Art and Computing: a Marriage Made in Mathematics

S Barry Cooper[1]

Abstract. The art and computing dichotomy became deeply ingrained in post-Enlightenment culture. It was not just different perspectives, priorities, interests and political expressions: the information age, with its new computational understandings, has brought us both a reinforcement of the rationalist world view, while at the same time an emergent understanding of the role of different models of computation. Of course, computability as a theory is a very young development, and it is only recently that the importance, and real world relevance, of higher order computation has become better understood.
But this higher order has increasingly become the 'elephant in the room' - manifesting itself via emergent phenomena in nature; via neuroscience and the persistence of conceptual consciousness and mental autonomy; through quantum ambiguity and non-locality; and the omnipresent computational inconveniences of turbulence, chaotic settings supporting strange attractors, and phase transitions working against the reversibility of computation and time. Most importantly, we have the persistent usefulness and sense of real significance of natural language - shared in the arts, humanities and science - which was discovered in the 1930s to take us far beyond the computable world.
On the computational side, we have the dominance of the Turing model, accompanied by less well-known work of Alan Turing and others relating to higher type computation, morphogenesis, and the underpinnings of artificial intelligence - see [4]. In this short sketch, we look at how the mathematics of different computational models appears to be embodied in the structure of the human brain. And how the conceptual clarification arising can be carried into the wider context, bringing both direction and optimism to the future of art and computing.

1 ART VERSUS COMPUTING

Why should we be so interested in the future of art and computing? Surely - the argument goes - art is best pursued in visceral autonomy, with computing at best a useful tool for materialising the creativity of the human mind and body? The algorithmic drive associated with the information age is anathema to those who still value humanity per se, and who see normalising rules as ones to be broken. On the other hand - adopting a different standpoint - computing is the pursuit of clarity, offering freedom from mistakes, a project devoted to something a little more serious than entertainment and the indulgence of human weakness and fallibility? Nothing less than the survival of the species is at stake, and it is scientific understanding that will give us direction and a new found harmony of utilitarian achievement!
Clearly, both these - rather crudely outlined - viewpoints have very different, and very valid roles to play in our complex world. And in an age where marriage itself is in question, what is to be gained from the mixing of such different perspectives? And the ages of the partners! The birth of the former is seen as being lost in pre-history, and the latter having arrived in the world less than 70 years ago.
In fact, it is the human brain itself - and new eyes provided by early pioneers of the computer age - which turns all this from pumpkin into golden coach. And it is the mathematics that so informatively accommodates our intuitions, and provides the basis for a truly magical marriage contract. We see how the magic and materialism may combine, as it does so powerfully in *Turing - A Staged Case History*, delivered to a packed Milanese theatre for a week in November 2012. Created and embodied by artists who worked with the technology, and spent many hours researching, and visiting Bletchley Park - the unique workspace of so many early computing pioneers - nothing could have demonstrated more immediately the rewards of the new burgeoning synergies.

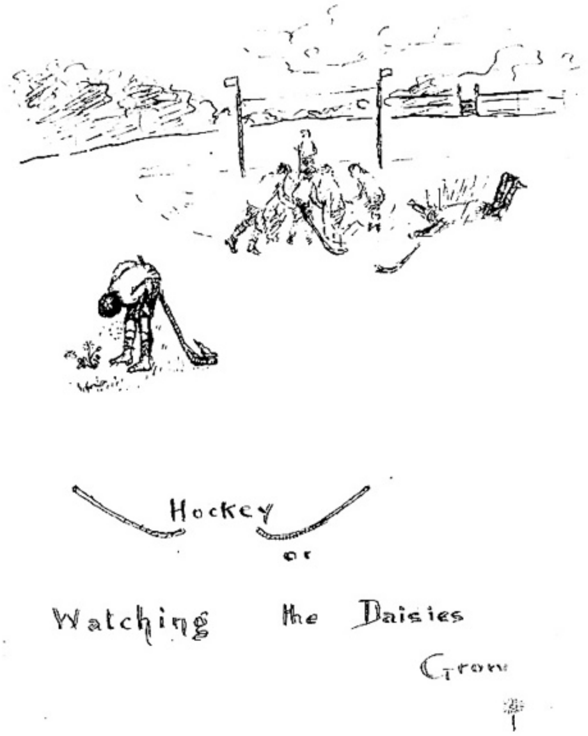

Figure 1. Drawing of Alan Turing by his mother, at his preparatory school, Hazelhurst, Sussex. Annotated by Mrs Turing 'sent to Miss Dunwall, matron at Hazelhurst. Date Spring term 1923.' Courtesy of Sherborne School.

The complexity and intimacy of the relationship is illustrated by Sara Turing's sketch of her son 'Watching the Daisies Grow' (Figure 1). A clear dichotomy between 'art' and 'computation': Hockey, social, interactive, distinctively human, and creative at its best, being determinedly ignored by the geekish Alan with his studious botanical interests. But look again. The hockey is of course a faltering application of rules of the game, a game pursued according to learned, stereotypical (programmed) procedures. While it is rule-breaker Alan we observe, focussed on the mysteries of emergent patterns in nature, a mystery he would immerse himself in and clarify so creatively thirty years later at the University of Manchester.

And what about our role as observers? We are certainly computing - in some sense - a description of Mrs Turing observing her son. But the computation has different levels, assisted by mental framings previously applied in different contexts. The process is partly learned, partly experimental application of conceptual tools successfully applied previously, and partly bringing together of contextual observation, from inside and out, in a formative and informative manner. We are handling the syntactics and semantics of a complex world with an aplomb born of different Levels of Abstraction (as outlined in Luciano Floridi's recent book [5] on *The Philosophy of Information*) within which we can apply what we know.

What becomes clear is that it is what happens in the cracks between these epistemological/ontological stepping stones that is important to our understanding. And that the relationships we need to deal with involve computation of a scientifically problematic nature. Paradoxically, we do not recognise much of this as computational, even though it provides a familiar and invaluable part of our everyday mental activity. Specially important is the computational role played by natural language, generating a fragmented and troublesome informational terrain, and at the same time reducing the uncertainties to a form which can be handled within the classical computational setting.

2 THE COMPUTATIONAL BRAIN

In his 2009 Yale University Press book [9] *The Master and his Emissary: The Divided Brain and the Making of the Western World,* Iain McGilchrist describes how:

> *The world of the left hemisphere, dependent on denotative language and abstraction, yields clarity and power to manipulate things that are known, fixed, static, isolated, decontextualised, explicit, disembodied, general in nature, but ultimately lifeless. The right hemisphere by contrast, yields a world of individual, changing, evolving, interconnected, implicit, incarnate, living beings within the context of the lived world, but in the nature of things never fully graspable, always imperfectly known — and to this world it exists in a relationship of care. The knowledge that is mediated by the left hemisphere is knowledge within a closed system. It has the advantage of perfection, but such perfection is bought ultimately at the price of emptiness, of self- reference. It can mediate knowledge only in terms of a mechanical rearrangement of other things already known. It can never really 'break out' to know anything new, because its knowledge is of its own representations only. Where the thing itself is present to the right hemisphere, it is only 're-presented' by the left hemisphere, now become an idea of a thing. Where the right hemisphere is conscious of the Other, whatever it may be, the left hemisphere's consciousness is of itself.*

A key aspect of the brain architecture of placental mammals, such as humans, is the corpus callosum, connecting and mediating the functionality of the separate hemispheres. McGilchrist comments (pp. 18-19):

> *... the evidence is that the primary effect of callosal transmission is to produce functional inhibition.*
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> *... it turns out that the evolution both of brain size and of hemisphere asymmetry went hand in hand with a reduction in interhemispheric connectivity. And, in the ultimate case of the modern human brain, its twin hemispheres have been characterised as two autonomous systems.*

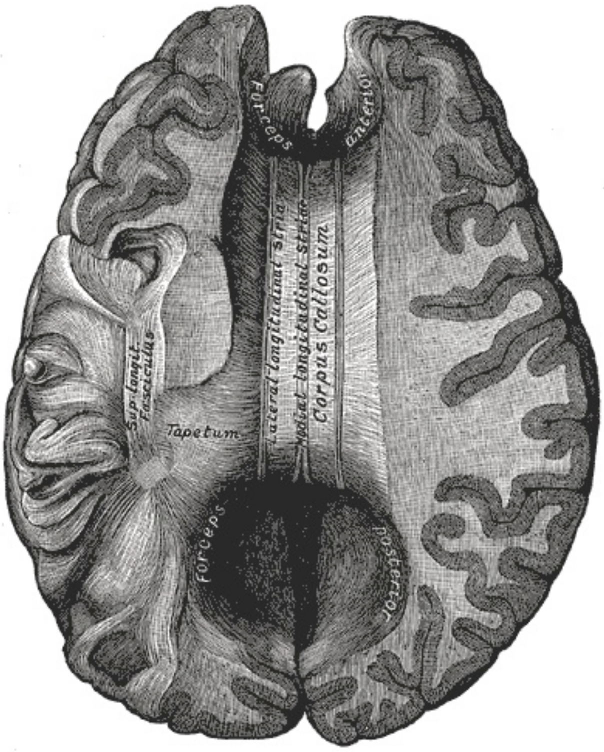

Figure 2. Corpus Callosum, from 20th U.S. edition of *Gray's Anatomy of the Human Body*, originally published in 1918.

So is there actually some purpose in the division of neuronal, and therefore, mental processes? If so, what could that be? We might further ask: Given this division, to be found in animals generally, what is the benefit of the moderated reconnection via the corpus callosum?

Left brain-right brain features commonly in popular writings. The point of mentioning it here is that there is a huge body of reputable work pointing to contrasting types of thinking, the types (a term we shall see, with relevant mathematical meaning) broadly corresponding to those in the folk culture. And that these types do correspond, on one hand, to differing computational frameworks: and on the other, to how we view the sorts of thinking predominating in the arts and humanities on one side, and in mathematics and the sciences on the other.

Part of the more serious literature is a basket of terminology relating to what is problematic for the classical Turing model of computation, and its embodiment in today's computing machines. In particular, the notion of *emergence,* without a clear definition of what it is, brings with it terms from many different fields. Listing in no special order, and with no hope of comprehensiveness, we have: morphogenesis, phase transition, self-organisation, supervenience, synergetics, chaos, strange attractors, randomness, turbulence, non-locality, decoherence, intuition, commonsense, creativity, semantics, inspiration, imagination, intelligence, fractal, wave collapse, entropy, downward causation, incomputability, consciousness, qualia, free will, strong determinism, networks, complexity, etc.

Most of the literature is either technically concerned with micro-analysis of particular aspects of the theory or the observables, where it is possible to do 'normal science', to simulate computationally, or to treat descriptively, building up intuitions and connections. This is work prone to duplication and permutation of known bodies of work: an epistemological serpent eating its own tail. One might identify the different approaches as predominantly left or right brain: analytic and scientific, with limited reach; or descriptive and artistic, without overarching coherence. Though it is increasingly apparent that there is no non-trivial representation and rationality without embodied non-local computation hosted by the brain (left or right), and no effective utilisation of wide-ranging cognitive resources without an underlying level of coordinating structure.

We might point to the situation in physics before Isaac Newton, when many of the ingredients of the coming scientific revolution were in place. All that was missing was the specificity of modeling, and the appropriate mathematics to handle it.

For more on this range of topics, see the recent approachable articles [2] and [3], and the more challenging *From Descartes to Turing: The Computational Content of Supervenience* [1].

3 A MATHEMATICAL MARRIAGE

Computability theory as a mathematical field had its genesis in the work of a number of seminal figures in the 1930s. These included Emil Post, Rózsa Péter, Kurt Gödel, Alonzo Church, Stephen Kleene, and, of course, the young Alan Mathison Turing. In fact, they were *all* relatively young, all less than forty years old in 1936, the year the first papers pointing to the reality incomputability appeared. They entered our world in the fifteen years 1897 (Post) to 1912 (Turing), and all were gone by 1995, having left it unrecognisable from the world they were born into.

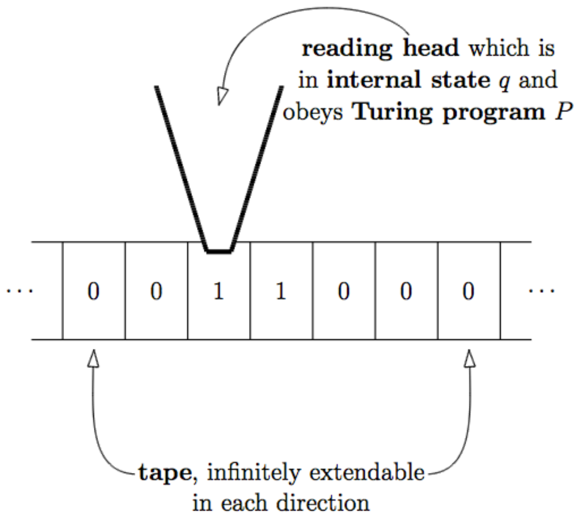

Figure 3. Tape & reading head host for the Turing machine model of computation.

It is Turing's classical model of computability around which much of our thinking about the subject has centred. The 2012 CACM article [3] on *Turing's Titanic Machine?* gives due credit to the role of the universal Turing machine in the post-war computing revolution, and much technology-assisted creative work arising from it delivers magnificently. In fact, the success of such work, in many ways epitomising Turing's vision of a partnership between human and machine, causes one to wonder what further understanding of our computational universe can deliver.

Figure 4. Alan Turing in 1928, aged 16. Photograph courtesy of Sherborne School.

What one has to remember though is that art is constrained by and subject to the values of the larger society. And the artist has a responsibility to address negative developments in the wider social context. At a time when corporate structures are becoming increasingly reliant on algorithmic control mechanisms, and human intelligence is diverted from positive expressions into outplaying the system, art has a key role. We are more than ever dependent on its unique power of dislocation of rigid patterns of thinking, while defending the autonomy and economic base of the artist from uncomprehending attack. The computer is a valuable resource, it should not be in charge - yet.

The main aim of the 1936 universal computing 'machine' was not the 'invention' of the modern computer, though it provided the essential 'stored program' ingredient. The aim was, at the same time as providing a programmable model of algorithmic computation, to show that David Hilbert was mistaken, and that unsolvable problems did exist, in fact, existed very close to home. To put it quite briefly: Having codified rationality and algorithmic computation, Turing used this clarity to show that computers, when they arrived, would be theoretically limited. "As thick as two short planks" as a colleague (who prefers to remain anonymous) put it a little while back. His trenchant remark outstrips even Marvin Minsky's Boston University comment (in May 2003) that:

AI has been brain-dead since the 1970s.

Turing himself did have occasionally expressed ambitions to 'build a brain', but the history of AI has deflated much of the early grandiosity.

What emerges from Alan Turing's 1936 description of an incomputable data-set, and his 1939 attempt to computationally outflank Gödel's Incompleteness Theorem, is the way in which information is subject to all sorts of mathematical hierarchies, with the levels having intimate relationships with natural language. The quantification allowed in the language corresponds to the scope and quality of the observer-status of the computing agent. Once again, Floridi's Levels of Abstraction appear on the scene, but now within a formal context. The role of statistical sampling and approximation is important in bringing the abstraction within a computationally familiar setting - though one which admits mistakes and blurs relationships between different computational settings.

Turing himself showed a prescient awareness of this extended world in his late work on the emergence of patterns in nature, defining morphogenic phenomena such as the dappling on a Fresian cow's back via the 'natural language' of differential equations. Importantly, buried away in his 1939 Princeton paper was a simple extension of his Turing machine model to encompass interactivity, providing a model of globally connected computational agents with the power to cooperate in raising informational and computational content way beyond what our computer can accurately handle.

An interesting take on real-world definability and how it both transcends the classical computational model and is hosted by the human brain is provided by the book [6] of Hofstadter and Sander. Their 'big idea' is the way in which the human brain packages 'big data' into analogies which can be judiciously applied in an ad hoc way across a wide range of previously unfamiliar contexts. Here they are discussing the computational difference between an analogy and a categorization, and the power and necessity of the analogy as a higher order aid to human mentality. The topic is:

- the seemingly trivial case of the recognition of a cup as a cup. Suppose you are at a friend's house and want to fix yourself a cup of tea. You go into the kitchen, open a couple of cupboards, and at some point you think, "Aha, here's a cup." Have you just made an analogy? If, like most people, you're inclined to answer, "Obviously not — this was a categorization, not an analogy!", we would understand the intuition, but we would propose another point of view. Indeed, there is an equally compelling "analogy" scenario, in which you would have just constructed inside your head a mental entity that represents the object seen in your friend's cupboard. In this scenario, you would have created a mental link between that mental representation and a pre-existing mental structure in your head — namely, your concept named "cup". In short, you would have created a bridge linking two mental entities inside your head.

Beyond definitions within natural language, there are more abstract models of higher order computation. Thes, for all their power, turn out to be far less well-behaved than their classical analogues. This fits well with the earlier questioning of the dividing of the brain, and the coordinating role of the corpus callosum. Or, for that matter, says a lot about why we do not want our reasonably trustworthy computer to take on characteristics of the so-called 'intelligent' operator, without some careful constraints. There is an eagerly anticipated new book [8] on such matters by John Longley and Dag Normann. For now, here is Longley describing how the notions have multiplied, and the computational frameworks confirms much of what we observe regarding the uncertainties attendant on the bringing of the mathematics of turbulence, emergence or big data into our classical computational comfort zone:

It is ... clear that very many approaches to defining higher type computability are possible, but it is not obvious a priori whether some approaches are more sensible than others, or which approaches lead to equivalent notions of computability. In short, it is unclear in advance whether at higher types there is really just one natural notion of computability (as in ordinary recursion theory), or several, or no really natural notions at all.

The human brain does handle higher order computation, and art and culture are the guardians of the autonomy of both. The partnership between art and computation still has much to deliver.

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