Creative Computing II

Fourier Analysis: Signals and Systems

26th January 2011

This lab sheet covers the basic properties of the Fourier Transform, and its use in computing system responses. You may wish to check you understand the labsheet for 1st December 2010 before proceeding.

1. This first part of the lab sheet is about the extraction of information for a single frequency from a signal.

   (a) Construct the *Octave* vector corresponding to the complex exponential with frequency 1Hz, at a sample rate of 100Hz, over a time of 4s.

   (b) Construct the *Octave* vector corresponding to a cosine wave with amplitude 1 and frequency 1Hz, at a sample rate of 100Hz, over a time of 4s.

   (c) Compute the dot product of your signals from parts 1a and 1b; note your answer.

   (d) Repeat parts 1b and 1c with a sine wave rather than a cosine wave. Verify that you understand the relationship between the two answers.

   (e) Repeat parts 1b to 1d with cosine and sine waves with frequency 2Hz (but everything else, including the frequency of the complex exponential, unchanged). Comment on your answers.

   (f) Repeat parts 1b to 1d with a vector representing the signal $\sin(f_0 t) + \frac{1}{3}\sin(3f_0 t) + \frac{1}{5}\sin(5f_0 t)$ over 4s, where f_0 is 1Hz.

2. This next part introduces the Fourier Transform and how to interpret the results of *Octave*'s `fft` operator.

   (a) Take the signal from part 1f, and stem plot the modulus of the vector returned from `fft` acting on that signal. Check that you understand

      i. the locations of the first three peaks;

      ii. the relative heights of the first three peaks;

      iii. the intensity of the first peak;

      iv. the existence of the second three peaks.

   (b) take the inverse Fourier Transform (using `ifft`) of the Fourier-Transformed signal, and verify that the original signal is recovered.

3. This final part introduces the basic use of the Fast Fourier Transform for computing system responses to signals; we will be making use of this in future weeks.

   (a) Compute and plot the modulus of the Fourier Transform of the unit delay system, using a window (second parameter to `fft`) of length 10;

   (b) Generate a random signal of length 9, and compute its Fourier Transform over a window of length 10.

(c) Verify that the inverse Fourier Transform of the elementwise product of your answers from parts 3a and 3b produces your signal delayed by one vector element, just as the use of `conv` would have done.

Other resources:

- Stephenson, G., *Mathematical Methods for Science Students*, material on Fourier Series;
- Boas, M. L., *Mathematical Methods in the Physical Sciences*, material on Fourier Series;