Emergence of Euclidian Geometry in a Computational Universe

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Abstract: In previous publications we resolved several challenging issues, which could make special relativity and quantum mechanics incompatible with a computational universe vision. These publications left untreated the emergence of Euclidian space, which concerns classic physics, but is also necessary for completing our computational model for special relativity. This paper determines the largest family of interaction laws enabling this emergence, and demonstrates this claim.

Keywords: Pan-computationalism, computational universe.

1 INTRODUCTION

The computational universe idea introduced by Konrad Zuse [1][2], and further developed by Jurgen Schmidhuber [3], considers thatÒ the universe is engendered by a computation. Another motivation in looking on physics from the computational view point was expressed by Richard Feynman in on of his famous articles [4]: "There are interesting philosophical questions about reasoning, and relationship, observation, and measurement and so on, which computers have stimulated us to think about anew, with new types of thinking. And all I was doing was hoping that the computer-type of thinking would give us some new ideas, if any are really needed."

Based on these motivations, we presented a vision [5-10], in which the universe is engendered by a computation-like process. However, the current vision of physics considers that the particles of the universe evolve within a space (to be referred as *veritable space*). But as computations manipulate state variables, they cannot engender a *veritable space* in which will evolve particles with computed states. Thus, a computational universe has to fully emerge from the evolution of a set of state variables, and its space geometry has also to emerge as a byproduct of this evolution. For this to work, two conditions are required. First, the engendered space should be perceived as a *veritable space* by any internal observer emerging in a computational universe. This issue is resolved in [5-10] by showing the existence of an ultimate limit of knowledge for any internal observer of a system, which prevents him from distinguishing a *veritable space* from a *space engendered by a computation-like process*. Second*, a computational process should be able to engender a space obeying the space geometry of our universe*. Thus, demonstrating this possibility, as in the present paper and in our past work [5-6], should be considered as a fundamental goal of pan-computationalism. Furthermore, the computation rules should not copy-paste the current interpretations of the theories of physics. Instead, they should exclude aspects in these interpretations that lead in non-computational rules. However, the computational rules *have to produce the same observable behavior as these interpretations*. For instance, computers use a time reference (e.g. implemented by a clock signal in digital computers). This creates a fundamental time, which is not compatible with the current interpretation of relativity, where there are as many time references as inertial frames (i.e. infinite number). This and several other computational deadlocks related to the special relativity and quantum mechanisms were highlighted in previous our publications [5-10]. Also, while these deadlocks seemed incompatible with computational models, our previous work [5 – 10] succeeded resolving them by giving a very central role to the observers that are part of a universe, that is, observers composed of the elementary entities (i.e. elementary particles) composing any structure of a universe.

Our past work addresses challenging issues related with special relativity and quantum mechanisms, but does not treat classical physics and the related Euclidian space. In particular, concerning special relativity, we have shown that, if the computational rules implementing the interaction laws satisfy a certain constraint (referred as RCA), the internal observers of the computational universe will experience a space-time complying Lorentz transformations, even if the computation process is synchronous and is based on a unique time. The RCA constraint determines how the intensity of the interaction (i.e. the force or the acceleration induced on a destination particle) is modified when the velocity variable of the source particle is non-null. These results are valid whatever is the analytical expression determining the intensity of the interaction when the source particle is at rest (*basic-form of the interaction law*).

However, special relativity describes a space-time without gravity, which has a flat (affine) structure. Therefore, in addition to the Lorentz transformations, spatial measurements should be compliant to the Euclidian geometry. However, as our previous publications [5-7] were focused on the more challenging goal of the emergence of Lorentz transformations in a computational universe, this issue was not treated. This problem is addressed in the subsequent sections, which provide a computational universe model enabling the emergence of classical (i.e. Euclidian) space, and completing our work on special relativity as: combining *RCA* with an *Euclidian basic-form of interaction laws* results in a space-time that obeys Lorentz transformations (due to RCA as shown in [6]), and has Euclidian flat structure (due to the *Euclidian basic-form of interaction laws)* as shown in this paper).

2 EMERGENCE OF EUCLIDIAN SPACE

In our model all structures of the computational universe should emerge from the values that take during its evolution the state variables of the elementary entities (particles) composing it. Space and its structure have to be engendered by the evolution of a particular state variable of the particles, which for convenience will be labeled to as P and referred as the position state-variable. To be able to engender space, P has to obey certain conditions:

Dimensionality: P should have the same number of dimensions as the space it engenders. Thus, to engender a 3D space, P should be a 3-dimensional variable described by three scalar variables labeled to as x, y, z. Thus, it will be represented as a 3-dimensional variable $P = (x, y, z)$ **(1)**

The derivative V of P will also be a state variable (velocity state-variable), and will be represented as $V = (v_x, v_y, v_z)$ **(2)**

where scalars v_x, v_y, v_z are the derivatives of x, y, z.

The derivative of V will be represented as $A = (a_x, a_y, a_z)$ **(3)**

where scalars a_x, a_y, a_z are the derivatives of v_x, v_y, v_z.

For each elementary entity, certain computational rules (to be referred as interaction laws) will determine the value of $A = (a_x, a_y, a_z)$. Then, the next value of V is determined by its present value and the present value of A: $V = V + A.dT$ **(4)**

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Finally, the next value of P is determined by its present value and the present value of V: $P = P + V.dT$ **(5)**

2.1 Euclidian Interaction Laws

Note that A is not a state variable: its next value does not depend on its past value. It is fully determined by *computation rules corresponding to the interaction laws*. These rules have a fundamental implication, as their form will determine the structure of space emerging in the computational universe: we find that *to engender Euclidian space, the interaction laws should have of the form*: $A = A(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$ [2] **(6)** where $r_x = x2\text{-}x1$, $r_y = y2\text{-}y1$, $r_z = z2\text{-}z1$, and $P1 = (x1, y1, z1)$, $P2 = (x2, y2, z2)$ are the position state-variables of the two interacting particles.

This form is obtained by considering that we are looking for laws that engender space which: is homogenous (thus, the value of A should not depend in the exact values of the position state-variables of the particles, but the relative ones (i.e. their difference); is isotropic (thus, the expression of A should be invariant to the change of the orientation of (r_x, r_y, r_z); and should engender objects obeying the Euclidian metric. These reasons will become clearer along the course of this paper.

Note that, each kind of interaction must be similar to all particles affected by this interaction. Thus, the accelerations A_{a1} and A_{a2} induced by any given kind of interaction *a* to any pair of interacting particles 1 and 2 will differ only by some constant factor. Hence, we will have $A_{a1} = c_{a1}(A_a(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$, and $A_{a2} = c_{a2}(A_a(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$.

Note also that, if we set $R = (r_x, r_y, r_z)$, $||R|| = (r_x^2 + r_y^2 + r_z^2)^{-1/2}$, (6) is written as $A = A(||R||^2)R$ **(6')**

Setting $R_0 = R/||R||$ and $A'(||R||) = A(||R||^2)||R||$ gives

$A = A'(||R||)R_0$ **(6")**

We remark that expression (6 – 6") is quite generic, as A(t) is an arbitrary function of t. Thus, it encompasses various forms of interaction laws. It includes the classical ones, where the intensity of the interaction is proportional to the inverse square of the distance $||R||$ separating the interacting particles, as well as functions like: $A'(||R||) = c||R||^{-n}$, able to represent the repulsive and attractive parts of the Lennard-Jones force that approximate the interaction between a pair of neutral atoms or molecules; or inverse exponentials $A'(||R||) = c.\exp(^-||R||)$, which can represent the R. A. Buckingham modification of the repulsive part of Lennard-Jones force, as well as the residual strong force; or more exotic laws where $A'(||R||)$ could be any arbitrary function of $||R||$.

2.2 Stable Objects and Euclidian Invariance

The existence of objects with stable dimensions (to be referred hereafter as rigid objects) is necessary if the computational universe has to resemble to ours. Also, performing length measurements requires using objects with constant length as length units. Let us consider an object of stable length. For simplicity we will use a simple object composed of two particles 1 and 2 that are subject to two interactions (interaction a and interaction b). Let $A_{a1} = c_{a1}A_a(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$, $A_{b1} = c_{b1}A_b(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$ be the accelerations induced by the interactions a and b on particle 1, and $A_{a2} = c_{a2}A_a(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$, $A_{b2} = c_{b2}A_b(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$ be the accelerations induced by the interactions a and b on particle 2. If at some instant the velocity state-variables of the two particles are equal and their position state-variables are such that their accelerations are null, then, the distance of the two particles will remain constant for the rest of the time (i.e. as long as they are not perturbed by interactions with other particles). Thus, the two particles will form a stable configuration (hereafter referred to as rigid object O). As in this configuration the acceleration induced to each particle by the two interactions is null, then, the value $||R||^2 = r_x^2 + r_y^2 + r_z^2$ will be determined by the solutions of the equations $A_{a1} + A_{b1} = (0, 0, 0)$ and $A_{a2} + A_{b2} = (0, 0, 0)$. From these solutions we have to exclude $(r_x, r_y, r_z) = 0, 0, 0)$, which corresponds to the case where the two particles occupy the same position. Then $A_{a1} + A_{b1} = (0, 0, 0)$ is equivalent to $c_{a1}A_a(r_x^2 + r_y^2 + r_z^2) + c_{b1}A_b(r_x^2 + r_y^2 + r_z^2) = 0$, and $c_{a2}A_a(r_x^2 + r_y^2 + r_z^2) + c_{b2}A_b(r_x^2 + r_y^2 + r_z^2) = 0$. These solutions must be equal, otherwise the acceleration induced on particle 1 and the acceleration induced on particle 2 will not become null at the same distance separating the two particles, and stable configurations could not be formed. Thus, the existence of particle configurations forming objects of constant dimensions implies the equality $c_{a1}/c_{b1} = c_{a2}/c_{b2}$. Thus, hereafter we will consider that the interactions satisfy the relation $c_{a1}/c_{b1} = c_{a2}/c_{b2}$. As a consequence, we need to solve only one of the above two equations, e.g. $c_{a1}A_a(r_x^2 + r_y^2 + r_z^2) + c_{b1}A_b(r_x^2 + r_y^2 + r_z^2) = 0$ **(7)**

Let us consider that the position state-variables of the two particles forming O are $P1' = (x1', y1', z1')$ and $P2' = (x2', y2', z2')$. Then, (7) gives $c_{a1}A_{a1}(r_x'^2 + r_y'^2 + r_z'^2) + c_{b1}A_{b1}(r_x'^2 + r_y'^2 + r_z'^2) = 0$ **(8)**

Equation (8) is identical to (7): it is obtained by substituting $r_x'^2 + r_y'^2 + r_z'^2$ to $r_x^2 + r_y^2 + r_z^2$. Thus, the solutions of (7) and (8) give identical values to $r_x^2 + r_y^2 + r_z^2$ and $r_x'^2 + r_y'^2 + r_z'^2$, resulting in the relation $r_x^2 + r_y^2 + r_z^2 = r_x'^2 + r_y'^2 + r_z'^2 = E_O$, where E_O is a constant value for object O. Thus, the expression $(r_x^2 + r_y^2 + r_z^2)$ is invariant regardless to the values of the position state-variables for which the two particles forming the object O reach equilibrium, and to the orientation of the line segment formed by the object O. Hereafter this property will be referred to as *the Euclidian invariance for rigid objects*, and E_O as *the Euclidian-invariance constant* of rigid object O.

Note also that equation (7) is of the form $F(r_x^2 + r_y^2 + r_z^2)$ and equation (8) is of the form $F(r_x'^2 + r_y'^2 + r_z'^2)$. They are in fact identical equations in which $r_x^2 + r_y^2 + r_z^2$ is replaced by $r_x'^2 + r_y'^2 + r_z'^2$. If the particles interact through more than two interactions obeying expression (6), then, again we will have two identical equations of the form $G(r_x^2 + r_y^2 + r_z^2)$ and $G(r_x'^2 + r_y'^2 + r_z'^2)$, in which $r_x^2 + r_y^2 + r_z^2$ is replaced by $r_x'^2 + r_y'^2 + r_z'^2$. Thus, we will find again $r_x^2 + r_y^2 + r_z^2 = r_x'^2 + r_y'^2 +$

[2] $A(r_x^2 + r_y^2 + r_z^2)$ represents any arbitrary function A(t) defined for all real values of t, in which t is replaced by $r_x^2 + r_y^2 + r_z^2$

$r_z'^2$. Hence, $r_x^2 + r_y^2 + r_z^2$ is invariant also for particles interacting through more than two interactions.

2.3 Emergence of Inertial Mass.

In section 2.2 we found that the existence of objects with stable dimensions imply the equality $c_{a1}/c_{a2} = c_{b1}/c_{b2}$. Then, thanks to this equality we can select four constants a1, a2, b1, b2 such that $a1a2/b1b2 = c_{a1}/c_{b1} = c_{a2}/c_{b2}$. Then, by setting $m1 = a1a2/c_{a1} = b1b2/c_{b1}$ and $m2 = a1a2/c_{a2} = b1b2/c_{b2}$, and $F_{a1} = A_{a1}m1$, $F_{a2} = A_{a2}m2$, $F_{b1} = A_{b1}m1$, $F_{b2} = A_{b2}m2$, we find:

$F_{a1} = F_{a2} = a1a2(A_a(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z) = F_a$.

$F_{b1} = F_{b2} = b1b2A_b(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z) = F_b$.

Therefore, for interaction *a* we have $F_{a1} = F_{a2} = F_a = A_{a1}m1 = A_{a2}m2$, which corresponds to the relations between: the force induced by an interaction to the interacting particles, the induced accelerations, and the inertial masses. Indeed, the forces induced by interaction *a* to the two particles are equal to each other ($F_{a1} = F_{a2} = F_a$), and the acceleration induced to each particle is equal to the force divided by the inertial masse ($A_{a1} = F_a/m1$, $A_{a2} = F_a/m2$). The same holds true for interaction *b*.

Thus, *we obtain the remarkable result that the existence of particle configurations forming objects of constant dimensions implies the emergence of the force and of the inertial masse*.

2.4 Emergence of the Euclidian Geometry Structure

We have to prove that if the interaction laws satisfy expression (6) the computational universe engenders a space complying the Euclidean geometry. To accomplish this task, we need to prove that these interactions imply the 5 postulates:

1. A straight line may be drawn from any point to any other.
2. A straight line may be extended to any finite length.
3. A circle may be described with any given point as its center and any distance as its radius.
4. All right angles are congruent[3].
5. At most one line can be drawn through any point not on a given line parallel to the given line in a plane.

Postulate 5 (known as the parallel postulate), can be replaced by the theorem of Pythagoras [11][12]. Thus, we just need to show the four first postulates plus the theorem of Pythagoras.

The challenge is to derive the above 5 basic principles of Euclidian geometry from a unique principle (the form of interaction laws). To simplify this task, we will first derive this geometry by using simple means, such as line segments created by two particles in equilibrium, and triangles formed by 3 particles in equilibrium. Then, we will use this geometry as a tool to simplify the generalization of our proof to all possible structures formed by the particles of the computational universe.

Note that, while in our proofs we employ similar skills as those used in analytical geometry, deriving Euclidian geometry from a single principle (the form of interaction laws) is totally new.

2.4.1 1st and 2nd postulates

Drawing Straight Lines: In modern physics the trajectories of particles that are not subjects to interactions are considered to be straight lines or their generalization for curved space (geodesics). Thus, we will use free particles to draw straight lines. Let us consider two position state-variables $Pa = (x_a, y_a, z_a)$ and $Pb = (x_b, y_b, z_b)$. Let Pa be the value of the position state-variables of a particle at a time instant T = 0, and let us suppose that this particle is not subject to interactions with other particles. At instant T1 let the components of the velocity state-variable of the particle to be equal to $vx = c(x_b - x_a)$, $vy = c(y_b - y_a)$, $vz = c(z_b - z_a)$. As the particle is not subject to interactions, these components will be constant. Then, the particle's trajectory will create a line $\mathcal{L}$ consisting in the set of values of position state-variables $P(T) = (x, y, z)$, with $x = x_a + c(x_b - x_a)T$, $y = y_a + c(y_b - y_a)T$, $z = z_a + c(z_b - z_a)T$, $T \geq 0$

For T=1/c we have $P(1/c)=(x_b, y_b, z_b)$. Thus, Pb also belongs to $\mathcal{L}$. Thus, the trajectory of this particle draws a line segment which passes from the two points of the computational universe corresponding to the position variables $Pa = (x_a, y_a, z_a)$ and $Pb = (x_b, y_b, z_b)$. For simplicity, we set $cT = \lambda$, resulting in the straight-line equation $x = x_a + \lambda(x_b - x_a)$, $y = y_a + \lambda(y_b - y_a)$, $z = z_a + (z_b - z_a)$, $\lambda \geq 0$. Setting P = (x, y, z), this is written as:

$P = Pa + \lambda(Pb-Pa)$ **(9)**

We expect that this line segment is straight as it is drawn by a particle free of interactions. However, as the fundamental property of straight-line segments is to have minimal length we should show this property. But the notions of length and distance are not yet determined in the computational universe context. To determine them in this context, the length of a line segment has to be measured by using as unit length an object O of the computational universe. To qualify as unit length, this object must have stable size (rigid object). Such objects were defined in section 2.2. Using a rigid object O as unit length, the following process can be used to measure the length of the segment of line $\mathcal{L}$ starting at Pa and finishing at Pb (segment Pa-Pb of $\mathcal{L}$):

- The one end of O is placed on the one end of the segment Pa-Pb of line $\mathcal{L}$ (point Pa) and its second end on another point of $\mathcal{L}$ (say point M1)[4].
- Then, the one end of O is placed on M1 and its second end on another point of $\mathcal{L}$ (say point M2).
- And so on...

If after k steps the second end of O coincides with Pb, then, the length of the segment Pa-Pb of $\mathcal{L}$ will be equal to k unit lengths. Note that, if the length of Pa-Pb is not a multiple of the length of O, the coincidence of the second end of O with Pb may never occur. However, this does not invalidate the measurement process, but only illustrates the need to employ smaller objects as subdivisions of the length unit, in order to use them when the measured length is not a multiple of the unit length. Thus, if Pb lays between the position of the second end of O at the k step and the position of the second end of O at the k+1 step, then, trivially, the k+1 step will be repeated by using an object that represents a subdivision of the length unit, until the measurement of the length of Pa-Pb of $\mathcal{L}$ is done with the required precision, or a new subdivision of the unit length will be used, ...

[3] As two angles are congruent if they have the same measure, for convenience we will use the term equal instead of congruent.

[4] In computational terms this means that the position variable of the one end of O is equal to (x_a, y_a, z_a), while the position variable of its second end is of the form (x, y, z) with $x = x_a + c(x_b - x_a)T$, $y = y_a + c(y_b - y_a)T$, $z = z_a + c(z_b - z_a)T$.

Now that we dispose a process enabling performing length measurements, we can use it to determine the metric of the engendered space.

Lemma 1: Let k be the result of the measurement of the length of a straight line segment whose ends are the points $Pa = (x_a, y_a, z_a)$ and $Pb = (x_b, y_b, z_b)$. Then, we will have $||P_b - P_a||^2 = (x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2 = k^2E_O$, where E_O is the *Euclidian Invariance constant* of the rigid object O used as unit length.

Proof[5]**:** Let $M_{r-1} = (x_{r-1}, y_{r-1}, z_{r-1})$ and $M_r = (x_r, y_r, z_r)$ be the positions on $\mathcal{L}$ of the two ends of O at the r_{th} step of the measurement process, and $M_{t-1} = (x_{t-1}, y_{t-1}, z_{t-1})$ and $M_t = (x_t, y_t, z_t)$ be the positions on $\mathcal{L}$ of the two ends of O at the t_{th} step of the measurement process. As M_{r-1} and M_r belong to $\mathcal{L}$, from (9) we $M_{r-1} = Pa + \lambda_{r-1}(Pb-Pa)$ and $M_r = Pa + \lambda_r(Pb-Pa)$. From these relations and the similar for M_{t-1} and M_t we find:

$M_r - M_{r-1} = (\lambda_r - \lambda_{r-1})(P_b-P_a)$, $M_t - M_{t-1} = (\lambda_t - \lambda_{t-1})(P_b-P_a)$ **(10)**

Relations (10) give: $||M_r - M_{r-1}||^2 = (\lambda_r - \lambda_{r-1})^2||Pb-Pa||^2$, and $||M_t - M_{t-1}||^2 = (\lambda_t - \lambda_{t-1})^2||Pb-Pa||^2$.

From section 2.2 the two ends of object O verify the *Euclidian Invariance* relationship. Then, as M_{r-1} and M_r are the two ends of rigid object O at the r_{th} step of the measurement process and M_{r-1} and M_r are the two ends of this object at the t_{th} step of the measurement, we have $||M_r - M_{r-1}||^2 = ||M_t - M_{t-1}||^2 = E_O$. This implies $(\lambda_r - \lambda_{r-1})^2||Pb-Pa||^2 = (\lambda_t - \lambda_{t-1})^2||Pb-Pa||^2 = E_O$. Thus, $\lambda_r - \lambda_{r-1} = \lambda_t - \lambda_{t-1}$ for any two steps of the measurement process. Then, for any steps r and t of the measurement process, (10) implies $M_r - M_{r-1} = M_t - M_{t-1} = R$. This relation implies that R is constant in all the k steps of the measurement process. Also, since $||M_r - M_{r-1}||^2 = ||M_t - M_{t-1}||^2 = E_O$, we have $||R||^2 = E_O$, Trivially we also have $P_b - P_a = (M_1 - P_a) + (M_2 - M_1) + \ldots + (M_{k-1} - M_{k-2}) + (P_b - M_{k-1})$. Thus, we have $P_b - P_a = kR$, which implies $||P_b - P_a||^2 = k^2E_O$ **(11)**

QED

Lemma 1 is expressed in computational terms (i.e. values that take the position variables), which are intrinsic to the process that engenders a computational universe. However, internal observers of a computational universe, as defined in [5 - 7], can access only values based on measurements. Thus, we need to express the outcome of lemma 1 in terms of measurement results. This is done by means of theorem 1.

Theorem 1: Let k be the result of the measurement of the length of a straight line segment whose ends are the points $Pa = (x_a, y_a, z_a)$ and $Pb = (x_b, y_b, z_b)$. Then, we will have $(k_{yb} - k_{xa})^2 + (k_{yb} - k_{ya})^2 + (k_{zb} - k_{za})^2 = k^2$, where k_{xa}, k_{ya}, k_{za}, k_{xb}, k_{yb}, k_{zb}, are the normalized coordinates x_a, y_a, z_a, x_b, y_b, z_b. That is, k_{xa} is the result of the measurements of the straight line segment staring at position (0, 0, 0) and ending at position $(x_a, 0, 0)$, k_{ya} is the result of the measurements of the straight line segment, staring at position (0, 0, 0) and ending at $(0, y_a, 0)$, and so on.

Proof: To replace the computational term x_a by a measured term, object O can be used to measure the straight line segment, staring at (0, 0, 0) and ending at $(x_a, 0, 0)$. Then, using lemma 1 we find $x_a^2 = k_{xa}^2E_O$. Similarly, we find $x_b^2 = k_{xb}^2E_O$, $y_a^2 = k_{ya}^2E_O$, $y_b^2 = k_{yb}^2E_O$, $z_a^2 = k_{za}^2E_O$, $z_b^2 = k_{zb}^2E_O$. Replacing these values in (11) we find: $(k_{yb} - k_{xa})^2 + (k_{yb} - k_{ya})^2 + (k_{zb} - k_{za})^2 = k^2$ **QED**

As mentioned earlier, computational terms are not accessible to the internal observers of the computational universe. Thus, as we are interested about the geometry of space perceived by these observers, in the rest of the paper we will use the normalized (i.e. measured) values of the positions, velocities, and accelerations. Thus, hereafter we will use the terms: position, and velocity instead of position state-variable, and velocity state-variable. To simplify their representation, hereafter we will represent them by means of the symbols used so far to represent the computational terms. For instance, instead of using the symbol k_{xa} to represent the normalized coordinate, we will use the symbol x_a. Similarly, we will use the symbols y_a and z_a will be used instead of the symbols k_{ya} and k_{za}, and so forth. Thus, using this representation, theorem 1 implies $[(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2]^{-1/2} = k$. As k is the result of the measurement of the length of the straight line segment starting at position $Pa = (x_a, y_a, z_a)$ and finishing at position $Pb = (x_b, y_b, z_b)$, this equation implies that the distance D(Pa, Pb) separating any two points $Pa = (x_a, y_a, z_a)$ and $Pb = (x_b, y_b, z_b)$, where x_a, y_a, z_a, x_b, y_b, z_b are normalized, is given by:

$D(Pa, Pb) = [(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2]^{-1/2}$ **(12)**

Also if in expression (6) we set $r_x = k_{rx}(E_O)^{-1/2}$, $r_y = k_{ry}(E_O)^{-1/2}$, $r_z = k_{rz}(E_O)^{-1/2}$, where k_{rx}, k_{ry}, k_{rz} are the measured lengths of r_x, r_y, r_z, we obtains $A = A(E_O(k_{rx}^2 + k_{ry}^2 + k_{rz}^2))E_O^{-1/2}(k_{rx}, k_{ry}, k_{rz})$. As $E_O^{-1/2}A(E_O(k_{rx}^2 + k_{ry}^2 + k_{rz}^2))$ is also a function of $k_{rx}^2 + k_{ry}^2 + k_{rz}^2$ the acceleration can be written as $A = F(k_{rx}^2 + k_{ry}^2 + k_{rz}^2)(k_{rx}, k_{ry}, k_{rz})$. This is of identical form as (6). Thus, hereafter we can use the expressions (6) and (6'), with r_x, r_y, r_z representing normalized values instead of computational ones.

Expression (12) is the Euclidian metric. Also, a well-established mathematical result is that equation (12) implies that[6]: line segments described by equations (9) satisfy the triangular inequality; and they also have the shortest length among all lines connecting two points. Thus, the lines we have drawn for implementing postulate 1 have the three attributes of straight lines: they correspond to trajectories of free particles;

[5] To avoid using cumbersome expressions containing three components for each position variable, our proofs will use vector representations. But we are cautious not to use surreptitiously any properties of vector spaces. Everything in our proofs is rigorously deduced from expression (6)-(6').

[6] The proofs of these properties are taught in elementary mathematics. Readers can also find them [15].

their length is given by the Euclidian metric; they have the shortest length among all line segments connecting two points.

The second postulate is realized trivially as a particle not subject to any interactions, and shown above to trace a straight line, it extends this line with the flow of time T to any finite length towards the direction of its velocity V = (vx, vy, vz). The extension towards the opposite direction is done by a particle not subject to interactions with velocity equal to (-vx, -vy, -vz).

2.4.2 The 4th postulate and the Theorem of Pythagoras

In this section we need to work with planes. Thus, we start with the definition of planes in the computational universe.

Defining Planes: As the other geometrical structures, planes too have to be created by the positions of particles belonging to the combinational universe (i.e. particles interacting according to expression (6)).

Let us consider a set Ps of interacting particles, which are not subjects to interactions with any particle external to Ps, and such that at an instant T0 the position Pi = (xi, yi, zi) of any particle i of Ps verifies the equation Pi = P0 + γP + γ'P' **(13)**
where P0 = (x0, y0, z0), P = (a, b, c), P'= (a', b', c'), in which x0, y0, z0, a, b, c, a', b', and c' are real constants, and γ and γ'are real variables. Let also the velocity of any particle in the set Ps to verify the equations V= γP + γ'P' **(14)**

Lemma 2: At any time instant T > T0, the position variable of each particle of set Ps verifies equation (13).

Proof: Let Pi and Pj be the positions at time T0 of any two particles i and j of set Ps. Since Pi and Pj verify the equations (13) they can be written as Pi = P0 + γiP + γi'P' and Pj = P0 + γjP + γj'P'. Then, from (6') the accelerations of particle i induced by its interaction with particle j is of the form $A = A(\|R_{ij}\|^2)R_{ij}$, with $R_{ij} = (\gamma j - \gamma i)P + (\gamma j' - \gamma i)P'$. Since $A(\|R_{ij}\|^2)$ is a scalar, by setting $A(\|R_{ij}\|^2)(\gamma j - \gamma i) = d_{ij}$ and $A(\|R_{ij}\|^2)(\gamma j' - \gamma i') = d_{ij}'$, we find $A = d_{ij}P + dij'P'$, where d_{ij} and d_{ij}' are scalars. Then, the total acceleration induced on particle i by its interactions with all the particles of Ps is $A = s_iP + s_i'P'$ **(15)**
where s_i and s_i' are the sums of the terms dij and dij' over all particles j of Ps, with j ≠ i. Thus, *(13) implies (15)*.

Step 1: Equations (13), (14) and (15) are valid at time T0 for each particle of set Ps.

Step 2: Let us suppose that these equations are also valid at time T for each particle of Ps. Due to the validity of (14) at time T, for each particle i of set Ps, equation (13) will also be valid at time T + dT for each particle i of Ps; due to the validity of (15) at time T for each particle i of Ps equations (14) will also be valid at time T + dT for each particle i of Ps; due to the validity of (13) at time T for each particle i of Ps, equation (15) will also be valid at time T +dT for each particle i of Ps.

Based on continuous mathematical induction[7], steps 1 and 2 imply that (13), (14) and (15) are valid at any time T > T0 for each particle i of Ps.

QED

Let $\mathcal{P}$ be the set of all possible positions verifying (13). As equation (13) has two real parameters (γ and γ') the set $\mathcal{P}$ is a two dimensional surface. In 3D Euclidian space such a surface is a plane if the following theorem holds true.

Theorem 2: If a straight-line $\mathcal{L}$ passes from two points of $\mathcal{P}$ all positions of $\mathcal{L}$ will also belong to $\mathcal{P}$.

Proof: Let us consider a straight line, which passes from two positions P1 and P2 belonging to $\mathcal{P}$. We found in section 2.4.1 (equations 9) that the positions $P_{\mathcal{L}} = (x_{\mathcal{L}}, y_{\mathcal{L}}, z_{\mathcal{L}})$ of a straight line $\mathcal{L}$ passing from two positions P1 = (x1, y1, z1) and P2 = (x2, y2, z2), are given by the equation $P_{\mathcal{L}}$ = P1 + λ(P2-P1)

Since P1 and P2 belong to the set $\mathcal{P}$, then, they will verify the equations (13). Thus we will have P1 = P0 + γ1P + γ1'P', and P2 = P0 + γ2P + γ2'P'. Replacing these values in the equations of the straight line $\mathcal{L}$ we find $P_{\mathcal{L}}$ = P0 + (γ2λ + γ1 - γ1λ)P + (γ2'λ + γ1' - γ1'λ)P', which gives positions satisfying equations (13) for any value of λ. **QED**

Lemma 2 and theorem 2 imply that: the potential positions that can take a closed set of particles with initial conditions verifying (13) and (14), are described by equation (13) and define a plane. Furthermore, a *plane that contains 3 positions* P1 = (x1, y1, z1), P2 = (x2, y2, z2), P3 = (x3, y3, z3) is descried by the equation:
P = P1 + (P2-P1)t + (P3-P1)t' **(16)**

Indeed: This equation has the form of (13). Thus, it describes a plane. This plane contains P1, P2, and P3 since t = 0 and t' = 0 give P1; t= 1, t' = 0 gives P2; t=0, t' = 1 gives P3.

In the following we have to employ triangles and angles of stable size. To reflect the space structure of the computational universe, these objects too have to be created by particles belonging to this universe (i.e. their interactions satisfy (6)). To simplify our task, we use the simplest possible objects for creating triangles and angles (objects composed of 3 particles).

Creating rigid triangles: Let us consider 3 particles 1, 2, and 3, such that each of these particles interact with the two other particles and is not subject to interactions with any other particle. Let P1, P2, and P3 be the positions of particles 1, 2, and 3. From section 2.2, the accelerations induced to particles 1 and 2 by their mutual interactions will become null at a distance D(P1, P2) = $E_{O12}^{-1/2}$ determined by the *Euclidian invariance constant* E_{O12} of the rigid object formed particles 1 and 2. Similarly, the accelerations induced to particles 1 and 3 by their mutual interaction will become null at a distance D(P1, P3) = $E_{O13}^{-1/2}$, and the accelerations induced to particles 2 and 3 by their mutual interaction will become null at a distance D(P2, P3) = $E_{O23}^{-1/2}$. Thus, the three particles will reach equilibrium and will form a rigid triangle whose sides will have constant length whatever is the position and orientation of this triangle, if there exist position triplets P1, P2, P3 whose distances satisfy the relations D(P1, P2) = $E_{O12}^{-1/2}$, D(P1, P3) = $E_{O13}^{-1/2}$, D(P2, P3) = $E_{O23}^{-1/2}$. Thus, in order for this to happen, these distances must verify the triangular inequalities (shown in section 2.4.1 to be valid in the computational universe). Then, as the values of $E_{O12}^{-1/2}$, $E_{O13}^{-1/2}$, and $E_{O23}^{-1/2}$ are determined by the intensities of the interactions of the three particles, these intensities should be such that $E_{O12}^{-1/2} + E_{O13}^{-1/2} > E_{O23}^{-1/2}$; $E_{O12}^{-1/2} + E_{O23}^{-1/2} > E_{O13}^{-1/2}$, $E_{O13}^{-1/2} + E_{O23}^{-1/2} > E_{O12}^{-1/2}$ (which also imply

[7] Note that, continuous mathematical induction is addressed by several authors [13][14]. However, published approaches are not convenient for the problem we treat here, as step 2 in these approaches requires that there is Δ > 0 such that the validity of the property under investigation for t = T implies its validity for all values t ∈ (x−Δ, x+Δ). Thus, in [15] we propose and prove a different principle of continuous mathematical induction, which is convenient for our needs.

$E_{O12}^{-1/2} \neq 0, E_{O13}^{-1/2} \neq 0, E_{O23}^{-1/2} \neq 0$).

Let as denote by A, B, and C the vertices of the triangle corresponding respectively to the position of the particles 1, 2, and 3. Then we will denote this triangle as $\mathcal{T}$ABC. We will also denote as: ^BAC the angle at vertex A, ^ABC the angle at vertex B, and ^ACB the angle at vertex C.

Our goal in introducing rigid triangles is to use them for creating right angles. We will do it by using triangles verifying the relation $D(P1, P2)^2 + D(P1, P3)^2 = D(P1, P3)^2$. We also need to be able to compare any two angles ^BAC and ^B'A'C'. This will be realized in the following manner.

Arrangement for comparing two angles ^BAC and ^B'A'C':

- The positions of vertices A and A' coincide.
- Segment A'B' belongs to the same straight line $\mathcal{L}$ as AB, with B and B' being at the same side of the common position occupied by A and A'.
- C' belongs to the same plane as line $\mathcal{L}$ and position C, with C and C' being at the same side of $\mathcal{L}$.

Then, if we find that C' belongs to the straight line passing from A and C, we will say that the two angles are equal.

In geometry if two angles are compared once and are found equal, they will be equal any time they are compared, regardless to their space location and orientation. This fact is necessary in order to proceed further in our developments, but it has first to be proven in the context of the computational universe.

Lemma 3: Let us consider two rigid triangles $\mathcal{T}$ABC and $\mathcal{T}$A'B'C'. If the angles ^BAC and ^B'A'C' are found to be equal at some instance of comparison, then, they will be found to be equal at any other instance of comparison.

Principle proof: The detailed proof of this lemma is given in [15]. Here we just give some general principles supporting its validity: lemma 3 holds since the properties of the objects of the computational universe are determined by interaction laws described by expression (6). As this expression is homogenous with respect to the positions of the interacting particles and the time variable, and isomorphic with respect to the orientation of the line segment connecting these positions, then, the properties of objects will not depend on their spatial and temporal location and their spatial orientation. Thus, a property, like the equality of angles of two triangles, will hold whatever is the time of the comparison and the location and orientation of the triangles during the comparison. **QED**

The next step is to show the validity of the 4th postulate: all right angles are congruent. By definition, in Euclidian geometry right angles have the property to divide the total angle formed by two halves of a straight line $\mathcal{L}$ into two equal angles. The following arrangements will be useful in order to create right angles, and/or check if an angle of a triangle is right.

Let us consider a plane $\mathcal{P}$, a straight line $\mathcal{L}$ belonging to $\mathcal{P}$, and a point P1 = (x1, y1, z1) belonging to $\mathcal{L}$.

Straight-line Angle Splitting

Step 1: the vertex A of a rigid triangle $\mathcal{T}$ABC is at the position P1 = (x1, y1, z1) of the straight line $\mathcal{L}$.

Step 2: the vertex B of $\mathcal{T}$ABC is at a position P2 = (x2, y2, z2) of the straight line $\mathcal{L}$.

Step 3: the vertex C of $\mathcal{T}$ABC is at a position P3 = (x3, y3, z3) of the plane $\mathcal{P}$.

Complimentary Straight-line Angle Splitting

Step 1': the vertex A of $\mathcal{T}$ABC is at the position P1 = (x1, y1, z1) of the straight line $\mathcal{L}$.

Step 2': the vertex B of $\mathcal{T}$ABC is at a position P2' = (x2', y2', z2') belonging to the straight line $\mathcal{L}$ and being on the opposite side of P2 with respect to P1.

Step 3': the vertex C of $\mathcal{T}$ABC is at a position P3' = (x3', y3', z3') of the plane $\mathcal{P}$, and at the same side of $\mathcal{L}$ as P3.

As we use the same triangle in both arrangements and, from lemma 3, changing the location and orientation of triangles preserves the equality of angles, we will have ^P3'P1P2' = ^BAC = ^P3P1P2. Thus, if P3' coincides with P3, then, segment P1P3 will split the total angle formed by the two halves of $\mathcal{L}$ originated at P1, in two equal angles. Hence, these angles as well as the angle ^BAC of $\mathcal{T}$ABC will be right angles. To obtain the coincidence of P3' and P3, we will use a triangle formed by particles whose intensities of interactions results in sides AB, BC, and AC having lengths satisfying the relationship:

$$D(A, B)^2 + D(B, C)^2 = D(A, C)^2 \quad \textbf{(17)}$$

Using the above arrangements we obtain the following results.

Theorem 3: If triangle $\mathcal{T}$ABC satisfies (17), then, its angle ^BAC divides the total angle formed by the two halves of $\mathcal{L}$ in two equal angles.

To prove this theorem, we show that: if ^BAC is used to implement the *Straight-line Angle Splitting* and the *Complimentary Straight-line Angle Splitting,* it results in P3' = P3. The detailed proof is given in [15].

Theorem 3 shows that the angle ^BAC of triangles verifying (17), split the total angle formed by the two halves of $\mathcal{L}$ in two equal angles. The next step, addressed in theorem 4, is to show the inverse implication.

Theorem 4: If the angle ^BAC of a triangle $\mathcal{T}$ABC splits in two equal angles the total angle formed by the two halves of a straight line, then $\mathcal{T}$ABC satisfies (17).

To prove this theorem, we show that if ^BAC is used to implement the *Straight-line Angle Splitting* and the *Complimentary Straight-line Angle Splitting* and the outcome is P3' = P3, then $\mathcal{T}$ABC will satisfy (17). The detailed proof is given in [15].

Theorem 5 addresses the final step for proving postulate 4.

Theorem 5: If the sides of two triangles $\mathcal{T}$ABC and $\mathcal{T}$A'B'C' satisfy the (17) then ^BAC = ^B'A'C'.

The proof is given in [15].

Corollary 1: All right angles are equal.

Proof:

a. Right angles are defined to be equal to the half of the total angle formed by two halves of a straight line.
b. Theorems 3 and 4 imply that the angle ^BAC of a triangle $\mathcal{T}$ABC splits in equal parts the total angle formed by two halves of a straight line if and only if $\mathcal{T}$ABC satisfies (17).
c. From theorem 5 and point b. all angles in a. are equal. **QED**

Corollary 2: If the angle of a triangle is right then the triangle verifies the theorem of Pythagoras, and vice-versa.

Proof: Theorem 4 implies the one direction of this corollary and theorem 3 implies the other direction. **QED**

Thus, Corollary 1 implies the validity of postulate 4 and corollary 2 implies the validity of the theorem of Pythagoras, which can replace postulate 5 [11][12].

2.4.3 The 3rd postulate

Drawing circles: In the computational universe, to draw a circle on a plane $\mathcal{P}$ with a point P1 = (x1, y1, z1) as its center and a distance r as its radius, we will employ a system described in theorem 6.

Theorem 6: If two particles 1 and 2 satisfy the conditions:

1. At a time T0 their positions P1 = (x1, y1, z1) and P2 = (x2, y2, z2) are on the plane $\mathcal{P}$ and satisfy the relation D(P1, P2) = r.
2. The velocity of particle 1 is V1 = (0, 0, 0).
3. The velocity $V_2 = (v_x, v_y, v_z)$ of particle 2 satisfies the relations: position P3 = (P1 + R + V2) belongs to $\mathcal{P}$; $R \cdot V2 = r_x v_x + r_y v_y + r_z v_z = 0$ (with $r_x = x2 - x1$, $r_y = y2 - y1$, $r_z = z2 - z1$, $R = (r_x, r_y, r_z)$); and $\|V2\|^2 = \|R\|\|A\|$, where $\|R\| = (r_x^2 + r_y^2 + r_z^2)^{1/2}$, and $\|A\| = (a_x^2 + a_y^2 + a_z^2)^{1/2}$ is the norm of the acceleration of particle 2 due to its interaction with particle 1.
4. Particle 1 is attached, through strong interactions, to an object of very large mass[8], resulting in insignificant acceleration for particle 1.

Then, the trajectory of particle 2 is a circle having P1 as its center and r as its radius.

While the conditions satisfied in this theorem are similar in certain aspects to those satisfied in uniform circular motion of Newtonian mechanics, the proof of this theorem does not exist in the literature, as the acceleration is not described by an analytical function but by the generic expression (6) representing a wide family of interaction laws (Newtonian interactions are just a specific case). Thus, theorem 6 needs to be proven. Its proof is given in [15]. In this proof we notice that the trajectory of particle 2 will be a circle with radius $r = (r_x^2 + r_y^2 + r_z^2)^{1/2}$ because, when the acceleration is described by expression (6):

- its direction is radial, and
- its norm $\|A\| = A(r^2)(r_x^2 + r_y^2 + r_z^2)^{1/2}$, is constant along a circle of radius $r = (r_x^2 + r_y^2 + r_z^2)^{1/2}$.

These properties are not satisfied in a computational universe in which the acceleration is given by an expressions different than (6), since in this case the direction of the acceleration is not radial, and/or its norm will change along any circle of constant radius $r = (r_x^2 + r_y^2 + r_z^2)^{1/2}$. So, the trajectory of particle 2 could not be a circle of radius $r = (r_x^2 + r_y^2 + r_z^2)^{1/2}$.

Note that we ca also use other means for creating circles. For instance: we can use a rigid object O composed of two particles in equilibrium whose length is equal to r; then, place its one end on the center of the circle and move the second end around the first one. The result is a circle of radius $r = (r_x^2 + r_y^2 + r_z^2)^{1/2}$. This outcome is due to the interaction laws satisfying expression (6), which imply that the measured length of the rigid object is $r = (r_x^2 + r_y^2 + r_z^2)^{1/2}$ and is constant in all directions.

2.4.3 Generalization to any objects and processes

Previously, the positions of particles were determined by the 3 components of the position state-variables, used by the rules computing the evolution of the particles. This determines by default a coordinate system X, Y, Z, in which X is the straight line consisting in the set of positions (x, 0, 0), where x is a real number variable, Y is the straight consisting in the set of positions (0, y, 0) where y is a real number variable, and Z is the straight line consisting in the positions (0, 0, z). X, Y, Z are also mutually orthogonal as we find trivially that the dot product of any vectors laying on any two of these axes is 0 (this property of dot product is shown in [15]). Then, all positions, distances, velocity vectors and accelerations were represented by their components in the Cartesian system X, Y, Z.

The 5 postulates, proven in the previous sections, determine a 3D Euclidian framework. Thanks to this framework we can define Cartesian systems having their origin at any point P = (x, y, z), and having as axes A, B, C any three mutually orthogonal straight lines. Thus, positions, distances, velocity vectors and acceleration vectors can be represented by their components in any system of coordinates (i.e. the projections of the vector on the axes A, B, and C of any Cartesian system). As usually, the transformation of the representation [U] of a vector from the system X, Y, Z to its representation [U'] in a system A, B, C, can be done by the expression [U'] = [Q][U], where [Q] is a 3x3 matrix having as element Qij the dot product $\mathbf{e}_j'.\mathbf{e}_i$, where $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are the unit vectors codirectional to the axes X, Y, Z, and $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$ are the unit vectors codirectional to the axes A, B, C.

Thus, using the relation [U'] = [Q][U], we can obtain in the coordinates system A, B, C the representation $R = (r_a, r_b, r_c)$ of the vector connecting the positions of two particles from its representation $R = (r_x, r_y, r_z)$ in the system X, Y, Z, as well as the representation $V = (v_a, v_b, v_b)$ of the velocity vector of a particle in the system A, B, C from its representation $V = (v_x, v_y, v_z)$ in the system X, Y, Z. Similarly, we obtain the representation of the acceleration $A = (a_a, a_b, a_c)$ in the system A, B, C from its representation $A = (a_x, a_y, a_z)$ in the system X, Y, Z.

The first important question is whether the expression of the acceleration is invariant with respect to the transformation of the system of coordinates. The answer depends on the form of the expression describing the acceleration. Lemma 5 treats this question for interaction laws described by expression (6).

Lemma 5: The expression of the interaction laws described by (6) is invariant to the transformation of the system of Cartesian coordinates.

Proof: From (6) the acceleration is given by $A = c(A(r_x^2 + r_y^2 + r_z^2)(r_x, r_y, r_z)$ **(18)**

The transformation of expression (18) from the Cartesian system X, Y, Z to the Cartesian system A, B, C, is obtained by replacing (r_x, r_y, r_z) by (r_a, r_b, r_c), where r_a, r_b, r_c are the components expressed in A, B, C of the vector separating the positions of the interacting particles. Thus, in A, B, C the acceleration is given by

$A=cA(r_x^2+r_y^2+ r_z^2)(r_a, r_b, r_c)$ **(19)**

Note that $cA(r_x^2 + r_y^2 + r_z^2)$ is not modified by the coordinates transformation, because it is a scalar. However, as the transformation of Cartesian coordinates does not alter the Euclidian distance, we have $(r_x^2 + r_y^2 + r_z^2) = (r_a^2 + r_b^2 + r_c^2)$. Replacing this in 19 gives $A=c_1A(r_a^2+r_b^2+r_c^2)(r_a, r_b, r_c)$ **(20)**

The expressions (18) and (20) of the accelerations in the systems X, Y, Z and A, B, C are identical. **QED**

Theorem 7: The Euclidian distance of any two particles belonging to a rigid object, or to two identical rigid objects, and expressed in the same system of Cartesian coordinates, is the same regardless to its(their) spatial position and orientation(s).

[8] The notion of mass in the computational universe is introduced in section 2.3.

Proof: Let us consider a configuration Cf of an arbitrary set of particles, which are in equilibrium (their accelerations are null) forming a rigid object. Let us also consider another particle configuration Cf', such that:

1. For each particle i of Cf there is an identical particle i' in Cf';
2. The components of the vector connecting the positions of each pair of particles i' and j' of Cf' expressed in a Cartesian coordinates system A, B, C, are equal to the components of the vectors connecting the corresponding pair of particles i, j of Cf expressed in the Cartesian system X, Y, Z.

Let the acceleration induced on a particle i of Cf by its interaction with another particle j of Cf to be equal to $A_{ji} = c_{ji}A_{ji}(r_x{}^2 + r_y{}^2 + r_z{}^2)(r_x, r_y, r_z)$. Considering the corresponding particles i', j' of Cf', from lemma 5 we will $A_{j'i'} = c_{ji}A_{jj}(r_a{}^2 + r_b{}^2 + r_c{}^2)(r_a, r_b, r_c)$. From condition 2 $(r_a, r_b, r_c) = (r_x, r_y, r_z)$. Thus, $A_{ji} = c_{ji}A_{ji}(r_x{}^2 + r_y{}^2 + r_z{}^2)(r_x, r_y, r_z) = c_{ji}A_{jj}(r_a{}^2 + r_b{}^2 + r_c{}^2)(r_a, r_b, r_c) = A_{j'i'}$. As a consequence, the accelerations of the particles in configuration Cf' are equal to the accelerations of the particles in configuration Cf. As the particles in Cf are in equilibrium forming a rigid object O, the particles in Cf' will be in equilibrium too, and will form a rigid object O'. From condition 2, the distances of the particles of O' expressed in A, B, C, are equal to the distances of the particles of O expressed in X, Y, Z. As transforming the Cartesian coordinates does not modify the Euclidian distance. These distances will also be equal when they are expressed in the same Cartesian system.

Thus the following facts hold true: the particles forming O' are identical to the particles forming O; the Euclidian distance of any pair of particles of O' expressed in any system of Cartesian coordinates, is equal to the Euclidian distance of the corresponding pair of particles in O expressed in the same system of coordinates; all particles in O and all particles in O' are in equilibrium (null accelerations). Thus, O and O' are identical rigid objects placed at different spatial positions and having different orientations. In particular, their orientations are such that object O' is obtained from object O by a rotation identical to the one giving the axes A, B, C from the axes X, Y, Z. As in this proof there are no constraints concerning the spatial location of these objects in the two coordinate systems nor the orientations of the axes A, B, C, this result is valid for any spatial positions and orientations of the rigid objects O' and O. **QED**

Corollary 3: The 5 postulates of the Euclidian geometry are valid whatever is the choice of the rigid objects used as length units and as triangles.

Proof: From theorem 7, the Euclidian distance between any two particles in a rigid object or in two identical rigid objects, expressed in the same Cartesian system of coordinates (e.g. in X, Y, Z), is invariant to the spatial location and orientation of the object. Then, as this property was on the basis of the rigid-triangle properties and the related angle properties shown in section 2.4.2, these properties will also be satisfied for any rigid objects having the shape of triangles. Thus, replacing the rigid objects used as unit lengths and triangles by any other rigid objects with adequate shapes, will not affect the proofs of the validity of the 5 postulates. **QED**

Theorem 7 addresses rigid objects. The last issue is to extend its results to any identical systems of particles: i.e. systems starting from identical initial conditions and evolving under the influence of identical environments. The proof of this extension is given in [15]. (lemma 6, theorem 8 and theorem 9).

CONCLUSION

In this paper we propose a computational universe model, supporting the emergence of Euclidian space and completing our previous work on the emergence of relativistic space-time. Some interesting outcomes are:

- The Euclidian space is engendered by a single principle (the form of the interaction laws), while the Euclidian geometry is based on multiple principles (the 5 postulates).
- This unique principle also implies the emergence of the notions of inertial mass and force as a consequence of constraints concerning the form of the interaction laws, and required for the emergence of objects presenting spatial stability (rigid objects).

Thus, it results a new framework treating geometry, which does not require any postulates but deduces geometry from a unique principle corresponding to the form of interaction laws.

Future work will address other theories of physics, and the related geometries, including general relativity and unified theories of physics.

REFERENCES

[1] Konrad Zuse, Rechnender Raum. Braunschweig: Friedrich Vieweg & Sohn. 70 pp, 1969.

[2] Konrad Zuse, Calculating Space, MIT Technical Translation AZT-70-164-GEMIT, (MIT (Project MAC), Cambridge, Mass. 02139, 1970.

[3] Schmidhuber A., Computer Scientist's View of Life, the Universe, and Everything. LNCS 201-288, Springer, 1997

[4] Richard Feynman , Simulating physics with computers, In International Journal of Theoretical Physics, Vol 21, Nos. 6/7, pp. 467-488, 1982

[5] M. Nicolaidis, The scandal of the computational universe, First part: the qualitative concepts, Proc. AISB'13 Proceedings ISBN: 978-1-908187-30-7, April 3-5, 2013, Exeter, UK

[6] M. Nicolaidis, The scandal of the computational universe, Second part: relativity and quantum mechanics, Proc. AISB'13 Proceedings ISBN: 978-1-908187-30-7, April 3-5, 2013, Exeter, UK

[7] M. Nicolaidis, Computational Space, Time and Quantum Mechanics, chapter in Thinking Machines and the Philosophy of Computer Science: Concepts and Principles, J. Valverdu (Ed.), IGI Global Publishing, Information Science Reference, pp. 253-279, Hershey – New York, 2010. ISBN 978-1-61692-014-2

[8] M. Nicolaidis, Simulating Time, Computing and Philosophy Symposium, In proceedings 2nd Symposium on Computing and Philosophy, pp. 51-58, ISBN – 1902956826, AISB 2009 Convention (Adaptive & Emergent Behaviour & Complex Systems), April 6-9, Edinburgh, UK

[9] M. Nicolaidis, On the State of Superposition and the Parallel or not Parallel Nature of Quantum Computing: a controversy raising view point, European Computing and Philosophy Conference, (ECAP), June 2008, Montpellier, France

[10] M. Nicolaidis, Computational Space, Time and Quantum Mechanics, 7th European Computing and Philosophy Conference, (ECAP'09), June 2009, Barcelona, Spain

[11] Brodie, S. E., The Pythagorean Theorem Is Equivalent to the Parallel Postulate., http://www.cut-the-knot.org/triangle/pythpar/PTimpliesPP.shtml.

[12] Weisstein Eric W., *CRC concise encyclopedia of mathematics* (2nd ed.). p. 2147, 2003, ISBN 1-58488-347-2.)

[13] Yuen Ren Chao, A note on "Continuous mathematical induction". Bull. Amer. Math. Soc. Volume 26, Number 1 (1919), 17-18.

[14] Pete L. Clark, The Instructor's Guide to Real Induction, arXiv:1208.0973 [math.HO], August 5 2012

[15] M. Nicolaidis, Emergence of Euclidian Geometry in a Computational Universe, Tech. Report TIMA lab, http://tima.imag.fr/publications/files/rr/2014/eeg_183.pdf