, Chapters 2-7.

- Peitgen, H.-O., Jürgens, H. and Saupe, D., *Chaos and Fractals*

- http://en.wikipedia.org/wiki/Mandelbrot_set.