Efficient Algorithms for Freeform Geometric Models

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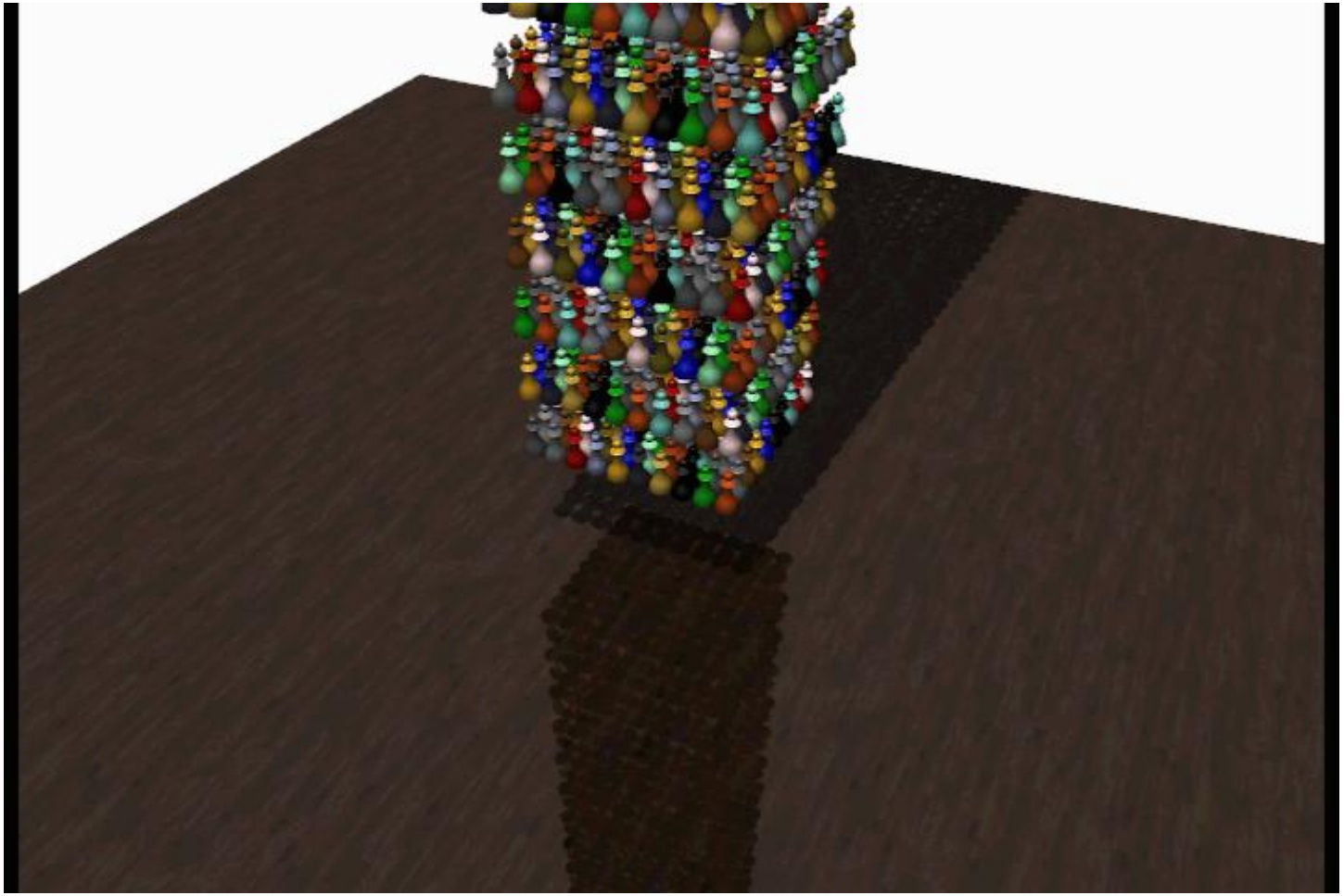
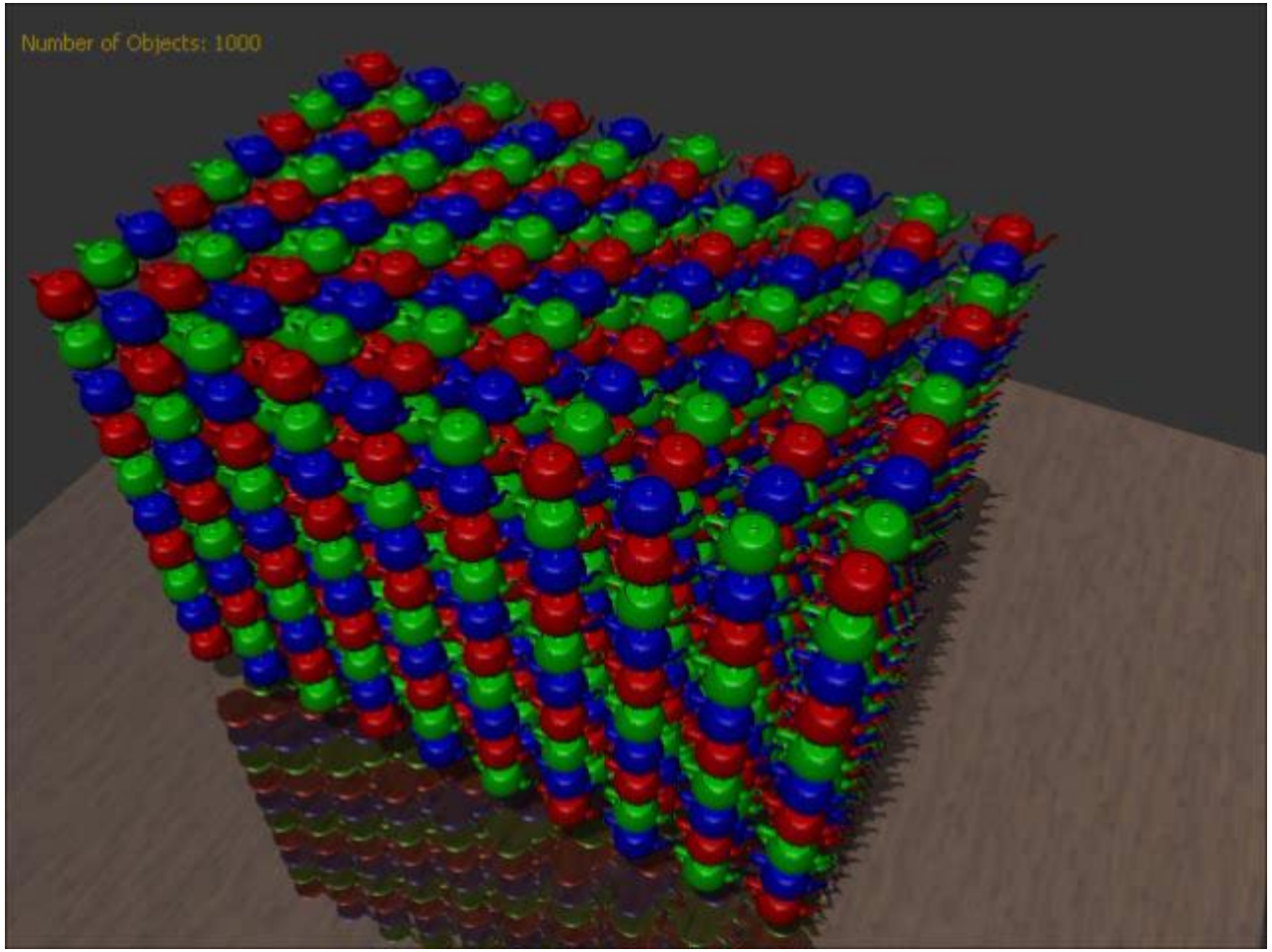
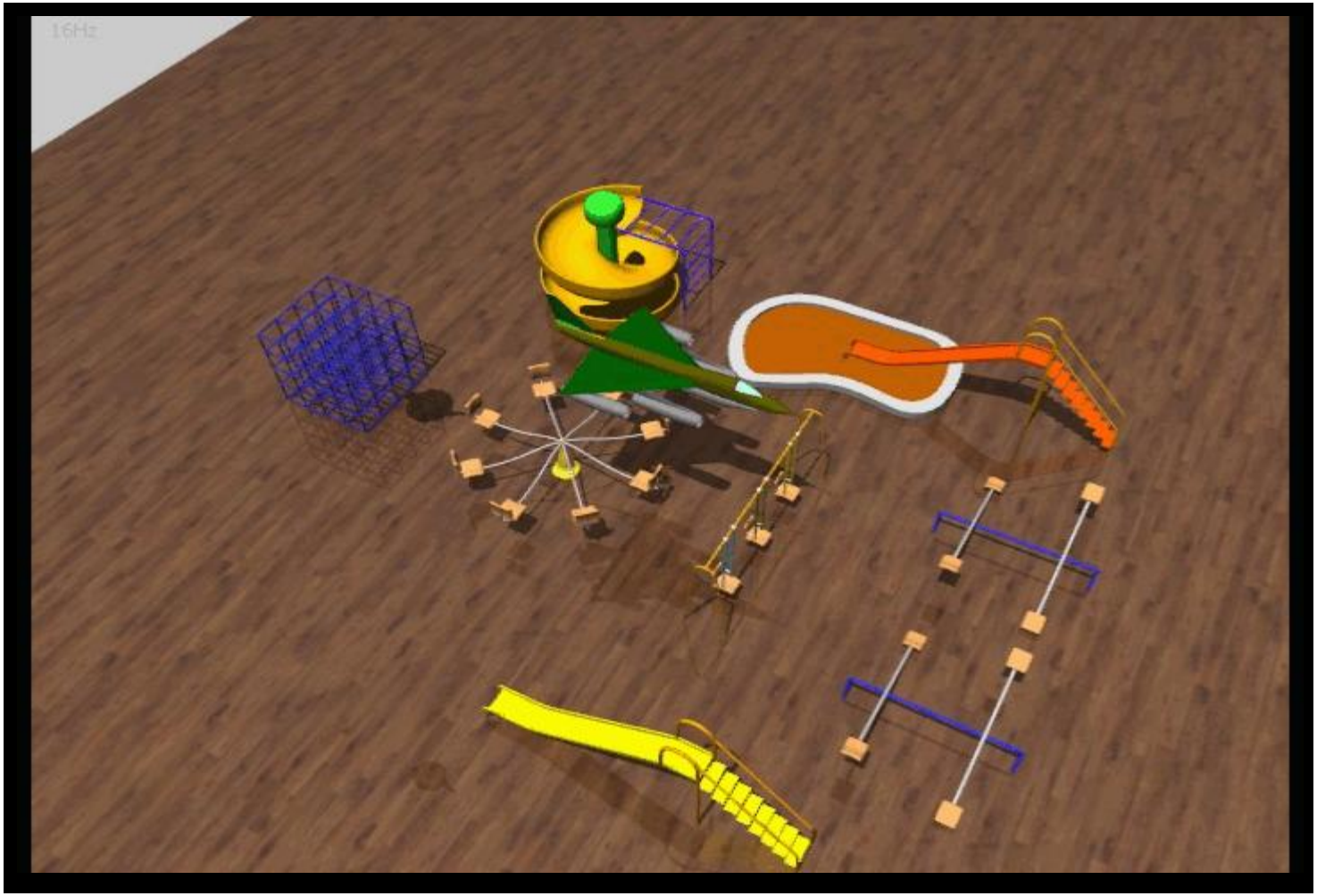
Collision Detection



Collision Detection

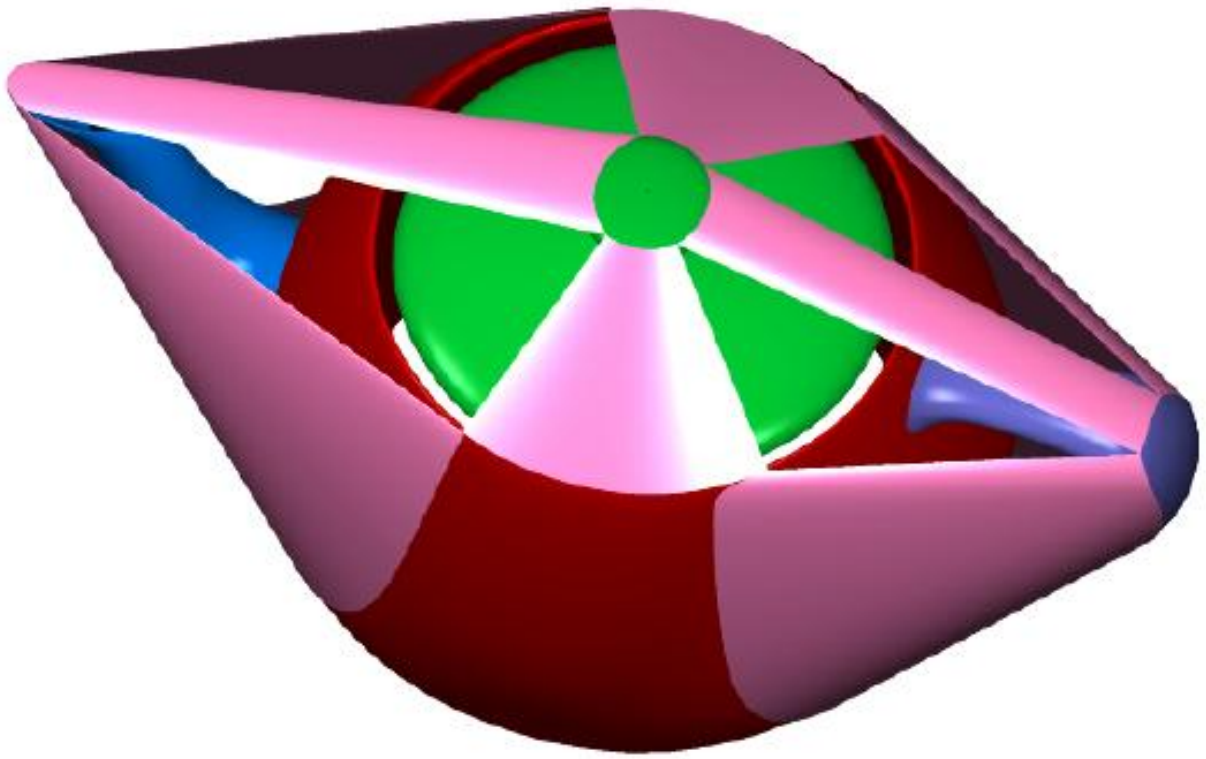


Minimum Distance



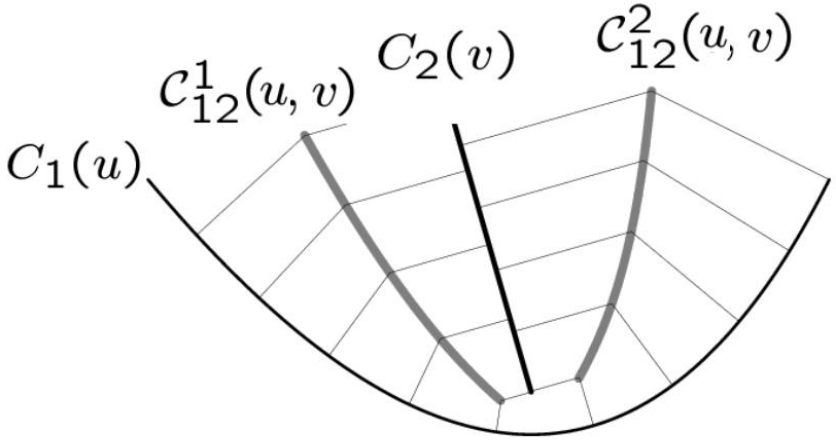
Hausdorff Distance

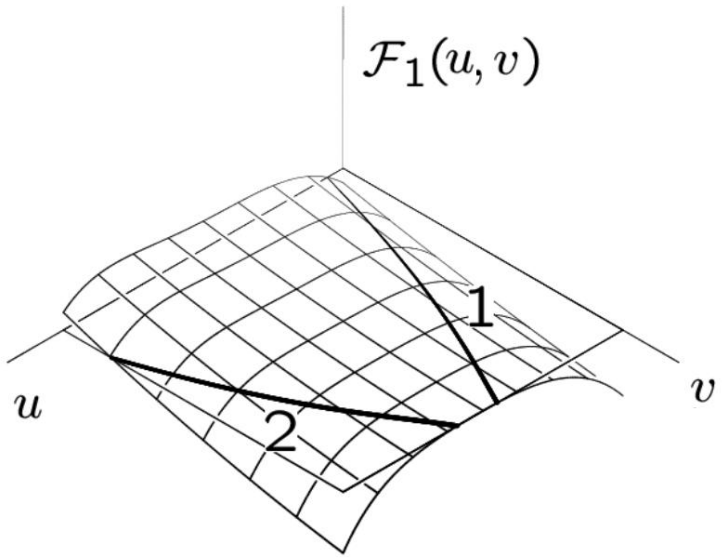
Convex Hull Computation



Previous Approach

$$\left\langle (x,y) - C_1(u), C_1'(u) \right\rangle = 0,$$
$$\left\langle (x,y) - C_2(v), C_2'(v) \right\rangle = 0,$$
$$\left\langle (x,y) - \frac{C_1(u) + C_2(v)}{2}, C_1(u) - C_2(v) \right\rangle = 0.$$





Problem Reduction to (u,v)

- The bisector curve of C(u) and D(v) is reduced to solving F(u,v)=0.
- Much lower degree than the bisector curve b(x,y)=0 itself in the xy-plane.
- Many other geometric problems can be solved in a similar way.
- But, this approach is too slow.

=> Preprocessing is needed!!!

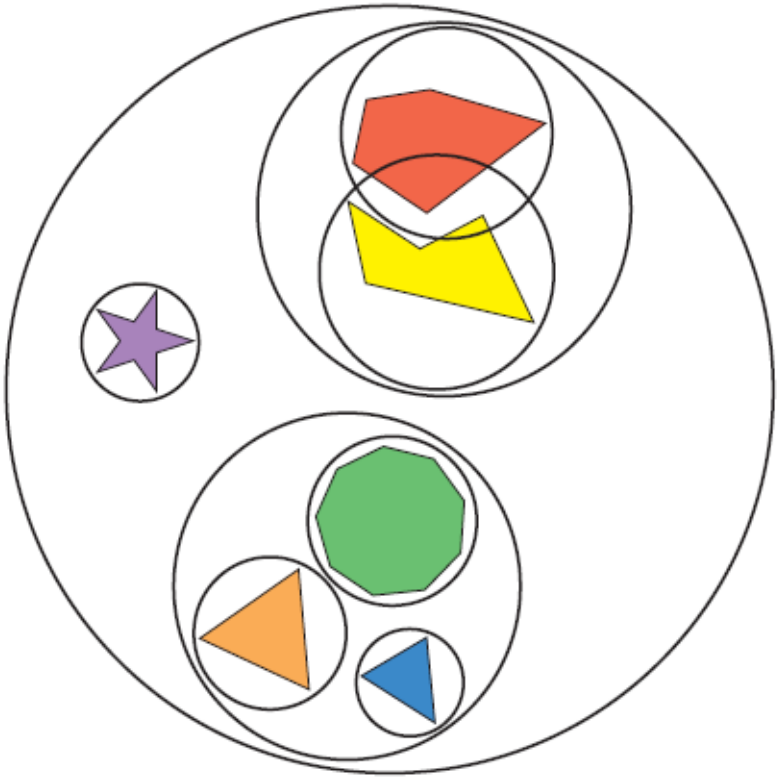
Preprocessing for Freeforms

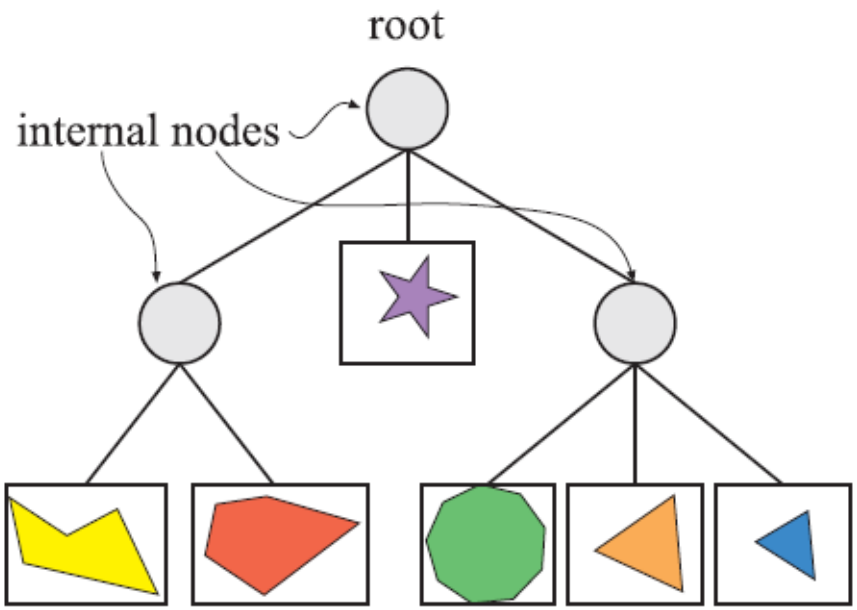
- Biarc approximation of planar curves
- Segmentation of planar curves to monotone spiral curves
- Support distance functions
- Approximation with simple surfaces

Background

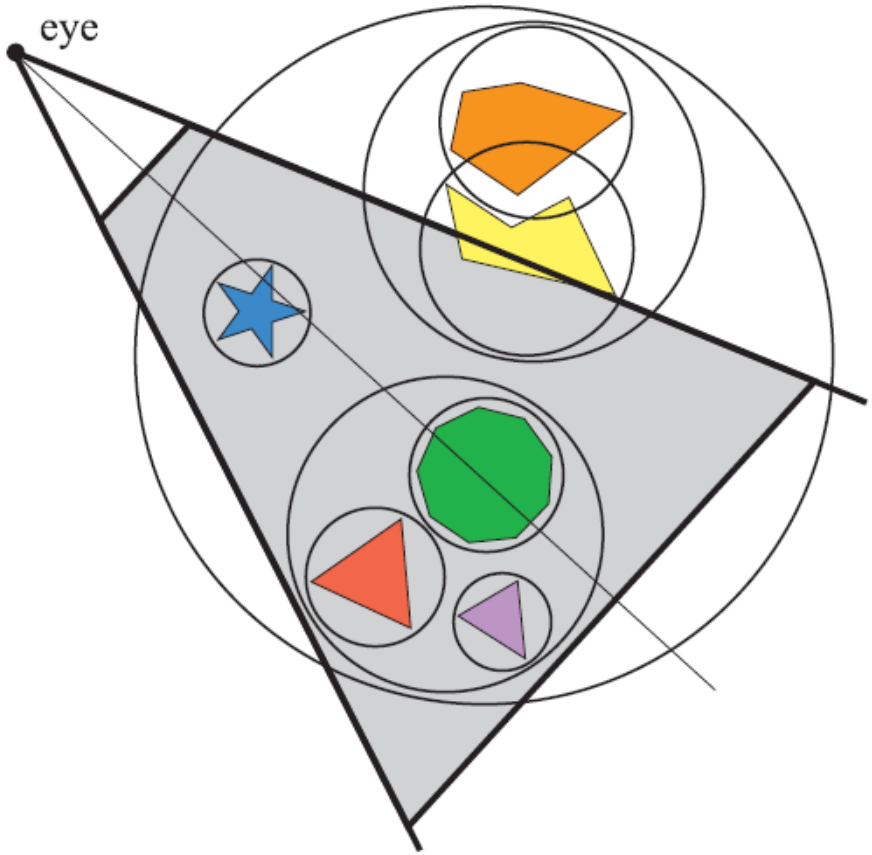
- IK Bi-National Grant (2007-2009)
  - Hausdorff distance computation
    for freeform curves and surfaces
- Tang, Lee, and Kim (SIGGRAPH 2009)
  - Real-time HD computation
    for triangular meshes using BVH
    (prebuilt hierarchical data structure)

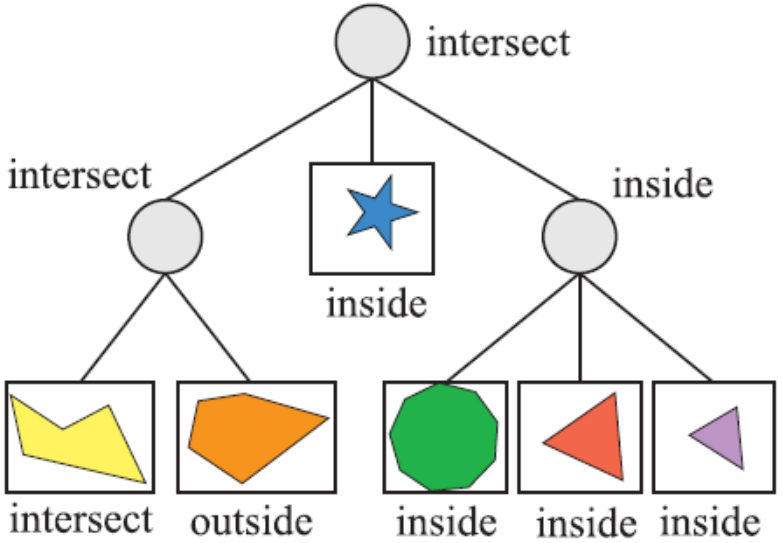
Bounding Volume Hierarchy



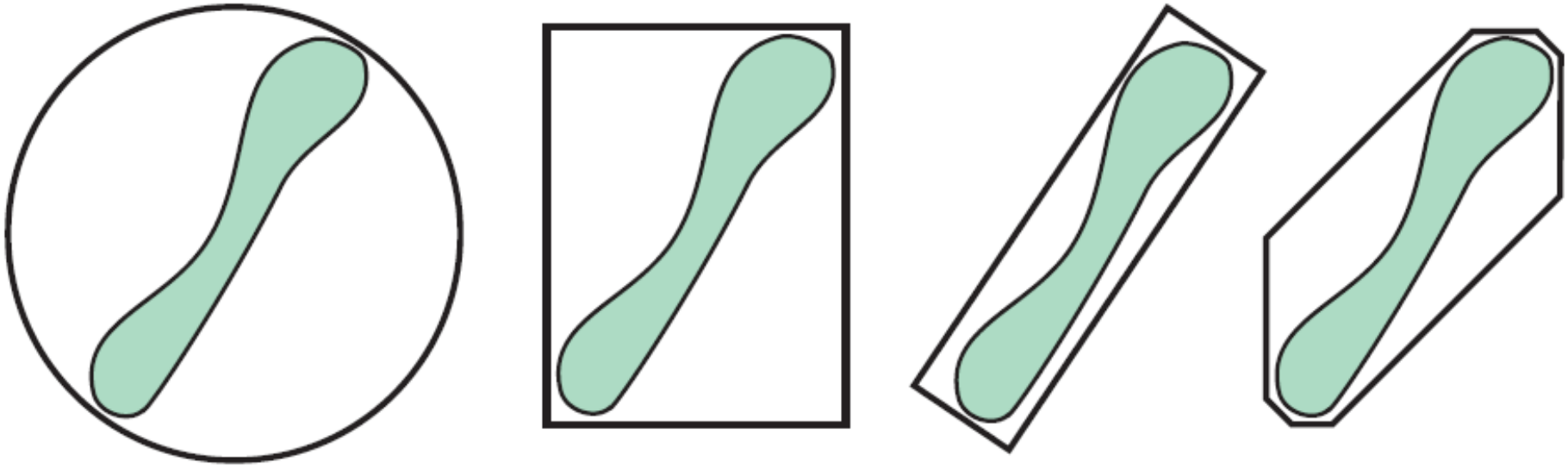


View Frustum Culling





Conventional Bounding Volume

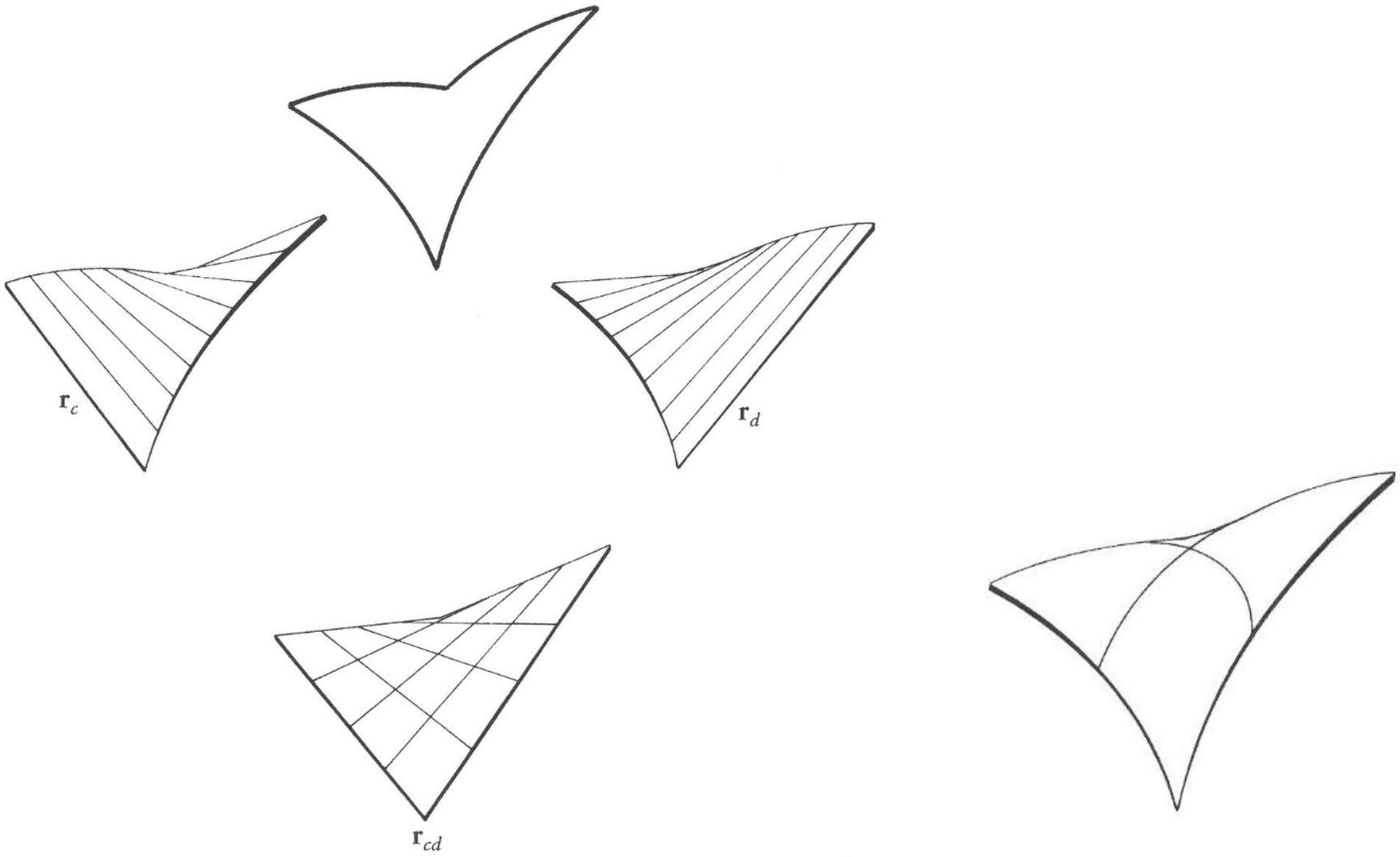


Conventional BVH Complexity
(Yoon and Manocha, EG2006)

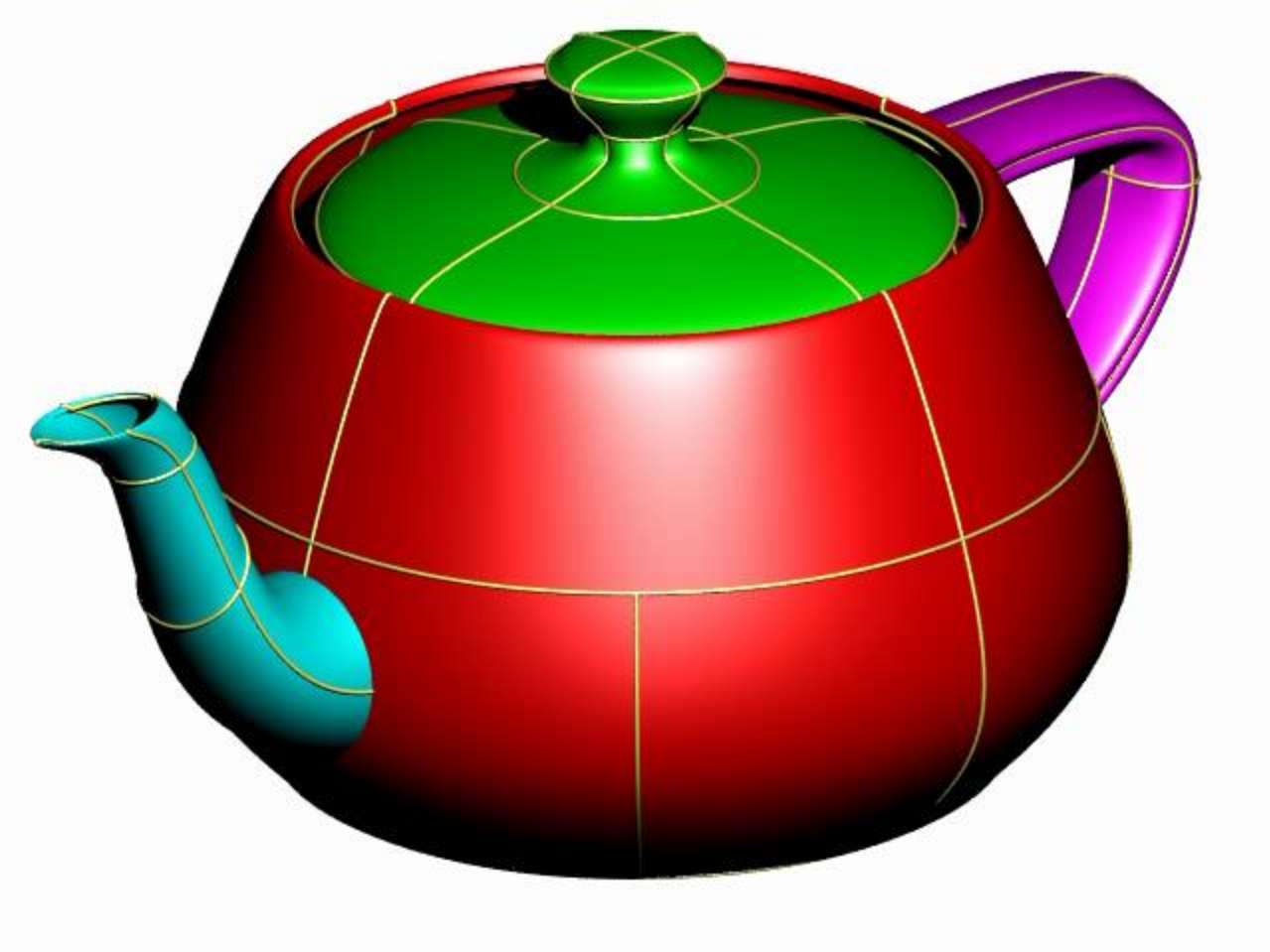
Model	Triangles (M)	Size of BVH (MB)	Mean and std of depth of leaves	Comp. time (min)
Hugo	0.02	2	16, 1.7	0.03
Bunny	0.07	8	17, 0.8	0.26
Dragon	0.8	108	21, 1.6	3
1M power plant	1.1	139	23, 2.9	6
Turbine	1.7	220	22, 0.7	8
Lucy	28	4,811	37, 3.4	34

Table 1: ***Benchmark Models:*** *Model complexity, sizes of BVHs, mean and standard deviation(std) of depth of leaf nodes, and computation time to compute cache-oblivious layouts are shown.*

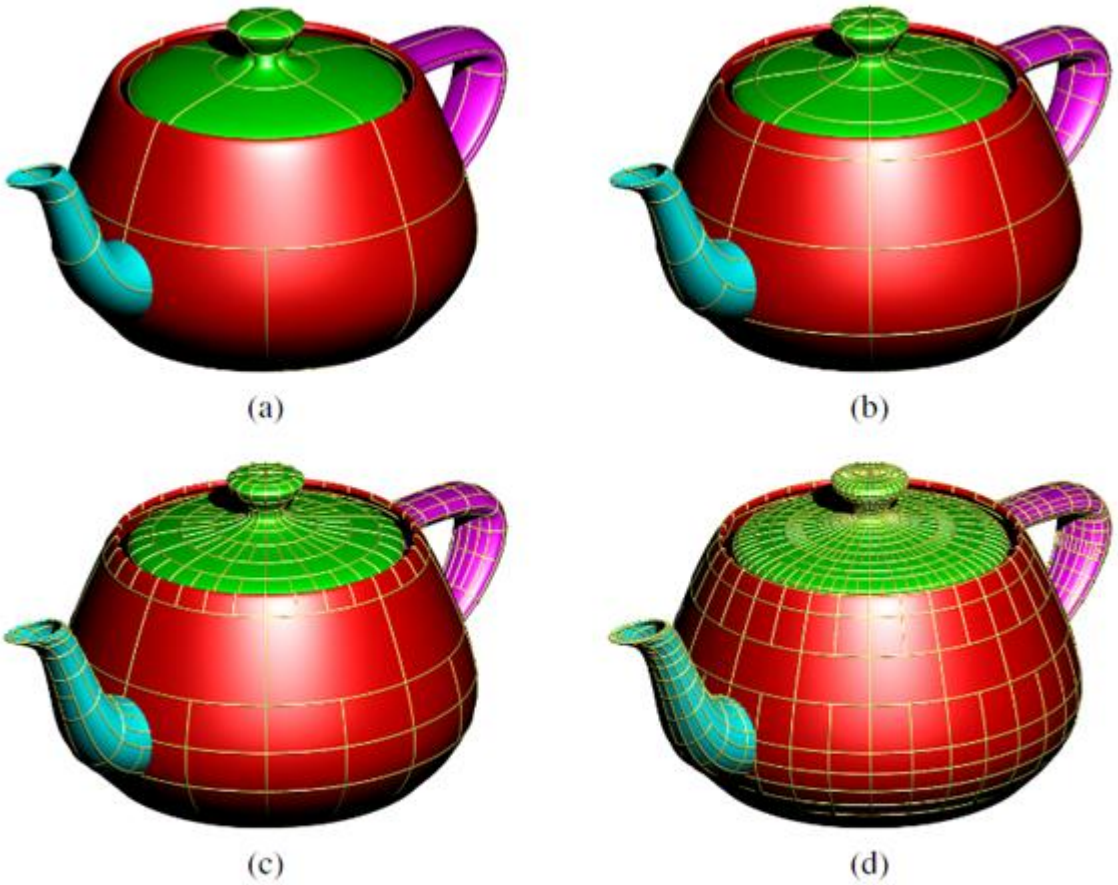
Coons Patch



Approx. with Coons Patches



Approx. with Coons Patches



Approximation of the Utah teapot using Coons patches: (a) 44 Coons patches within an error bound 10^{-4}, (b) 150 patches for 10^{-3}, (c) 492 patches for 10^{-4}, and (d) 1688 patches for 10^{-5}.

Bounding Coons Patches

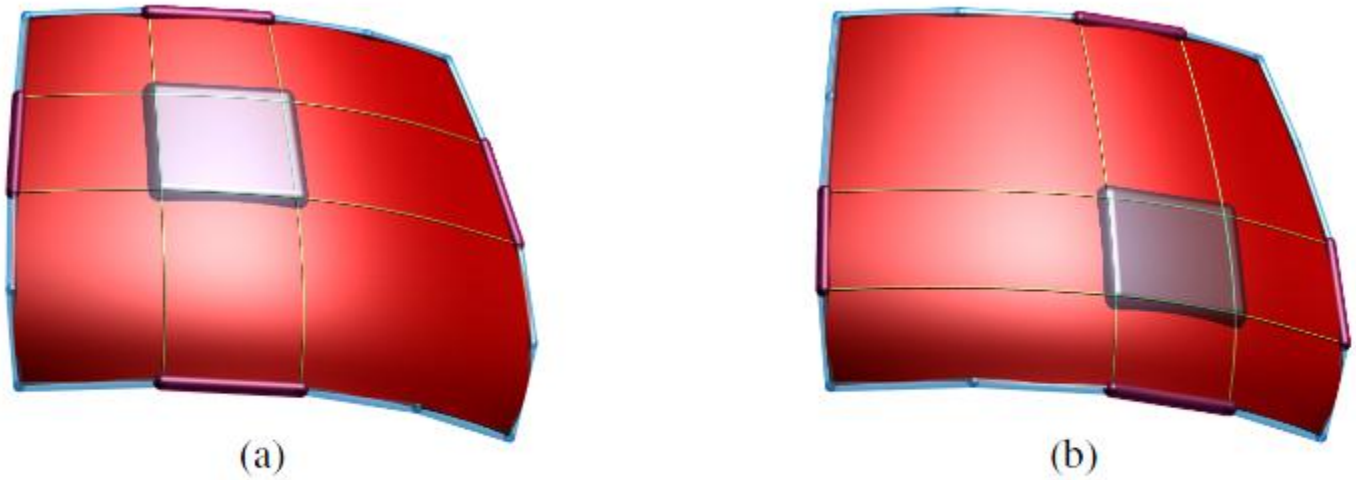


Figure 3: *Coons patch and pairs of opposite boundary curves.*

Two Steps of Approximation

- Bezier surface by Coons patches (ϵ
- Coons patch by bilinear surfaces (0.2ϵ by approximating the boundary curves

Bounding Volume

- Bounding bilinear surface by tetrahedron and offset by maximum error

Bounding Coons Patches

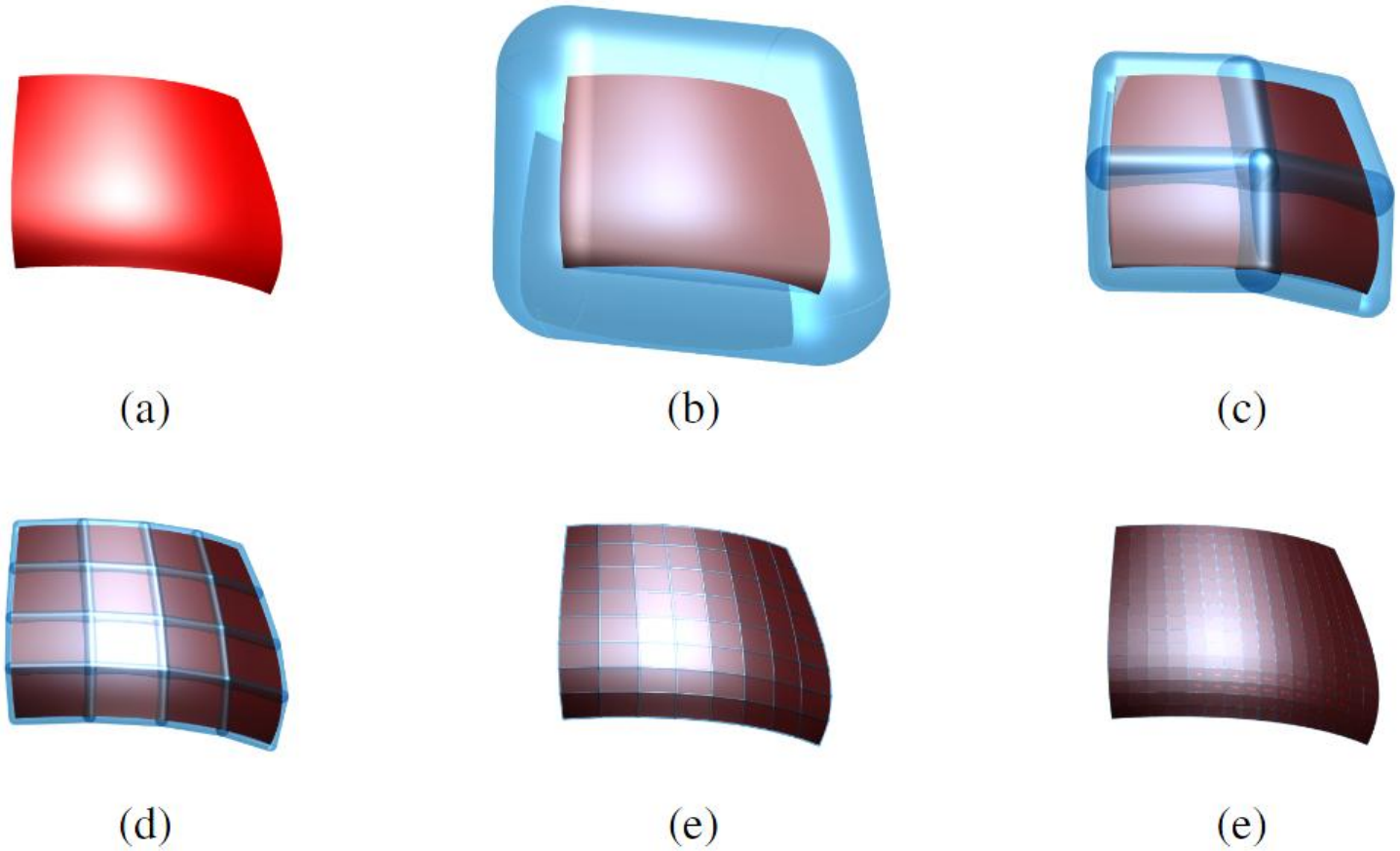
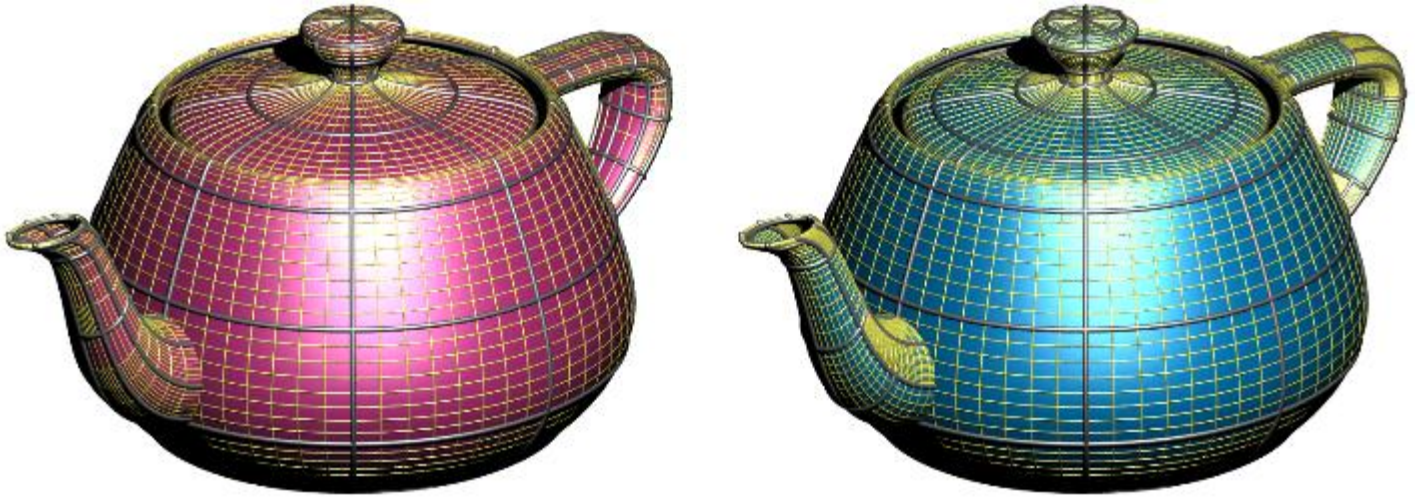


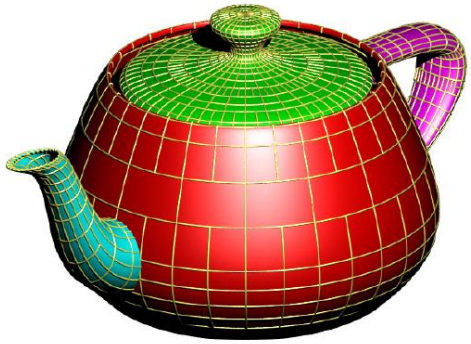
Figure 5: *Bounding Coons patch with offset volumes.*

Comparison



(a) (b)

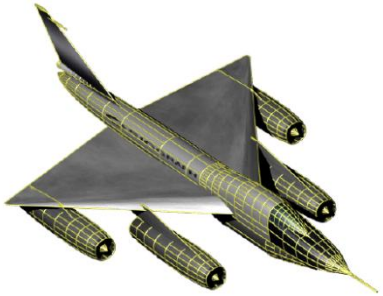
Figure 4: *Approximation of Bézier surfaces and Coons patches using bilinear surfaces: (a) 160 Bézier surfaces (precisely on the Utah teapot) approximated with 5074 bilinear surfaces within an error bound* $1.2 * 10^{-3}$ *and (b) 150 Coons patches (within a distance* 10^{-3} *from the Utah teapot) approximated with 12864 bilinear surfaces within an error bound* $2 * 10^{-4}$.



BVH Complexity

Teapot (4KB)	#NURBS(8)		#Bézier(160)	
	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Coons	44	150	492	1,688
#Bilinear	7.8K	51.8K	636K	5.6M
Coons BVH	0.73KB	2.7KB	9.7KB	55KB
Time (sec)	0.14	0.42	1.65	8.24

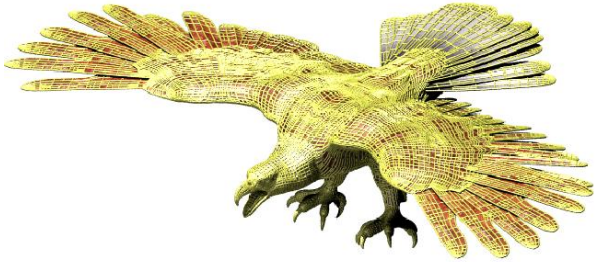
Teapot	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Triangles	1.2K	11K	99K	938K
BVH (RSS)	320KB	2.7MB	25MB	240MB
Time (sec)	0.009	0.094	0.95	9.7



BVH Complexity

B58 (12KB)	#NURBS(240)		#Bézier(266)	
	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Coons	210	276	530	1,552
#Bilinear	23.9K	75.4K	726K	10.1M
Coons BVH	2.3KB	4.1KB	9.6KB	31.9KB
Time (sec)	0.09	0.30	1.14	5.90

B58	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Triangles	1K	5K	45K	424K
BVH (RSS)	250KB	1.3MB	11MB	109MB
Time (sec)	0.006	0.047	0.41	4.2



BVH Complexity

Eagle (102KB)	#NURBS(273)		#Bézier(9786)	
	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Coons	348	1,313	6,314	24,594
#Bilinear	463K	3.4M	39.7M	235M
Coons BVH	5.8KB	23.8KB	118KB	482KB
Time (sec)	18.2	34.1	70.9	184.7

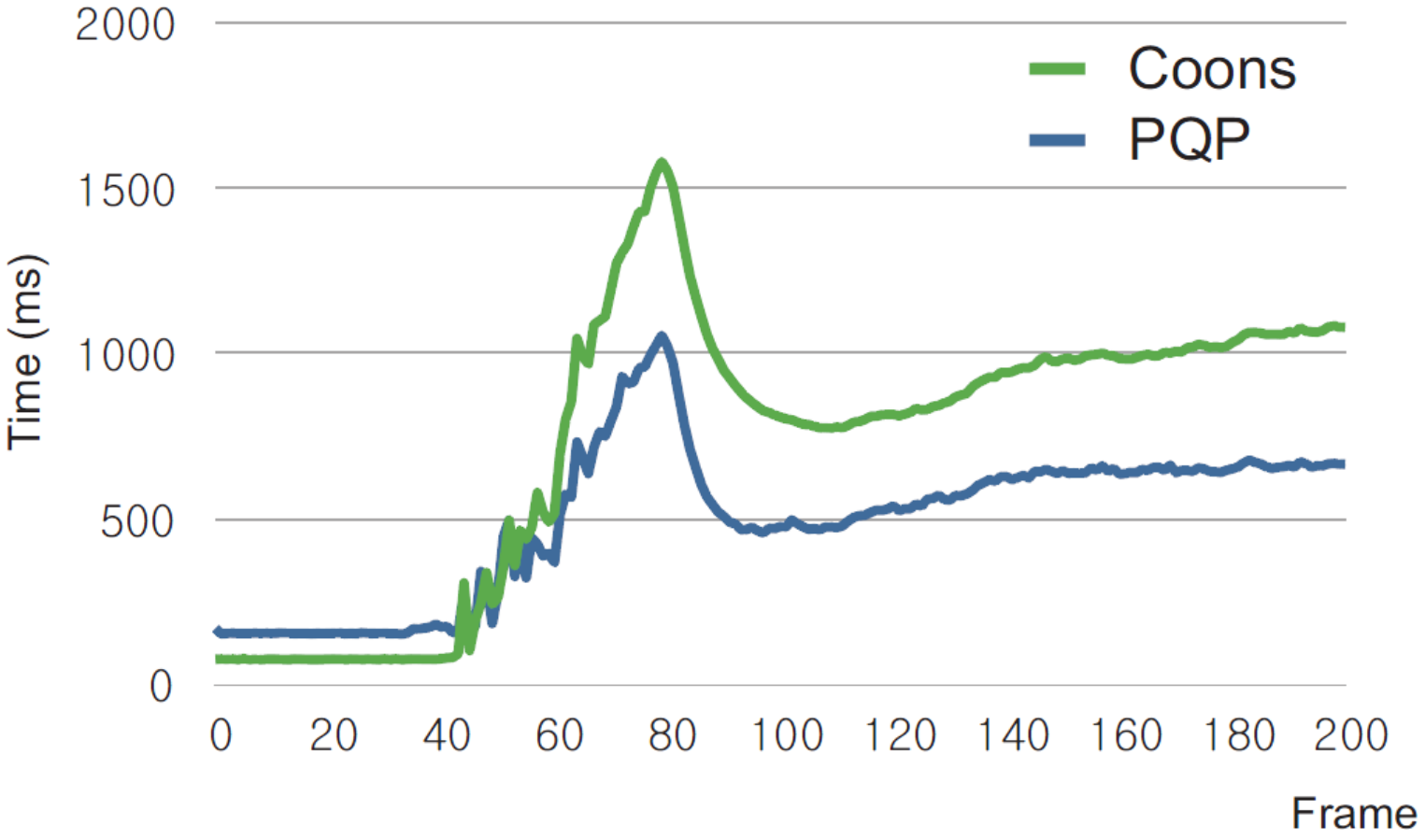
Eagle	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Triangles	2K	26K	199K	1.6M
BVH (RSS)	550KB	6.5MB	51MB	421MB
Time (sec)	0.016	0.24	2	18

BVH Complexity

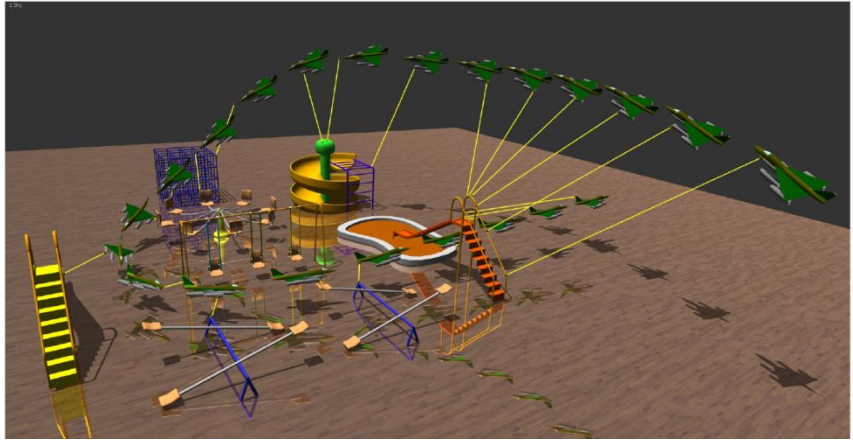
Playground (120KB)	#NURBS(984)		#Bézier(2264)	
	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Coons	1,243	1,723	4,476	12,526
#Bilinear	284K	2.5M	10.5M	98.4M
Coons BVH	13.8KB	26.6KB	82.5KB	258KB
Time (sec)	1.5	3.8	14.1	60.0

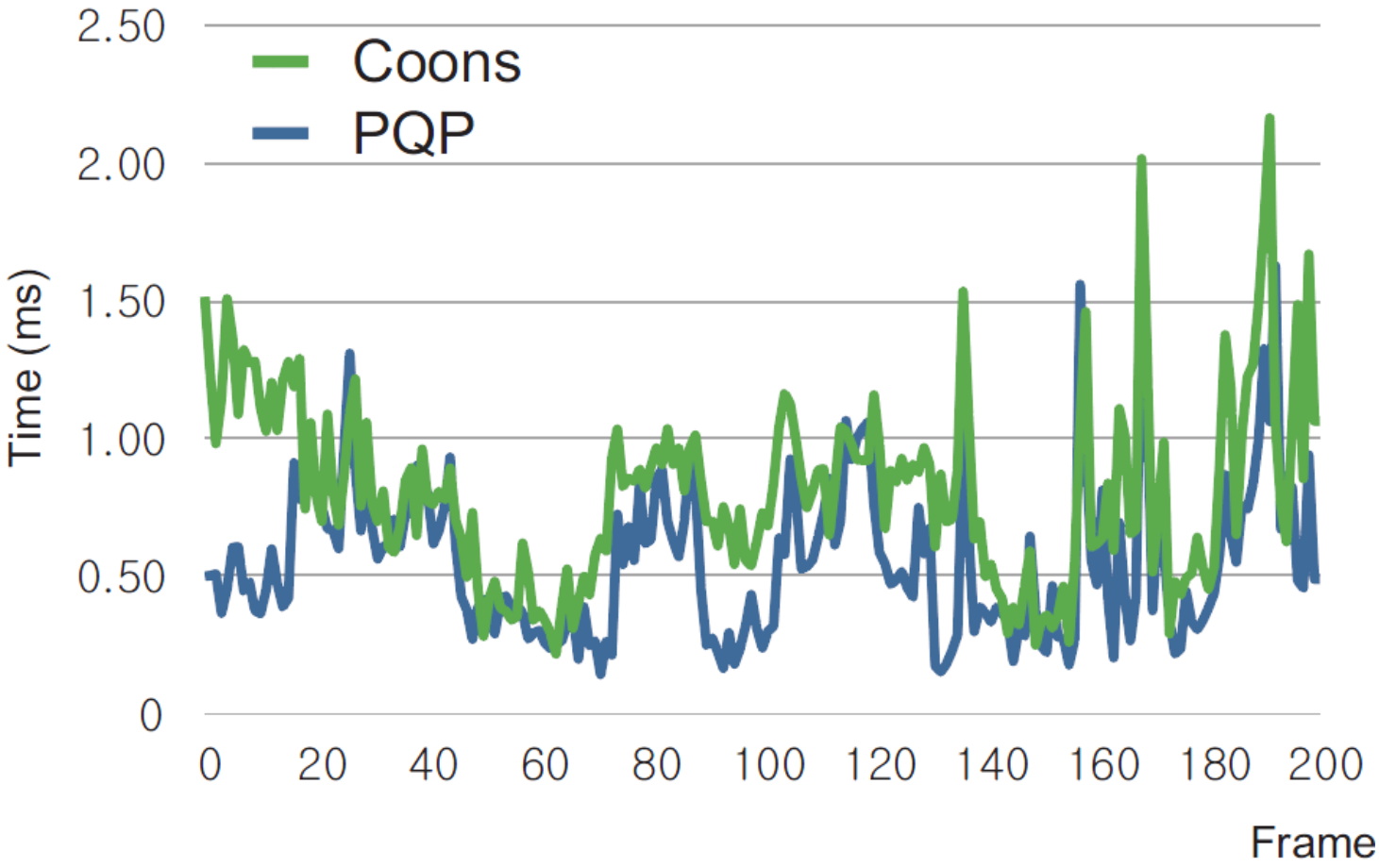
Playground	10^{-2}	10^{-3}	10^{-4}	10^{-5}
#Triangles	14K	112K	1.0M	9.3M
BVH (RSS)	3.6MB	29MB	259MB	2.4GB
Time (sec)	0.12	1.1	11	108

Performance Comparison



Performance Comparison





Conclusions

- Compact BVH for Freefrom Models
- Efficient Geometric Algorithms
  - Collision detection
  - Minimum distance computation
  - Hausdorff distance computation
  - Convex hull computation