

Creative Computing II

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Autumn 2010, Wednesdays:
10:00–12:00: RHB307 & 14:00–16:00: WB316
Winter 2011, TBC

Systematization of Colour

Maxwell Triangle

Device to represent *chromaticity*

- ▶ which colour;
- ▶ how much colour.

Brightness or **luminance** not represented.

Systematization of Colour

Maxwell Triangle

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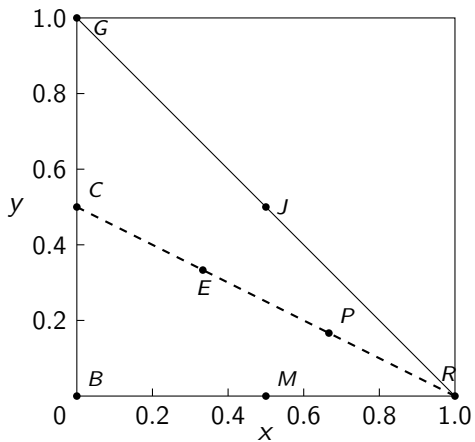
Brightness or **luminance** not represented.

- ▶ $C = rR + gG + bB$
- ▶ $x = \frac{r}{r+g+b}$
- ▶ $y = \frac{g}{r+g+b}$

Together with *luminance* $r + g + b$, makes a new three-dimensional colour space.

Systematization of Colour

Maxwell Triangle



Systematization of Colour

CIE 1931 Standard Observer

Problem:

- ▶ positive mixtures of three colours cannot represent all visible colours.

Systematization of Colour

CIE 1931 Standard Observer

Problem:

- ▶ positive mixtures of three colours cannot represent all visible colours.

Solution:

- ▶ do careful experiments with human observers
- ▶ use 'imaginary' primaries with useful properties.

Systematization of Colour

CIE 1931 Standard Observer

Properties of CIE 1931 XYZ primaries:

- ▶ Y represents **luminance** (brightness); X and Z independent of luminance;
- ▶ visible colours occupy as large a space as possible on the chromaticity diagram;
- ▶ 'equal-energy white' is at $x = y = z = \frac{1}{3}$.

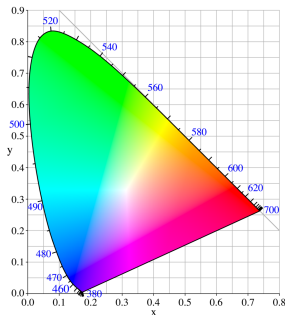
Note:

- ▶ pure- X , pure- Y and pure- Z are not visible colours.

Systematization of Colour

The CIE xyY Colour Space

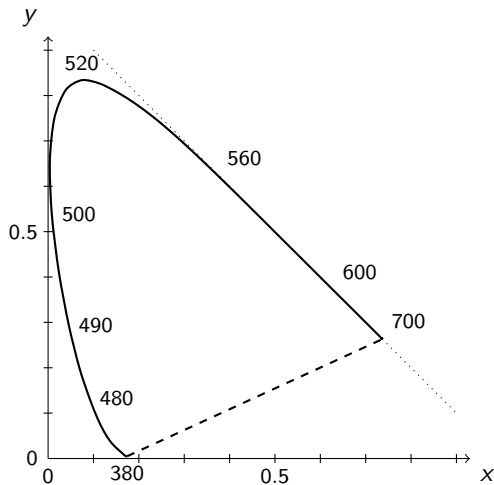
- ▶ take colour specified in CIE XYZ space;
- ▶ convert to chromaticity:
 - ▶ $x = \frac{X}{X+Y+Z}$
 - ▶ $y = \frac{Y}{X+Y+Z}$



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Systematization of Colour

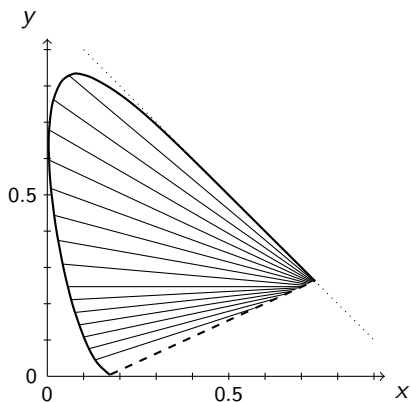
The CIE xyY Colour Space



Systematization of Colour

The CIE xyY Colour Space

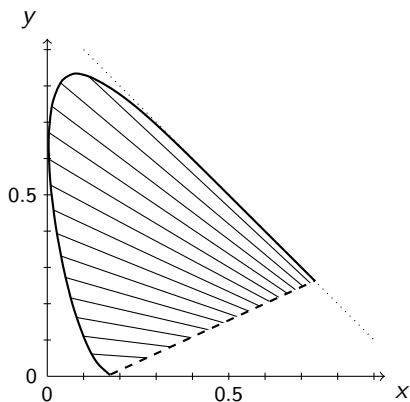
Confusion lines for *protanopes* meet at $(0.75, 0.25)$:



Systematization of Colour

The CIE xyY Colour Space

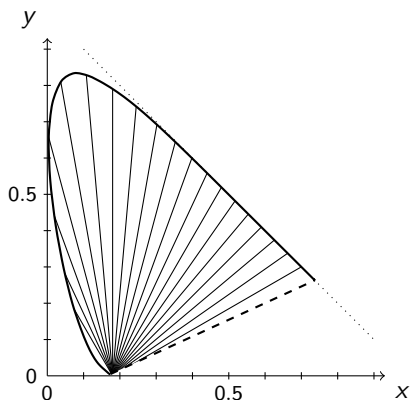
Confusion lines for *deuteranopes* meet at $(1.0,0.0)$:



Systematization of Colour

The CIE xyY Colour Space

Confusion lines for *tritanopes* meet at (0.18,0.0):



data from Judd, D. B., *Fundamental Studies of Color Vision from 1860 to 1960*, Proc. Nat'l Acad. Sci. 55:6 (1966)

Systematization of Colour

Digital Displays and sRGB

sRGB Colour space:

- ▶ developed by Hewlett-Packard and Microsoft;
- ▶ primaries the same as that used in display hardware;
- ▶ *transfer function* typical of display hardware;
- ▶ used as default colour space on
 - ▶ monitors;
 - ▶ some printers;
 - ▶ many file formats (PNG, TIFF);
 - ▶ the Internet (HTML, CSS colour specifications).

Systematization of Colour

Digital Displays and sRGB

CIE XYZ \rightarrow sRGB:

$$\blacktriangleright \begin{pmatrix} R_I \\ G_I \\ B_I \end{pmatrix} = \begin{pmatrix} 3.2410 & -1.5374 & -0.4986 \\ -0.9692 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0570 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

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$$\blacktriangleright C_{\text{sRGB}} = \begin{cases} 12.92 C_I & C_I < 0.00304 \\ 1.055 C_I^{1/2.4} - 0.055 & \text{otherwise} \end{cases}$$

Systematization of Colour

Digital Displays and sRGB

CIE XYZ \rightarrow sRGB:

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$$\rightarrow C_{\text{sRGB}} = \begin{cases} 12.92 C_I & C_I < 0.00304 \\ 1.055 C_I^{1/2.4} - 0.055 & \text{otherwise} \end{cases}$$

\rightarrow (for normal use)

- \rightarrow clamp C_{sRGB} to be between 0 and 1;
- \rightarrow scale to desired colour resolution (e.g. 0–255).

Systematization of Colour

Digital Displays and sRGB

sRGB \rightarrow CIE XYZ:

- ▶ Scale C_{sRGB} values to be between 0 and 1;

Systematization of Colour

Digital Displays and sRGB

sRGB \rightarrow CIE XYZ:

- ▶ Scale C_{sRGB} values to be between 0 and 1;
- ▶ $C_I = \begin{cases} \frac{C_{sRGB}}{12.92} & C_{sRGB} < 0.04045 \\ \left(\frac{C_{sRGB} + 0.055}{1.055} \right)^{2.4} & \text{otherwise} \end{cases}$

Systematization of Colour

Digital Displays and sRGB

sRGB \rightarrow CIE XYZ:

- ▶ Scale C_{sRGB} values to be between 0 and 1;

$$\text{▶ } C_I = \begin{cases} \frac{C_{\text{sRGB}}}{12.92} & C_{\text{sRGB}} < 0.04045 \\ \left(\frac{C_{\text{sRGB}} + 0.055}{1.055} \right)^{2.4} & \text{otherwise} \end{cases}$$

$$\text{▶ } \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{pmatrix} \begin{pmatrix} R_I \\ G_I \\ B_I \end{pmatrix}$$

Systematization of Colour

The CIE LAB Colour Space

Problem with CIE XYZ:

- ▶ 'distances' in colour space are not perceptually relevant;
- ▶ (just like RGB, HSB)

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The CIE LAB Colour Space

Problem with CIE XYZ:

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- ▶ (just like RGB, HSB)

Solution:

- ▶ transform CIE XYZ into a colour space where distances correlate with perceived colour differences
- ▶ CIE $L^*a^*b^*$ (CIELAB)
- ▶ (different spaces with the same aim: CIE Luv, Hunter Lab)
- ▶ L^* matches **lightness** (not the same as brightness);
- ▶ a^* and b^* are chromaticity components:
 - ▶ a^* : red/magenta vs green;
 - ▶ b^* : yellow vs blue.

Systematization of Colour

The CIE LAB Colour Space

Euclidean distance between CIE LAB colours

$$\Delta C = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

correlates with perceptual colour difference.

Systematization of Colour

The CIE LAB Colour Space

CIE XYZ \rightarrow CIE LAB:

▶ define $f(t) = \begin{cases} \sqrt[3]{t} & t > (\frac{6}{29})^3 \\ \frac{1}{3} (\frac{29}{6})^2 t + \frac{4}{29} & \text{otherwise} \end{cases}$

Systematization of Colour

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CIE XYZ \rightarrow CIE LAB:

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$$\text{▶ } L^* = 116f\left(\frac{Y}{Y_0}\right) - 16$$

$$\text{▶ } a^* = 500 \left[f\left(\frac{X}{X_0}\right) - f\left(\frac{Y}{Y_0}\right) \right]$$

$$\text{▶ } b^* = 200 \left[f\left(\frac{Y}{Y_0}\right) - f\left(\frac{Z}{Z_0}\right) \right]$$

Systematization of Colour

The CIE LAB Colour Space

CIE LAB \rightarrow CIE XYZ:

$$\blacktriangleright \text{define } f^{-1}(z) = \begin{cases} z^3 & z > \frac{6}{29} \\ (z - \frac{4}{29})^3 + (\frac{6}{29})^2 & \text{otherwise} \end{cases}$$

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▶ $f_y = \frac{L^* + 16}{116}$

▶ $f_x = f_y + \frac{a^*}{500}$

▶ $f_z = f_y - \frac{b^*}{200}$

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▶ $f_y = \frac{L^* + 16}{116}$

▶ $f_x = f_y + \frac{a^*}{500}$

▶ $f_z = f_y - \frac{b^*}{200}$

▶ $X = X_0 f^{-1}(f_x)$

▶ $Y = Y_0 f^{-1}(f_y)$

▶ $Z = Z_0 f^{-1}(f_z)$