Creative Computing II

Christophe Rhodes
c.rhodes@gold.ac.uk

Autumn 2010, Wednesdays:
10:00–12:00: RHB307 & 14:00–16:00: WB316
Winter 2011, Wednesdays:
10:00–12:00: RHB307 & 14:00–16:00: WB316

Course Information

Administrative matters

- Course website:
  `http://doc.gold.ac.uk/~mas01cr/teaching/cc227/`
- My e-mail address: `c.rhodes@gold.ac.uk`
- Feedback and Consultation hours:
  - Wednesday, 16:00–18:00, room 2.06, BPB
  - Use phone for entry to BPB 2nd floor

Course Information

Administrative matters

- Course website:
  `http://doc.gold.ac.uk/~mas01cr/teaching/cc227/`
- My e-mail address: `c.rhodes@gold.ac.uk`
- Feedback and Consultation hours:
  - Wednesday, 16:00–18:00, room 2.06, BPB
  - Use phone for entry to BPB 2nd floor
- Dr Marcus Pearce: `m.pearce@gold.ac.uk`

Course Outline

Syllabus

- Visual perception;
- Animation;
- Sound, hearing and music;
- Signals;
- Audio and image filtering;
- Multimedia information retrieval;

Course Outline

Syllabus

- **Visual perception**: cones, rods and the eye; optical illusions; colour vision; colour spaces and profiles; motion perception and Gestalt psychology.
- **Animation**: approaches to animation; perception in video and film; making animations; visualisation.
- **Sound, hearing and music**: sound and the ear; frequency, pitch and harmony; melody; rhythm; digital audio formats and compression.

Course Outline

Syllabus

- **Visual perception**: cones, rods and the eye; optical illusions; colour vision; colour spaces and profiles; motion perception and Gestalt psychology.
- **Animation**: approaches to animation; perception in video and film; making animations; visualisation.
- **Sound, hearing and music**: sound and the ear; frequency, pitch and harmony; melody; rhythm; digital audio formats and compression.

Course Outline

Syllabus

- **Signals**: the nature of signals; special signals; audio signals and sampling; frequency, amplitude and phase; the Fourier representation.
- **Systems**: linearity and time-invariance; impulse responses and convolution; spectral analysis; convolution by spectrum multiplication.
- **Audio and image filtering**: EQ; filter design; subtractive synthesis; echo and reverberation; resampling; image representation; two-dimensional convolution and image effects.
- **Multimedia information retrieval**: retrieval, fingerprinting and similarity; features and distance measures; systems for multimedia information retrieval.

Course Outline

Syllabus

- **Signals**: the nature of signals; special signals; audio signals and sampling; frequency, amplitude and phase; the Fourier representation.
- **Systems**: linearity and time-invariance; impulse responses and convolution; spectral analysis; convolution by spectrum multiplication.
- **Audio and image filtering**: EQ; filter design; subtractive synthesis; echo and reverberation; resampling; image representation; two-dimensional convolution and image effects.
- **Multimedia information retrieval**: retrieval, fingerprinting and similarity; features and distance measures; systems for multimedia information retrieval.

Systems

- ‘Systems’ construct new signals from existing ones.
- Examples:
  - computer monitor
    - input: electrical signals;
    - output: light emitted from screen.

Systems

- ‘Systems’ construct new signals from existing ones.
- Examples:
  - computer monitor
    - input: electrical signals;
    - output: light emitted from screen.
  - violin body
    - input: bow moving over the strings;
    - output: musical sound.

Systems

- 'Systems' construct new signals from existing ones.
- Examples:
  - computer monitor
    - input: electrical signals;
    - output: light emitted from screen.
  - violin body
    - input: bow moving over the strings;
    - output: musical sound.
  - car suspension
    - input: bumps on the road;
    - output: smoothness of ride.

Systems

Overview

- Properties and nomenclature
  - Linearity
  - Time-invariance

Systems

Overview

- Properties and nomenclature
  - Linearity
  - Time-invariance
- Impulse Response
  - System characterisation
  - Convolution

Systems

Overview

- Properties and nomenclature
  - Linearity
  - Time-invariance
- Impulse Response
  - System characterisation
  - Convolution
- Spectral Analysis
  - Complex Numbers
  - Complex Exponentials
  - Fourier Transform
  - Fast Fourier Transform

Systems

Notation

Given an input signal x, the action of a system H on that signal, producing an output signal y, is denoted

$$y = H\{x\}$$

Systems

Notation

Given an input signal x, the action of a system H on that signal, producing an output signal y, is denoted

$$y = H\{x\}$$

For a discrete-time signal $x[n]$,

$$y[n] = H\{x[n]\} \text{ or } y[n] = H\{x\}[n]$$

Systems

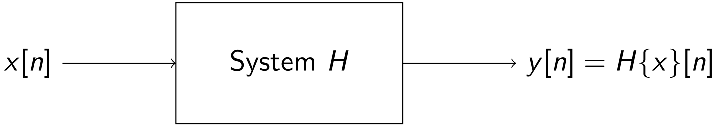
Notation

Given an input signal x, the action of a system H on that signal, producing an output signal y, is denoted

$$y = H\{x\}$$

For a discrete-time signal $x[n]$,

$$y[n] = H\{x[n]\} \text{ or } y[n] = H\{x\}[n]$$



Systems

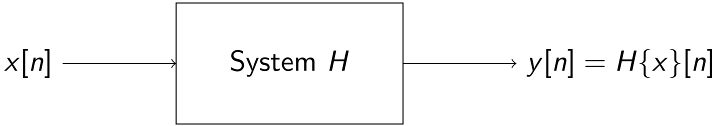
Notation

Given an input signal x, the action of a system H on that signal, producing an output signal y, is denoted

$$y = H\{x\}$$

For a discrete-time signal $x[n]$,

$$y[n] = H\{x[n]\} \text{ or } y[n] = H\{x\}[n]$$



(Health Warning: the notation can be confusing)

Systems

Linear Systems

Linear systems have the property that *superposition* and *scaling* of their input signals yield the corresponding scaled superposition of their outputs.

Linear systems have the property that *superposition* and *scaling* of their input signals yield the corresponding scaled superposition of their outputs.

For any input signals x_1 and x_2, if

$$y_1 = H\{x_1\}$$

and

$$y_2 = H\{x_2\},$$

a system H is linear if

$$H\{\alpha x_1 + \beta x_2\} = \alpha y_1 + \beta y_2$$

Systems

Linear Systems

Linear systems have the property that *superposition* and *scaling* of their input signals yield the corresponding scaled superposition of their outputs.

For any input signals x_1 and x_2, if

$$y_1 = H\{x_1\}$$

and

$$y_2 = H\{x_2\},$$

a system H is linear if

$$H\{\alpha x_1 + \beta x_2\} = \alpha y_1 + \beta y_2$$

(Almost) all systems in the real world are linear systems *for small enough signals*.

Systems

The Unit Delay System

We introduce a special linear system T_1 whose output signal is the input signal, but *delayed* by one time unit.

$$y = T_1\{x\}$$

or with discrete time explicitly represented

$$y[n] = x[n-1]$$

This **unit delay** system is the building block of the systems we will cover in this course.

Systems

The Unit Delay System

We introduce a special linear system T_1 whose output signal is the input signal, but *delayed* by one time unit.

$$y = T_1\{x\}$$

or with discrete time explicitly represented

$$y[n] = x[n-1]$$

This **unit delay** system is the building block of the systems we will cover in this course.

Octave:

- `shift` function (almost) is a direct implementation of a delay;
- we will instead be using a more general system implementation.

Time-invariant systems have the property that the output signal of the system for a given input signal does not depend explicitly on absolute time.

For any input signal x with $y = H\{x\}$, the system H is time-invariant if

$$H\{T_\delta\{x\}\} = T_\delta\{y\}$$

Where T_δ is a delay system for arbitrary delay.

Time-invariant systems have the property that the output signal of the system for a given input signal does not depend explicitly on absolute time.

For any input signal x with $y = H\{x\}$, the system H is time-invariant if

$$H\{T_\delta\{x\}\} = T_\delta\{y\}$$

Where T_δ is a delay system for arbitrary delay.
Many systems of interest in the real world are time-invariant systems.

Linear Time-Invariant or **LTI** Systems have both the linear property and the time-invariant property.

Systems

The Unit Impulse

The **unit impulse** is a signal such that

$$d[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

The unit impulse is a fundamental signal building block:

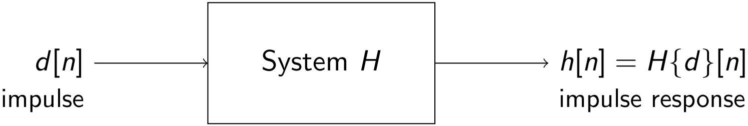
- *any* signal is the weighted sum of delayed unit impulses
- $x = \sum_{-\infty}^{\infty} x[n] T_n\{d\}$

Systems
Impulse Response

Since we can represent any signal as a sum of impulses

$$x = \sum_{-\infty}^{\infty} x[n] T_n\{d\}$$

if we know the response of an **LTI system** to the unit impulse, we know its response to any signal whatsoever.



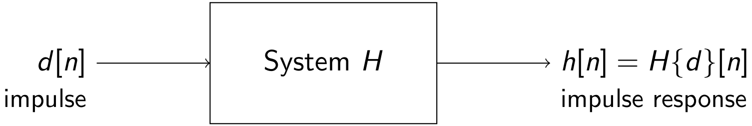
Systems

Impulse Response

Since we can represent any signal as a sum of impulses

$$x = \sum_{-\infty}^{\infty} x[n] T_n\{d\}$$

if we know the response of an **LTI system** to the unit impulse, we know its response to any signal whatsoever.



Octave: the `conv` operator

The convolution operation is notated

$$y(t) = (h * x)(t)$$

In discrete time we define the operation as

$$y[n] = (h * x)[n] = \sum_{k=-\infty}^{\infty} h[k] \times x[n-k]$$

Properties:

- commutative: $(a * b) = (b * a)$;
- associative: $(a * (b * c)) = ((a * b) * c)$;
- distributive over addition: $(a * (b + c)) = (a * b + a * c)$

Octave: `conv` function

- care required in interpreting output (time origin);
- `length(conv(a,b)) = length(a) + length(b) - 1`

Why convolution?

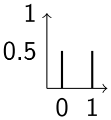
- implementation of LTI systems!

The output y of a system H for an input signal x is the convolution of the input and the impulse response of the system.

Why convolution?

- implementation of LTI systems!

The output y of a system H for an input signal x is the convolution of the input and the impulse response of the system.



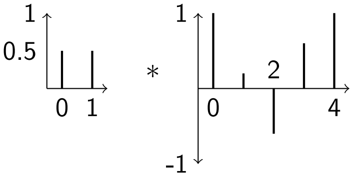
Systems

Convolution

Why convolution?

- implementation of LTI systems!

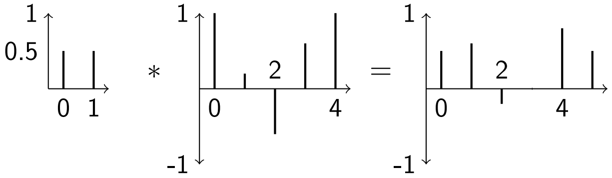
The output y of a system H for an input signal x is the convolution of the input and the impulse response of the system.



Why convolution?

- implementation of LTI systems!

The output y of a system H for an input signal x is the convolution of the input and the impulse response of the system.



Systems

The Unit Delay System, Again

Let

$$h[k] = \begin{cases} 1 & k = 1 \\ 0 & \text{otherwise} \end{cases}$$

The convolution of h with an input signal x:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

is equal to $x[n-1]$. Therefore $h[\tau]$ represents the unit delay system.

Systems

The Unit Delay System, Again

Let

$$h[k] = \begin{cases} 1 & k = 1 \\ 0 & \text{otherwise} \end{cases}$$

The convolution of h with an input signal x:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

is equal to $x[n-1]$. Therefore $h[\tau]$ represents the unit delay system.

Octave:

- `h = [0 1]`
- `[0 1]` is also the representation of the impulse response of the unit delay system.
- NB: vector index 1 corresponds to discrete time 0.

Systems

Scaled Delay Systems

General transformation built from scaling and time delay:

- delay by k time units;
- scale by a factor s.

System notated as $sT_k\{\}$.

Systems

Scaled Delay Systems

General transformation built from scaling and time delay:

- delay by k time units;
- scale by a factor s.

System notated as $sT_k\{\}$.

Octave:

- `[zeros(1,k) s]`
- exercise: check that convolving this kernel with a signal outputs the appropriate scaled delayed signal.

Systems

Convolution Efficiency

Consider a linear, time-invariant system H with Finite Impulse Response

- Impulse Response of length L (in samples);
- All other values 0

Implementation of that system with direct convolution for an input signal X of length N:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{L-1} h[k]x[n-k]$$

Computational Complexity:

- $O(NL)$
- … but we can do better …

Systems

Convolution Efficiency

Consider a linear, time-invariant system H with Finite Impulse Response

- Impulse Response of length L (in samples);
- All other values 0

Implementation of that system with direct convolution for an input signal X of length N:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{L-1} h[k]x[n-k]$$

Computational Complexity:

- $O(NL)$
- ... but we can do better ...
- ... with some more technical machinery.

Numbers

The Natural Numbers

The *Natural Numbers* are the (strictly) positive integers.

$$\underset{1}{\bullet} \quad \underset{2}{\bullet} \quad \underset{3}{\bullet} \quad \underset{4}{\bullet} \quad \underset{5}{\bullet} \quad \underset{6}{\bullet} \quad \underset{7}{\bullet} \quad \underset{8}{\bullet} \quad \cdots$$

- First recorded use (of tallies) c. 35000 BC;
- Place-value systems:
  - Babylonians (base 60, c. 3400 BC);
  - Egyptians (base 10, c. 3100 BC).
- The set of natural numbers is sometimes denoted $\mathbb{N}$

Numbers

The Number Zero

Zero as a number in an of itself:

- What number, added to 4, gives 4?
  - Solve $4 + x = 4$ for x.
- What do I have left if I subtract 7 from 7?
  - $7 - 7 = ?$

- NB: not the same as the 'no tens' in a place-value system;
- Historical:
  - India, China in 4th Century BC;
  - Greece: Ptolemy 130AD
  - Persia: Abū Àbdallāh Muḥammad ibn Mūsā al-Khwārizmī (c.780–c.850)
- The natural numbers and zero denoted by $\mathbb{N}_0$.

Numbers

Negative Numbers

- Solve $4x + 20 = 0$ for x.

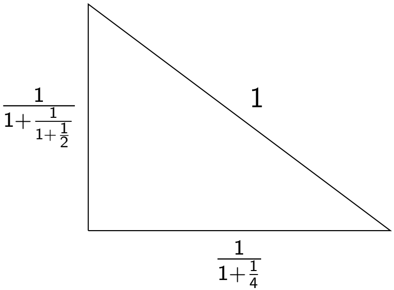
$$\cdots \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \cdots$$

- Historical:
  - China, 100BC;
  - Greece, 3rd C. AD (Diophantus, c.210–c.290);
  - Europe, 12th C. AD (Fibonacci, c.1170–c.1250)
- Uses:
  - Initially, as a calculation aid;
  - Given independent meaning in accounts as debts or losses.
- the set of integers is denoted $\mathbb{Z}$.

Numbers

Rationals

Rational numbers are those which can be expressed as fractions of two integers.

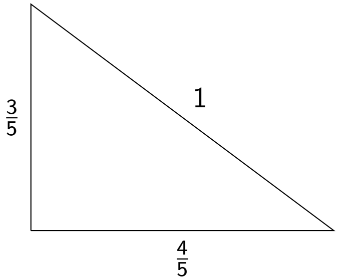


- Used since antiquity; recognized as numbers in Greek times;
- Connection to musical tuning (Pythagoras);
- The set of rationals is denoted $\mathbb{Q}$.

Numbers

Rationals

Rational numbers are those which can be expressed as fractions of two integers.



- Used since antiquity; recognized as numbers in Greek times;
- Connection to musical tuning (Pythagoras);
- The set of rationals is denoted $\mathbb{Q}$.

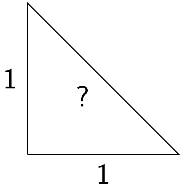
Numbers

Irrationals

System of numbers $\mathbb{Q}$ is *closed*:

- for linear equations;
- with the exception of division by zero.

Trouble in numerical paradise (I):



- Proof attributed to the Pythagorean school;
- The set of real numbers is denoted $\mathbb{R}$.

Numbers

Quadratic equations

Examples of quadratic equations:

- equilateral right-angled triangle: solve $x^2 - 2 = 0$ for x;
- general quadratic: solve $ax^2 + bx + c = 0$ for x.

Solution:

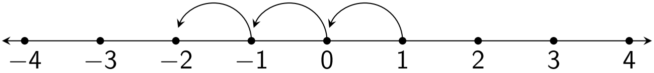
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- What if $b^2 - 4ac$ (the *discriminant*) is negative?
  - no solutions?
- History:
  - First known solution: Babylonian, c.2000 BC;
  - Geometrical interpretation: India, 8th C. BC; Euclid (Greece), 3rd C. BC;
  - First recorded general solution: Abraham bar Ḥiyya ha-Nasi (Catalonia, 12th C. AD)

Numbers

The Number Line

Just like at primary school: $1 - 3 = ?$



General Cubic:

- Solve $ax^3 + bx^2 + cx + d = 0$ for x.

Trouble in numerical paradise (II):

- Omar Khayyàm (Persia, 1048–1131)
  - Equation can have one or three real-number solutions;
- Gerolamo Cardano (Italy, 1501–1576)
  - Publishes method for solving cubic equation;
  - Method sometimes involves taking the square root of a negative number;
- Rafael Bombelli (Italy, 1526–1572)
  - publishes rules for dealing with square roots of negative numbers;

Numbers

Imaginary Numbers

Imaginary numbers (contrast with real numbers):

- Basic building block: $i = \sqrt{-1}$
- General imaginary number: $a \times i$ for $a \in \mathbb{R}$

Rules (after Bombelli):

- $a \times bi = abi$
- $-a \times bi = -abi$
- $a \times -bi = -abi$
- $-a \times -bi = abi$
- $ai \times bi = -ab$
- $ai \times -bi = ab$
- $-ai \times bi = ab$
- $-ai \times -bi = -ab$

Or: i behaves just like another number, but $i \times i = -1$

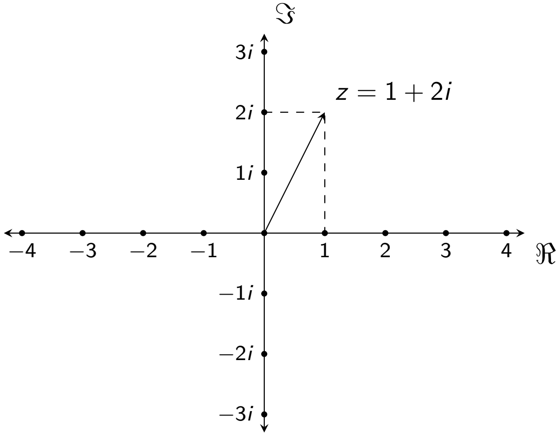
Numbers

Complex Numbers

Complex numbers:

- sum of a real and an imaginary part
- $z = a + ib$

Geometrical interpretation (Argand plane):



Arithmetic Rules:

- Add and subtract parts:
  - $(a + ib) + (c + id) = (a + c) + i(b + d)$
  - $(a + ib) - (c + id) = (a - c) + i(b - d)$
- Multiply out:
  - $(a+ib)\times(c+id) = ac+ibc+iad+iibd = (ac-bd)+i(bc+ad)$
- Divide using complex conjugate:
  - $(a + ib) \div (c + id)$
  - $= ((a + ib)(c - id)) \div ((c + id)(c - id))$
  - $= ((a + ib)(c - id)) \div (c^2 + d^2)$

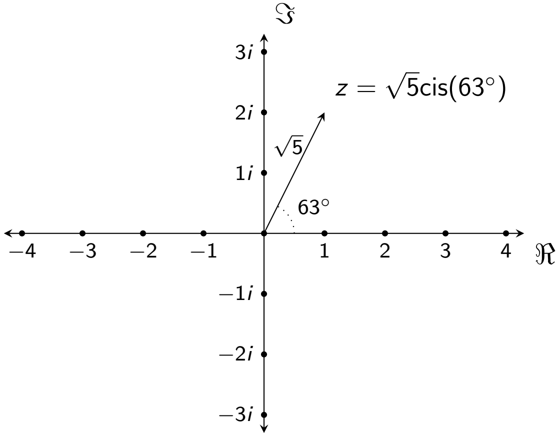
Numbers

Complex Numbers

Complex numbers:

- real length at an angle to the real axis
- $z = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

Geometrical interpretation (Argand plane):



Multiplication and Division revisited:

- $\mathrm{cis}(\theta_1) \times \mathrm{cis}(\theta_2) = \mathrm{cis}(\theta_1 + \theta_2)$
- $r_1\,\mathrm{cis}(\theta_1) \times r_2\,\mathrm{cis}(\theta_2) = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$
- $r_1\,\mathrm{cis}(\theta_1) \div r_2\,\mathrm{cis}(\theta_2) = \frac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$

Euler Identity:

$$e^{i\theta} = \mathrm{cis}(\theta) = \cos\theta + i\sin\theta$$

- ‘one of the most remarkable, almost astounding, formulas in all of mathematics’ (R.P. Feynman)
- central to *complex analysis*; used everywhere in physics, electrical engineering, signal processing...
- connection to signals’ amplitude and phase: Fourier analysis

Systems

Complex Numbers

- The Number Line;
- Pythagoras and the Rationals;
- The irrationality of $\sqrt{2}$;
- Solving quadratic equations;
- Cubic equations and Cardano;
- The Argand Plane;
- Arithmetic properties of Complex Numbers;
- Geometric interpretation of Complex Arithmetic;
- Phase and Logarithms;
- The Complex Exponential.