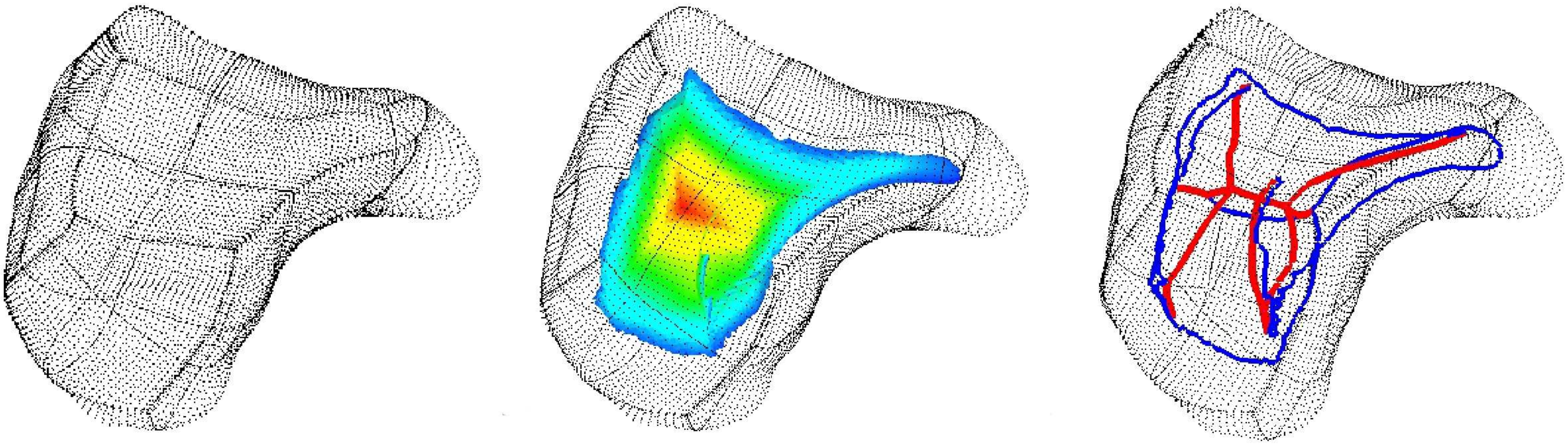
Medial Scaffolds for 3D data modelling: status and challenges



Frederic Fol Leymarie

Goldsmiths
UNIVERSITY OF LONDON

Outline

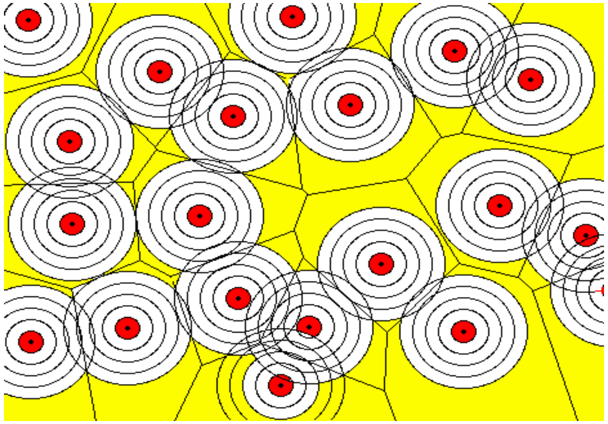
Background

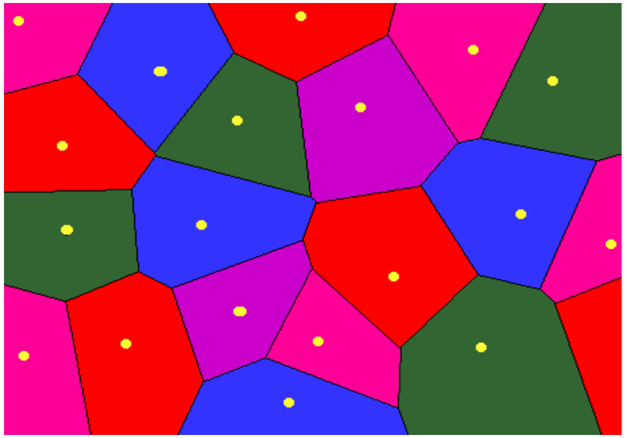
Method and some algorithmic details

Applications

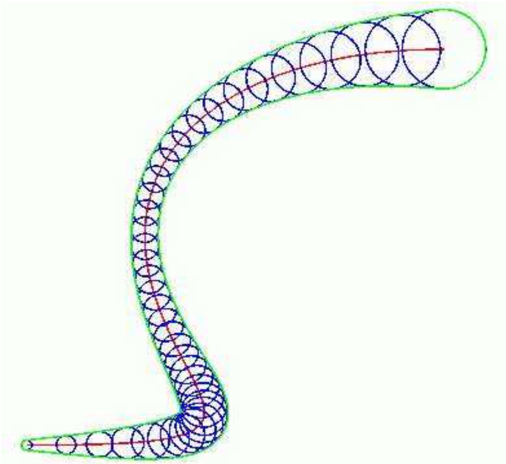
Shape representation: From the Medial Axis to the Medial Scaffold

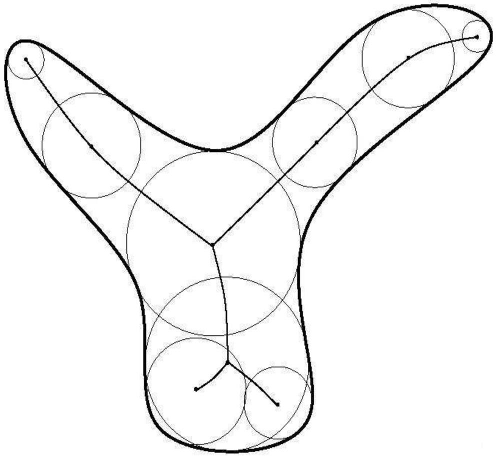
Wave propagation
Blum, Voronoi, Turing, *et al.*



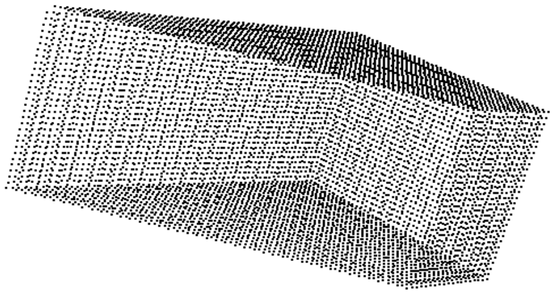


Maximal disks
Blum, Wolter, Leyton, Kimia, Giblin, *et al.*





Study 3D shape with minimal assumptions



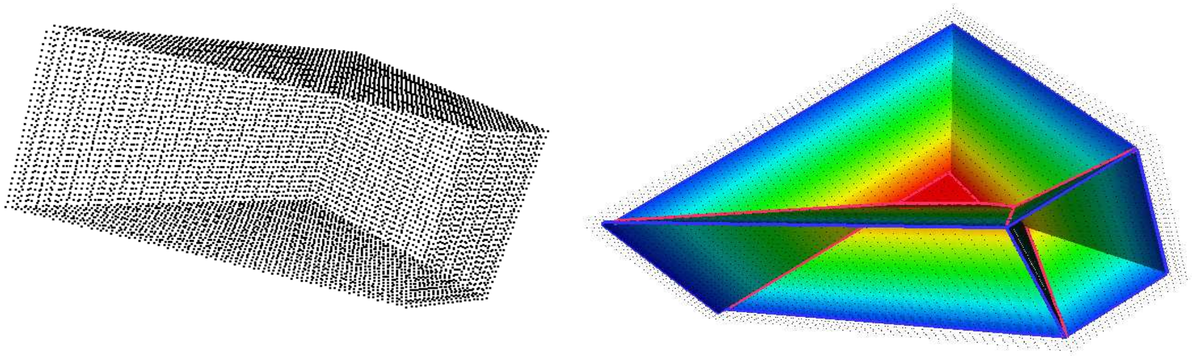
Context: 1st reconstruct a surface mesh from *unorganized* points, with a "minimal" set of assumptions:

the samples are nearby a "possible" surface

(thick volumetric traces not considered here).

Benefit: reconstruction across many types of surfaces.

Study 3D shape with minimal assumptions



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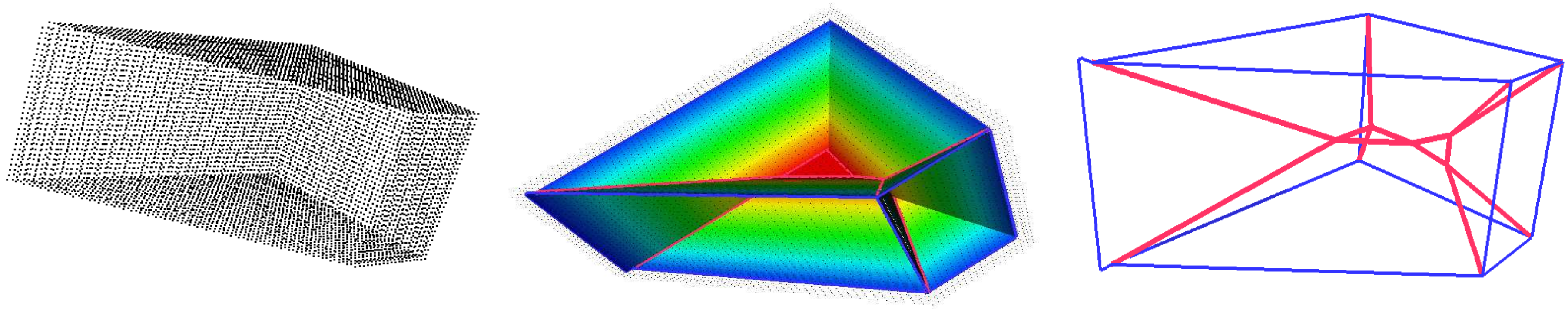

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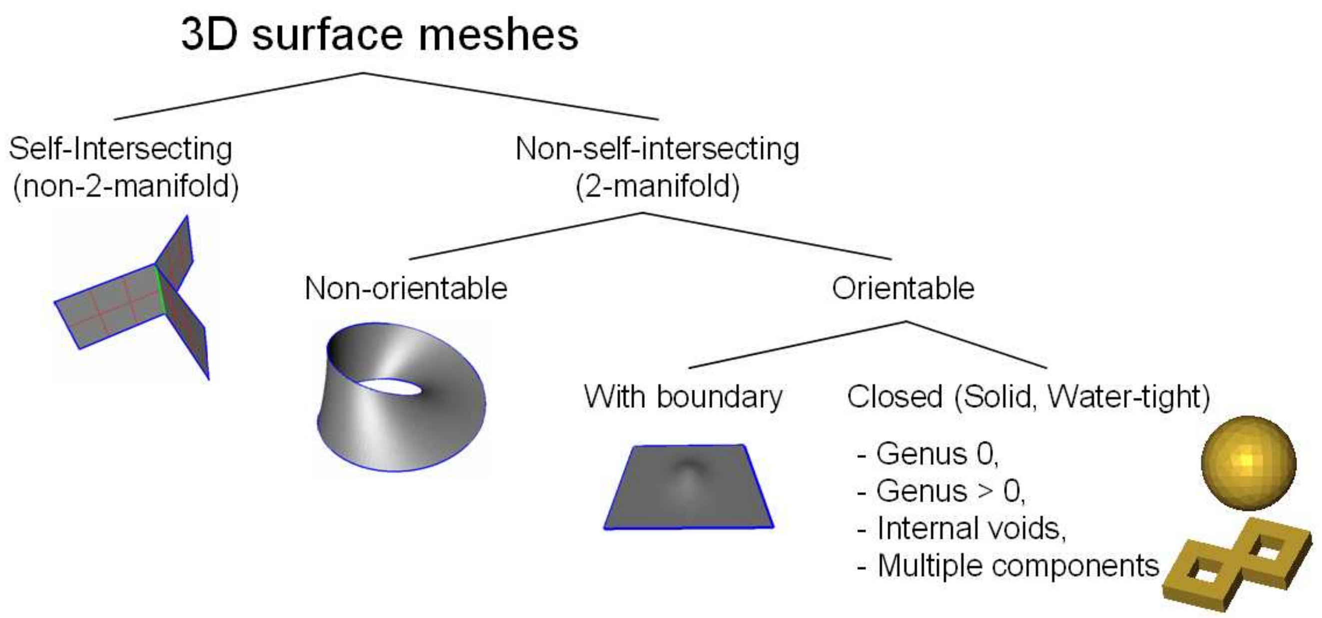
(thick volumetric traces not considered here).

Benefit: reconstruction across many types of surfaces.

Study shape with minimal assumptions

To find a *general* approach, applicable to various topologies, without assuming strong *input constraints*, *e.g.:*

- No surface normal information.
- Unknown topology (with boundary, for a solid, with holes, non-orientable).
- No a priori surface smoothness assumptions.
- Practical sampling condition: non-uniformity, with varying degrees of noise.
- Practical large input size (> millions of points).



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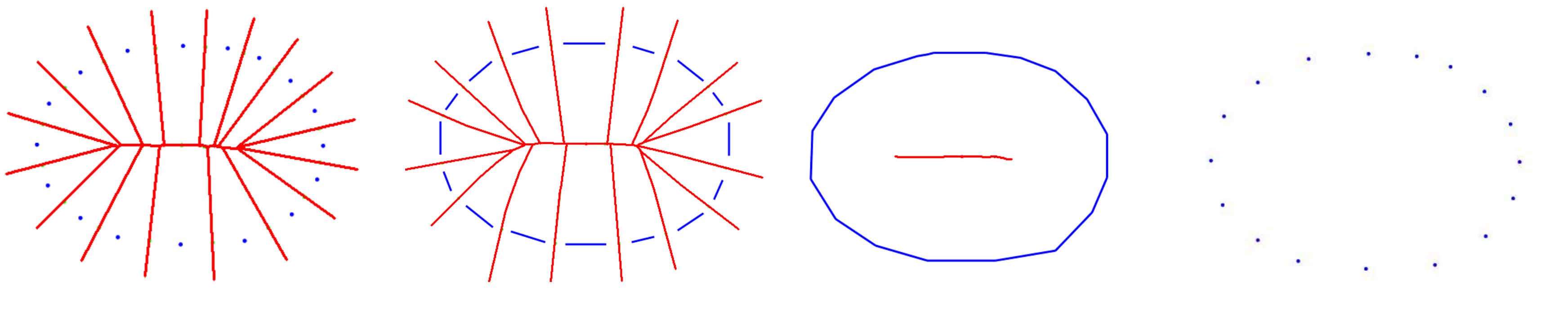
How: Overview of Our Approach (2D)

Not many clues from the assumed loose input constraints.

- Work on the shape itself to recover the sampling process.

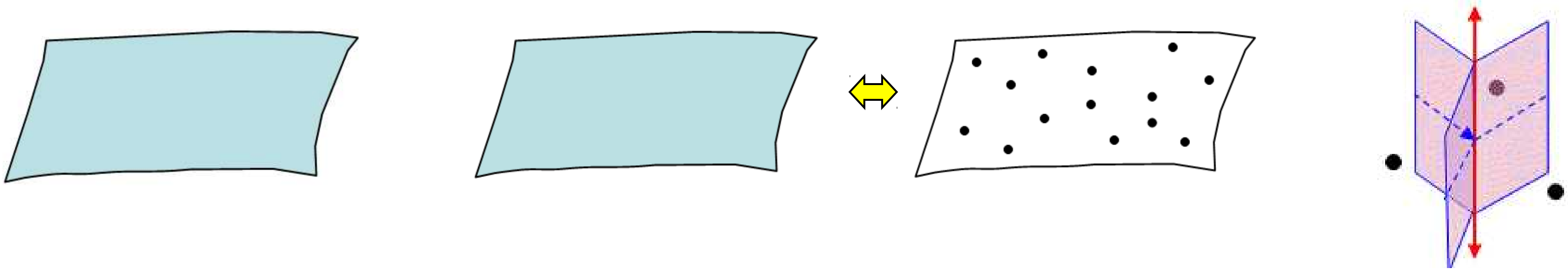


Key ideas:

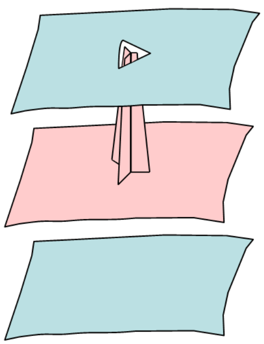
- Relate the sampled shape with the underlying (unknown) surface by a sequence of shape deformations (growing from samples).
- Represent (2D) shapes by their medial “shock graphs”. [Kimia *et al.*]
- Handle shock *transitions* across different shock topologies to recover gaps.

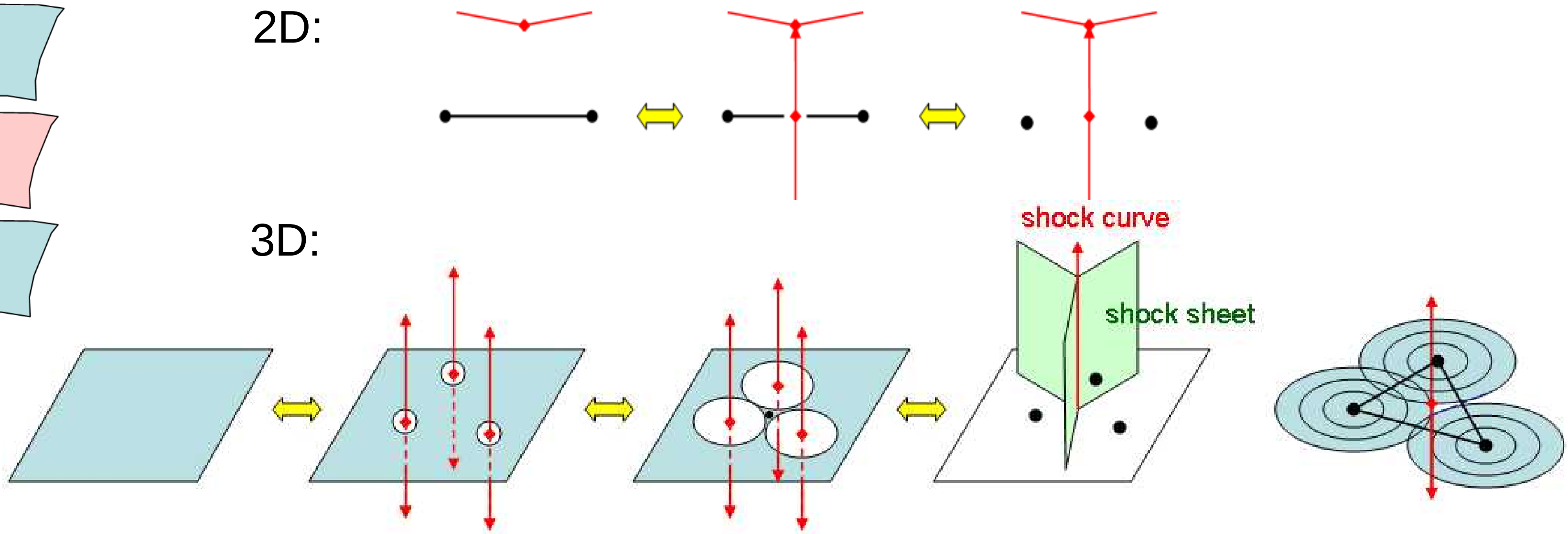
How: Sampling / Meshing as Deformations

Schematic view of sampling: infinitesimal holes grows, remaining are the samples.



We consider the removing of a patch from the surface as a Gap Transform.





Surface

Infinitesimal holes

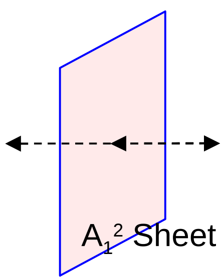
Holes merging, surface between holes degenerating into points

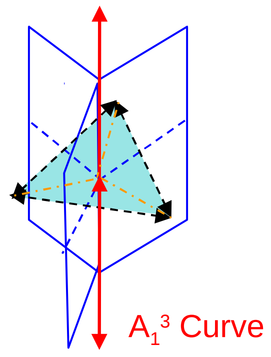
The shock curve with 3 shock sheets are used to recover gaps.

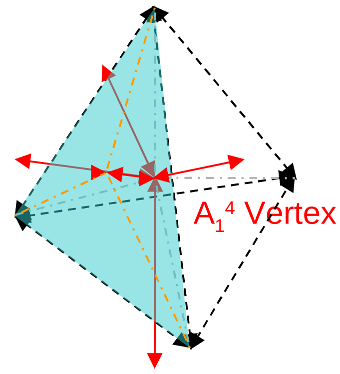
How: Sampling / Meshing as Deformations

Special case where input consists only of points (in 3D), then the Medial Scaffold consists of only:

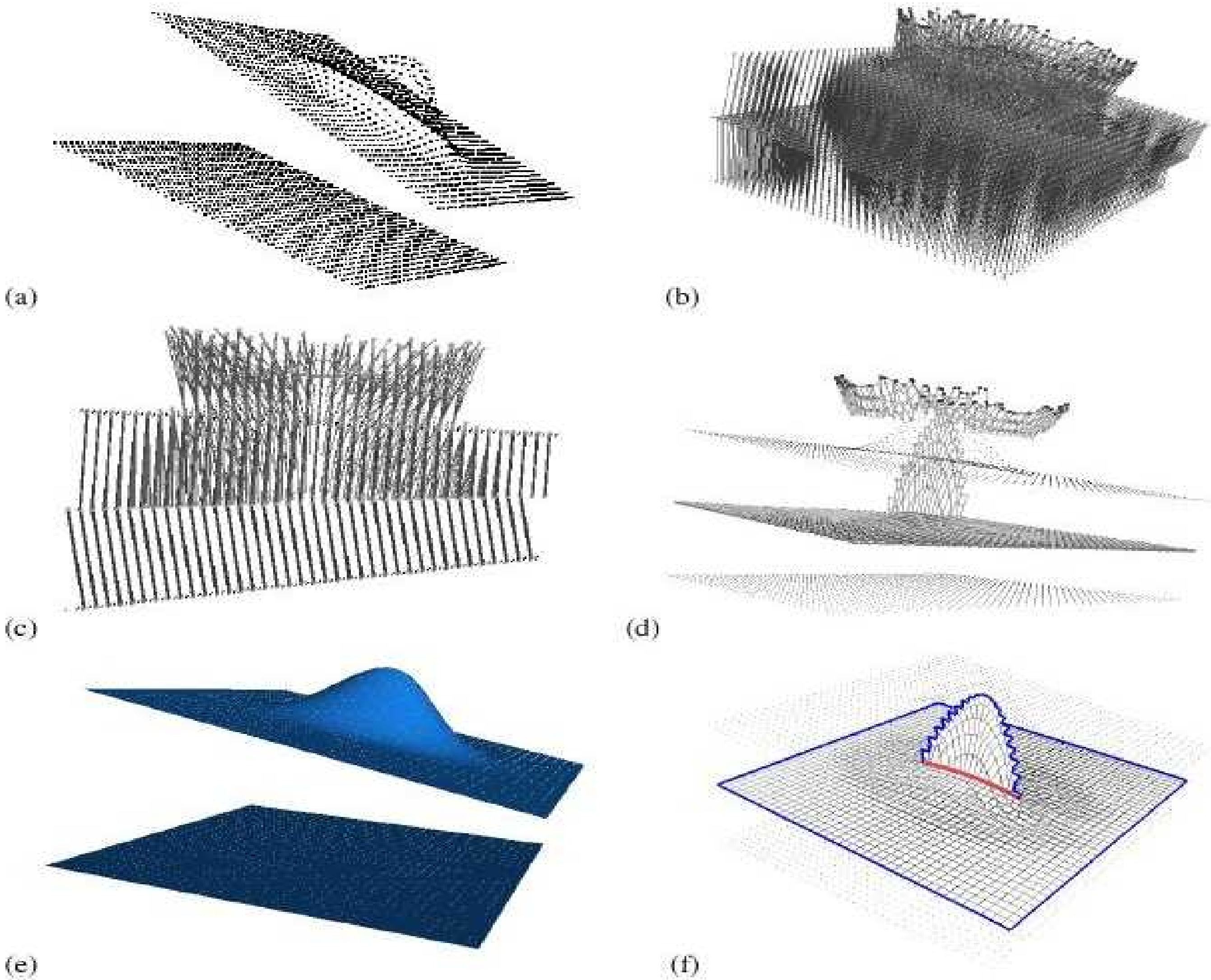
A_1^2 Sheets, A_1^3 Curves, A_1^4 Vertices.







How: Sampling / Meshing as Deformations



CVIU 2009, Chang, Fol Leymarie, Kimia.

How: Medial Scaffolds for **3D** Shapes

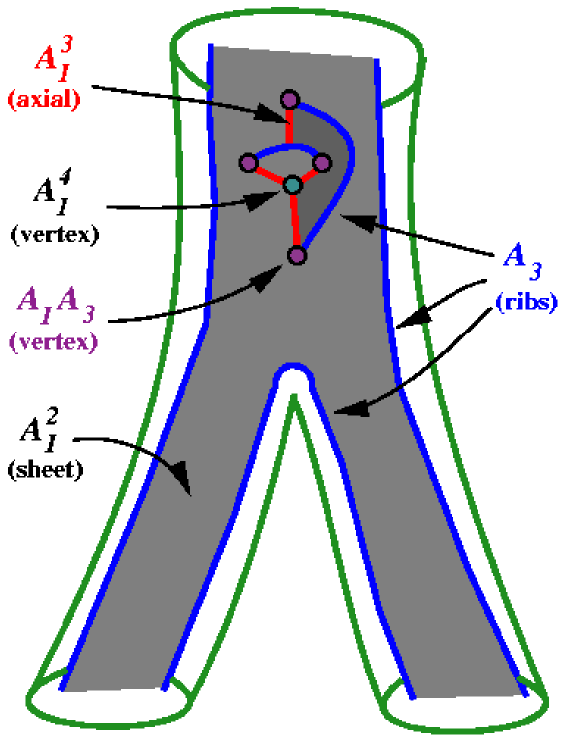
A graph structure for the 3D Medial Axis

Classify shock points into 5 general types,

and organized into a hyper-graph form

[Giblin&Kimia PAMI'04, Leymarie&Kimia PAMI'07]:

- Shock Sheet: A_1^2
- Shock Curves: A_1^3 (***Axial***), A_3 (***Rib***)
- Shock Vertices: A_1^4, A_1A_3



$\mathbf{A}_k^n$: contact (max. ball) at n distinct points, each with $k+1$ degree of contact.

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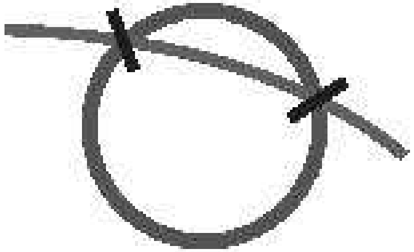
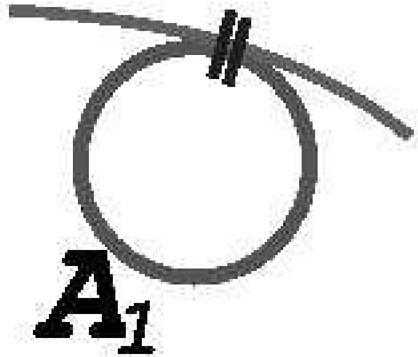
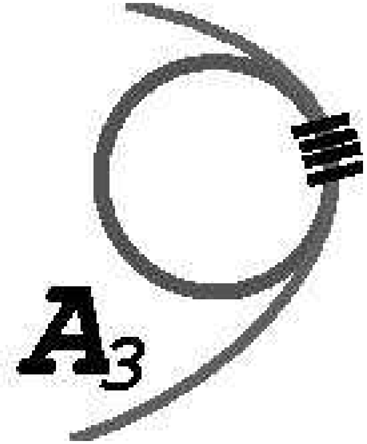
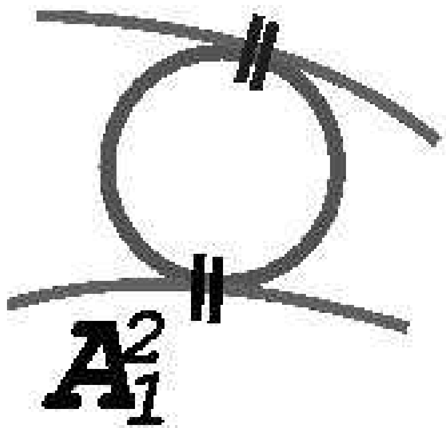
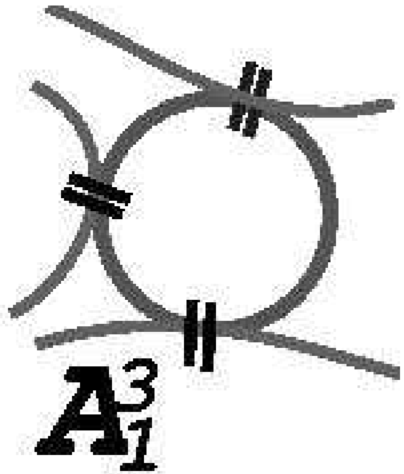
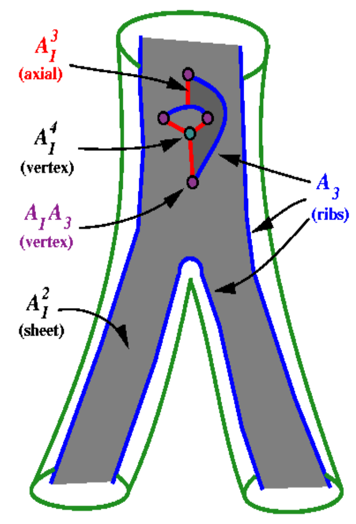






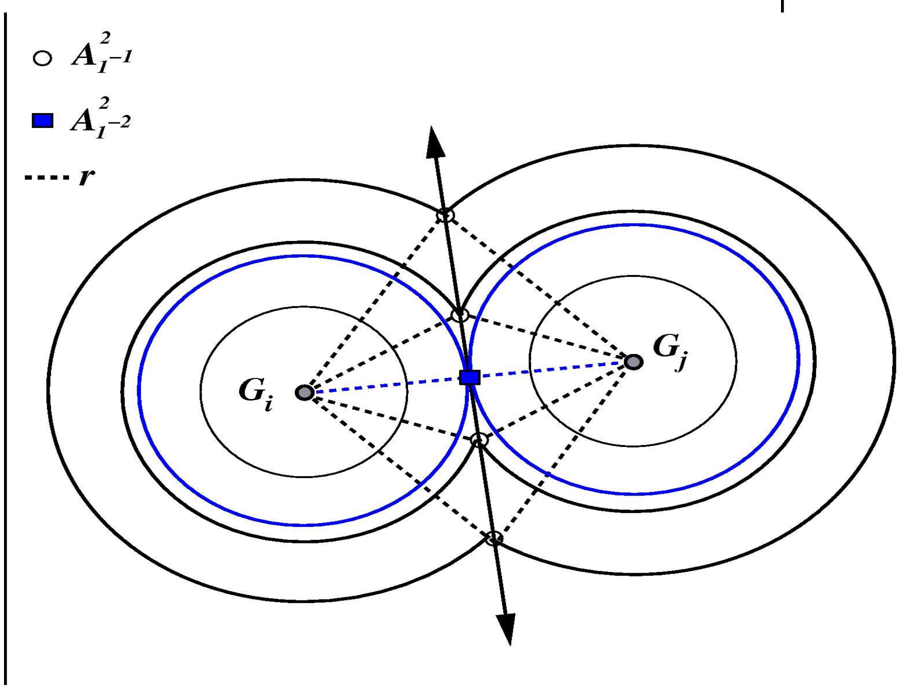


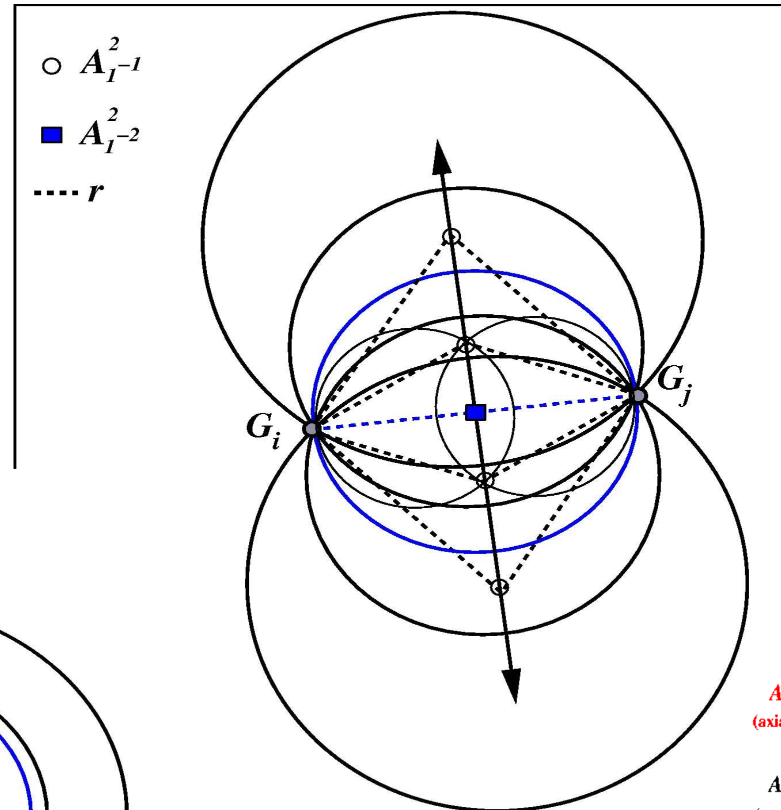


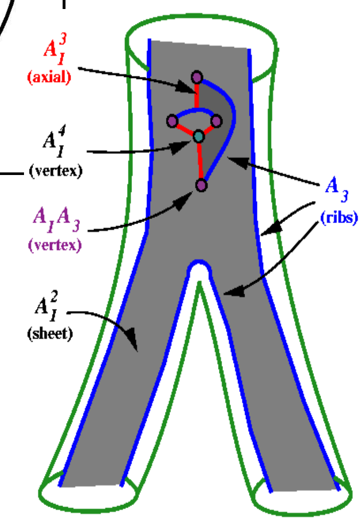


How: Medial Scaffolds for **3D** Shapes

A graph structure for the 3D Medial Axis







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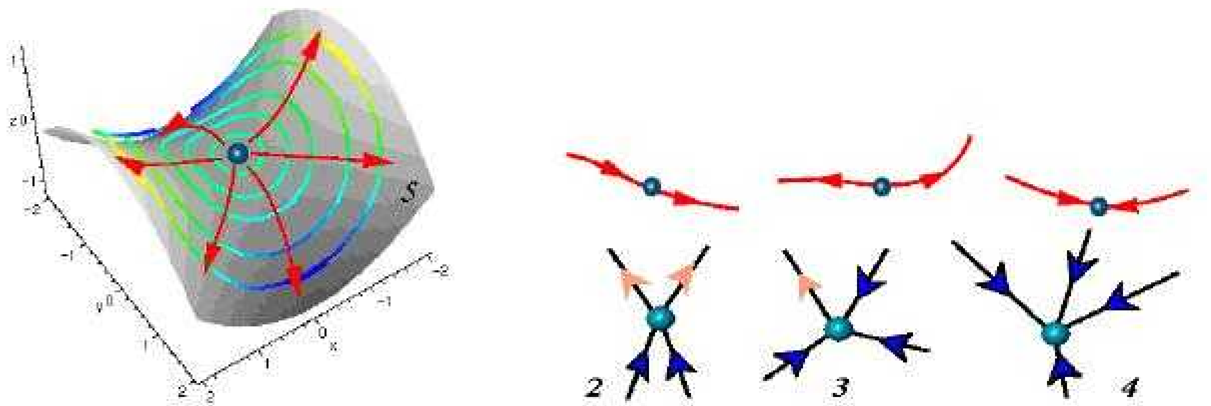
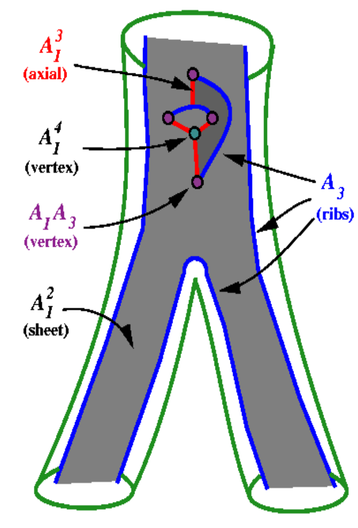


(a) (b)

Figure 1.3: Types of flows along $\mathcal{MA}$ structures. (a) For $\mathcal{MA}$ sheets, the flow is generally initiated at a single point, and the sheet is grown outward and radially from that point. (b) At the top are shown the typical flows along $\mathcal{MA}$ curves, *i.e.*, regular, initial and final. At the bottom are shown the typical sets of inward and outward flows (along $\mathcal{MA}$ curves) at $\mathcal{MA}$ vertices where the number of inward flows is indicated.



How: Medial Scaffolds for **3D** Shapes

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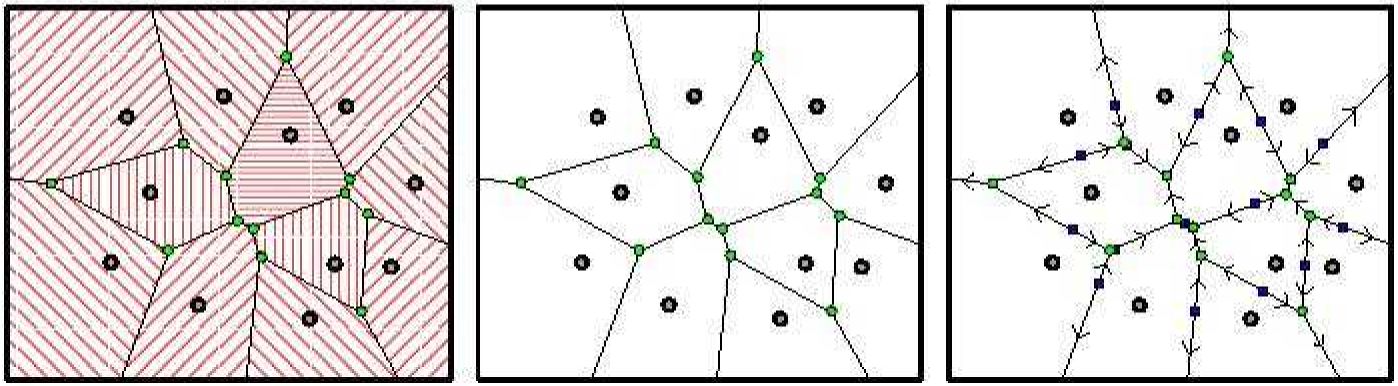
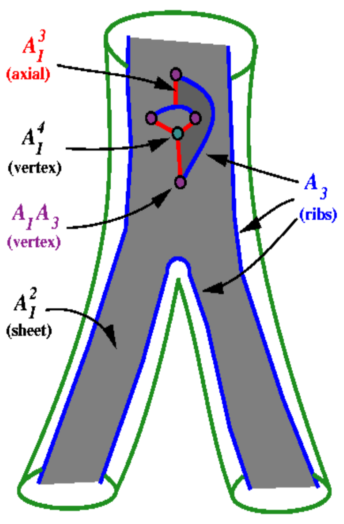
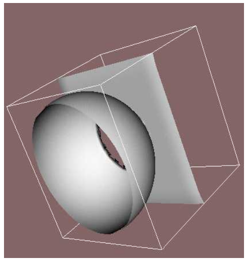


(a) $\mathcal{VD}$ (b) $\mathcal{MA}$ (c) $\mathcal{SC}$

Figure 3.14: Example of a 2D Voronoi diagram (a), medial axis (b), and shock scaffold (c) for a set of eleven point generators (large grey disks) in the plane. Voronoi or shock vertices are indicated as smaller green disks, Voronoi edges or shock curves are drawn as straight lines, Voronoi regions are hashed in **red**. In (c), $A_1^2\!-\!2$ shock sources are indicated as **blue** squares. In (b) we see that the $\mathcal{VD}$ minus the interior of its Voronoi regions coincides with the $\mathcal{MA}$.



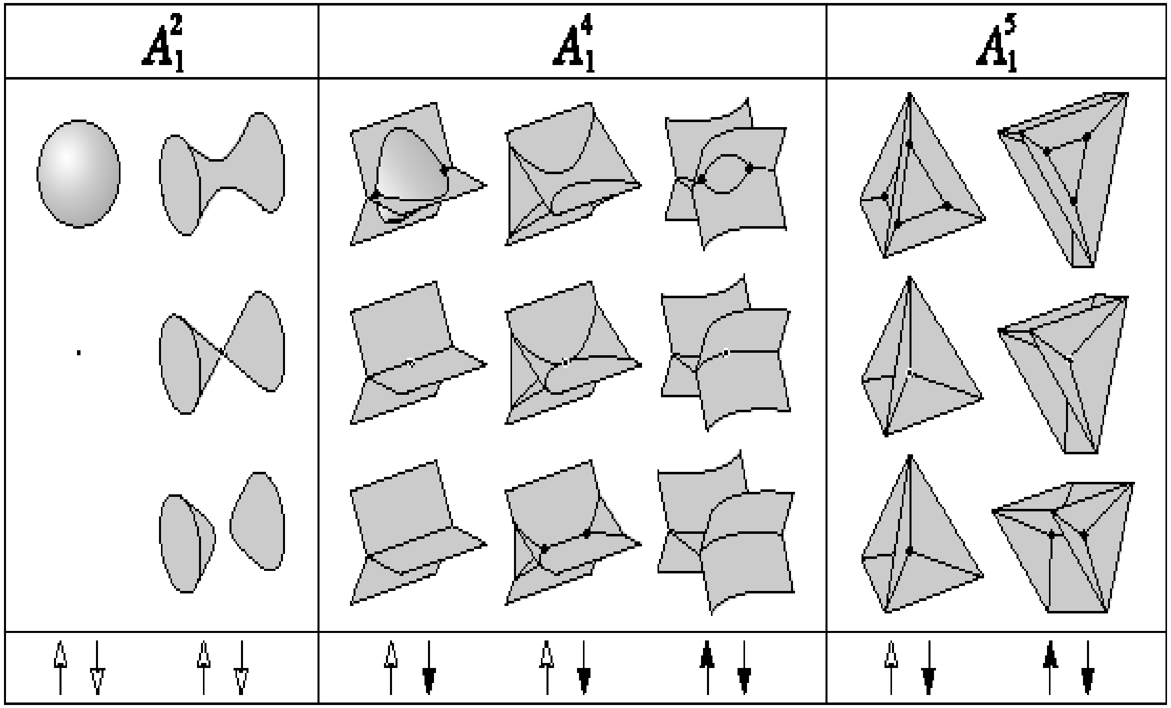
Transitions of the 3D graph structure



Study the topological events of the graph structure under perturbations and shape deformations.

Singularity theory (Arnold *et al.*, since the 1990's):

In 3D, 26 topologically different perestroikas of linear shock waves.



"Perestroikas of shocks and singularities of minimum functions"
I. Bogaevsky, 2002.

Transitions of the 3D graph structure

Study the topological events of the graph structure under **perturbations** and **shape deformations**.

Transitions of the MA (Giblin, Kimia, Pollit, PAMI 2009):

Under a 1-parameter family of deformations, only **seven transitions** are relevant.

Transition	Collision of Types
A_1^4	A_1^3 - A_1^3
A_1^5	A_1^4 - A_1^4, A_1^4 - A_1^3
A_5	A_1A_3 - A_1A_3, A_3 - A_3
$A_1A_3 - I$	A_1A_3 - A_1A_3
$A_1A_3 - II$	A_1A_3 - A_1A_3, A_1^3 - A_3
$A_1^2A_3 - I$	A_1^4 - A_1A_3
$A_1^2A_3 - II$	A_1^3 - A_1A_3

Transitions of the 3D graph structure

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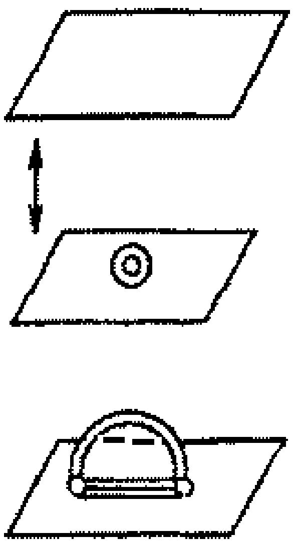

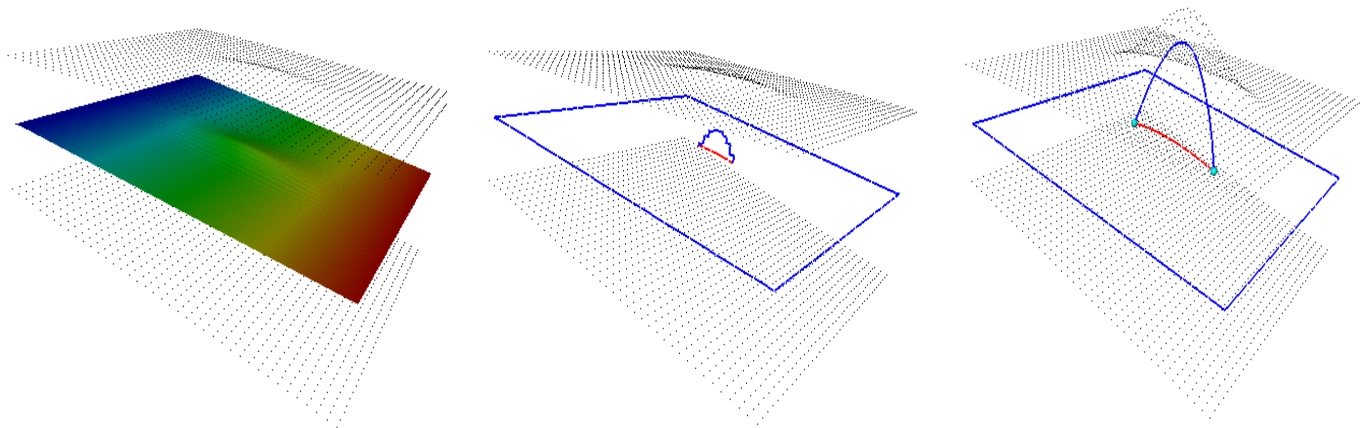

A_1A_3-I (protrusion-like, Leymarie, PhD, 2002)

Transitions of the 3D graph structure

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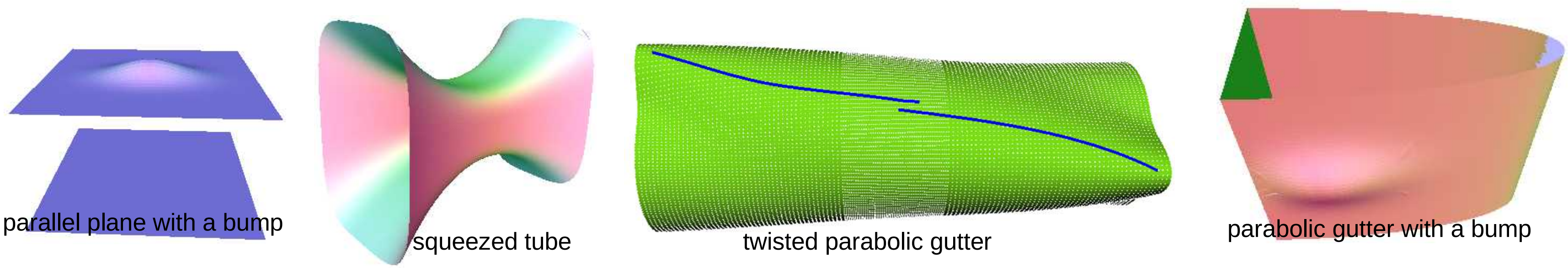
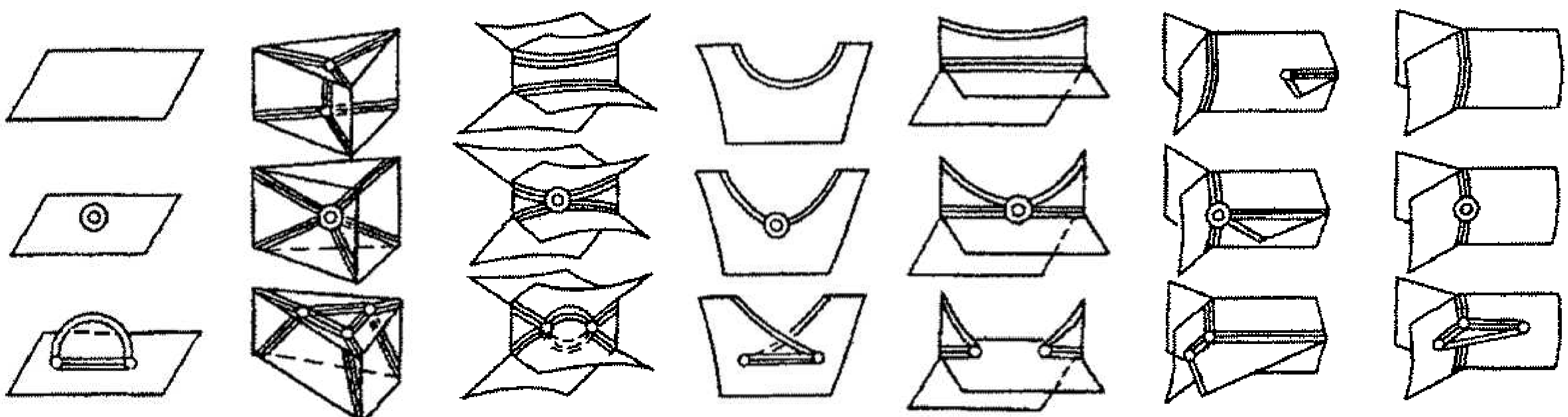


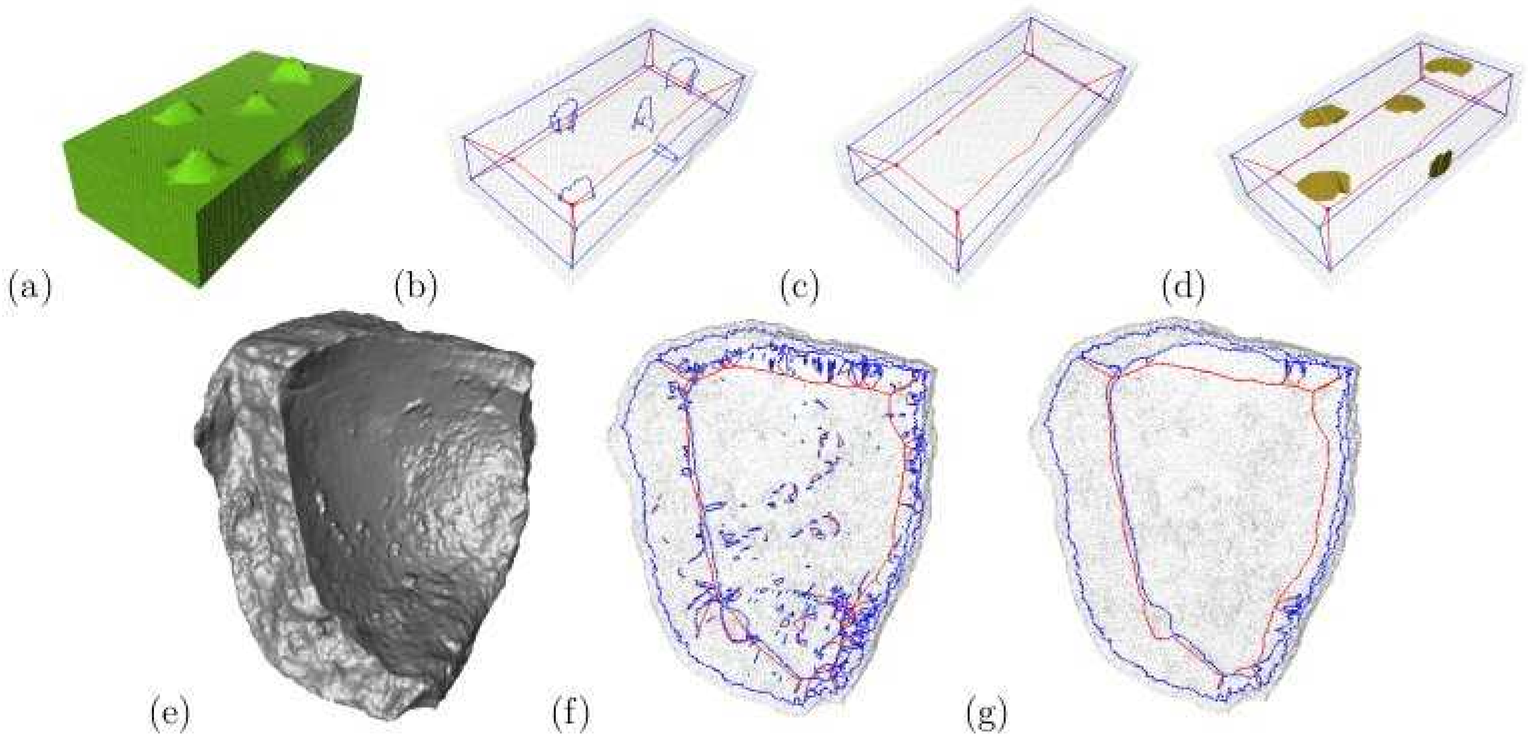


A_1A_3-I **A_1^5** **A_1^4** **A_5** **A_1A_3-II** **$A_1^2A_3$-I** **$A_1^2A_3$-II**

Total of 11 cases for regularization across transitions (M.C. Chang *et al.*)

Transitions of the 3D graph structure

Study the topological events of the graph structure
under **perturbations** and **shape deformations**.



Towards surface regularisations via transitions (Leymarie, Giblin, Kimia, 2004)

Transitions of the 3D graph structure

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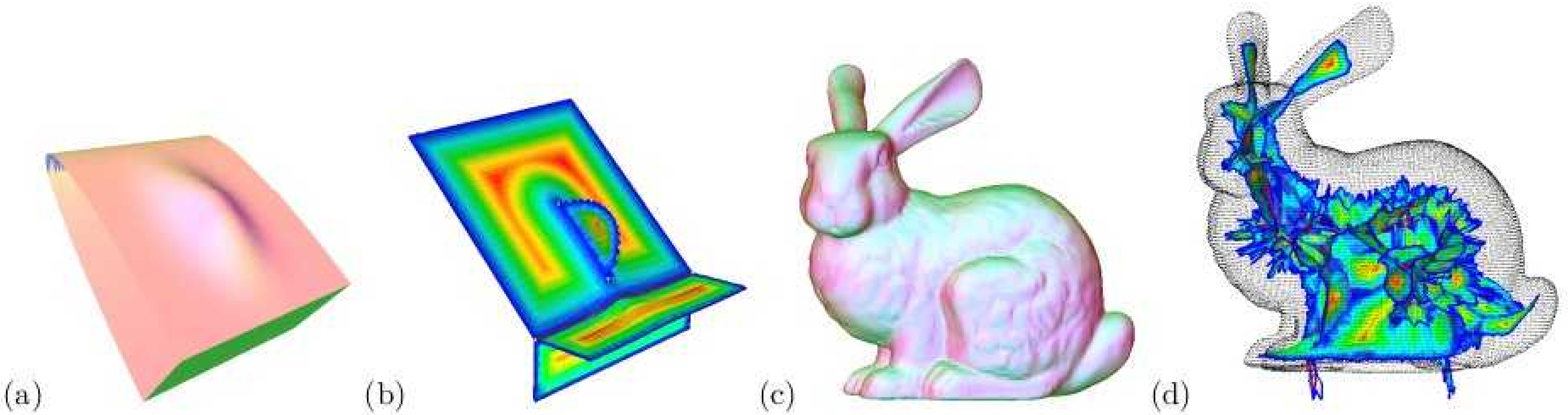

Capture transitions via geodesy on MA (Chang, Kimia, Leymarie, on-going)

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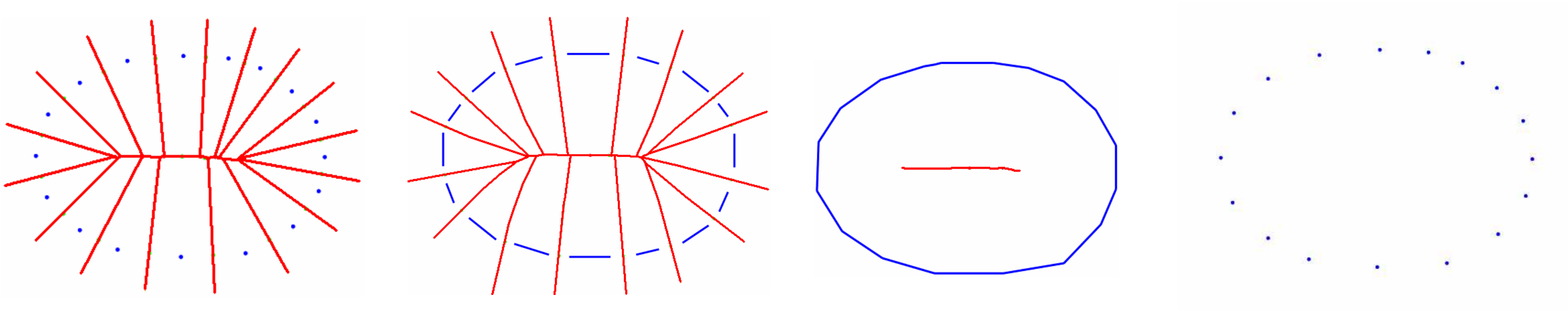
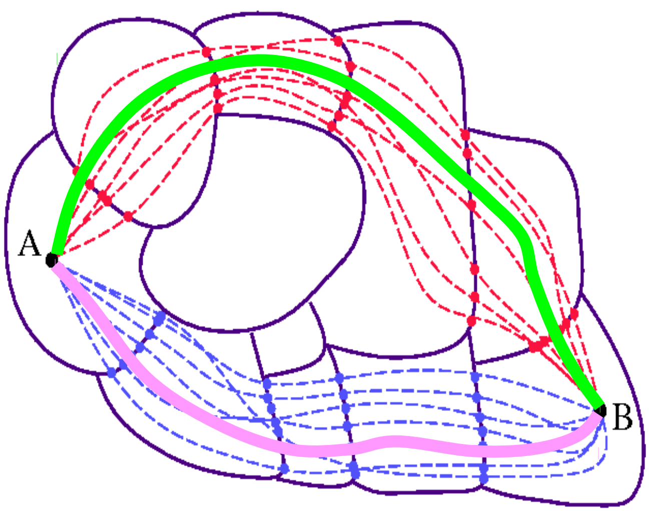
How: Organise/Order Deformations (**2D**)



A B

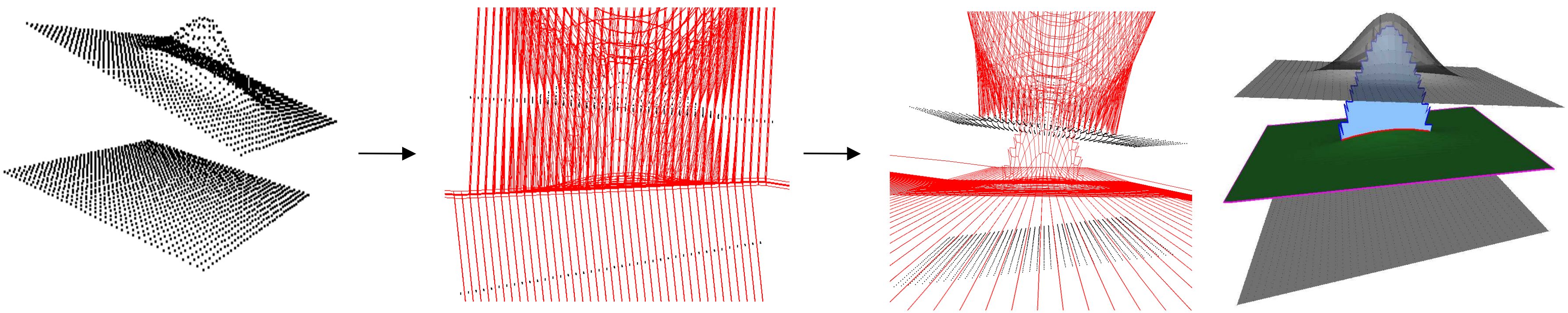


Deformation in shape space

NB: A & B share **object** symmetries. Symmetries due to the **sampling** need to be identified.

How: Organise/Order Deformations (**3D**)

- Recover a mesh (connectivity) structure by using Medial Axis transitions modelled via the Medial Scaffold (MS).
  - Meshing as *shape deformations* in the ‘*shape space*’.
- The Medial Scaffold of a point cloud includes both the symmetries due to sampling and the original object symmetries.
  - Rank order Medial Scaffold *edits* (gap transforms) to “segregate” and to simulate the recovery of sampling.



Sampling recovery

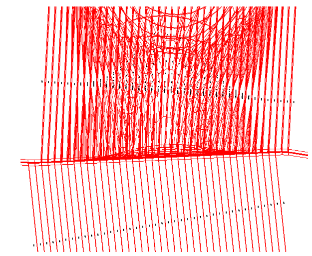
Object symmetry

Meshed Surface + Organized MA

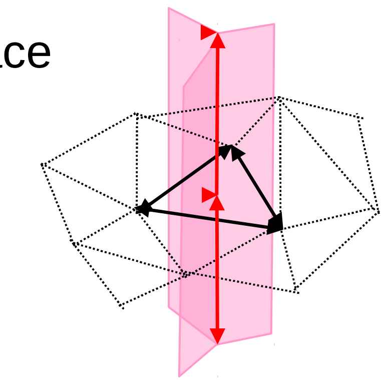
Shock Segregation [Leymarie, PhD’03], Surface reconstruction [CVIU'09]

Algorithmic Method

- Consider Gap Transforms on *all* A_1^3 shock curves in a ranked-order fashion:
  - best-first (greedy) with error recovery.
- Cost reflects:
  - Likelihood that a shock curve (triangle) represents a surface patch.
  - Consistency in the local context (neighboring triangles).
  - Allowable (local surface patch) topology.

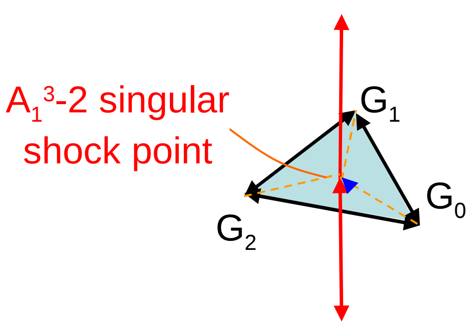


A_1^3 shock curve

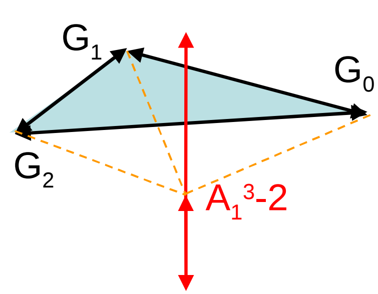


Three A_1^2 shock sheets

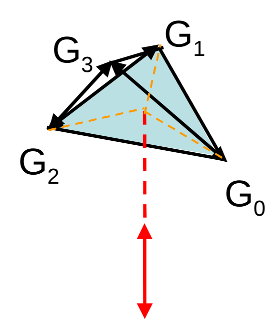
3 Types of A_1^3 shock curves (dual Delaunay triangles):
Represented in the MS by “singular shock points” (A_1^3-2)



Type I



Type II



Type III

(unlikely to be correct candidate)

Algorithmic Method

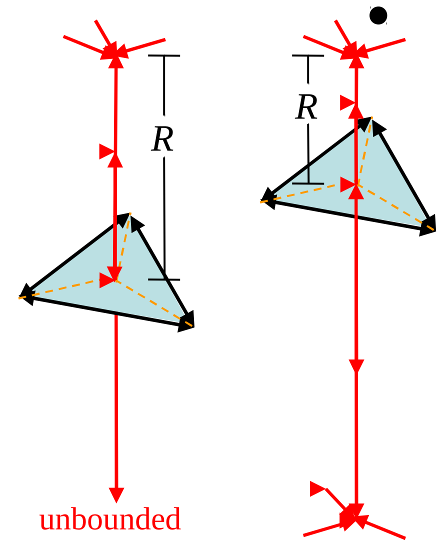
How we **order gap transforms**:

- Favor small “compact” triangles.
- Favor recovery in “nice” (simple) areas, *e.g.*, away from ridges, corners, necks.
- Favor simple local continuity (similar orientation).
- Favor simple local topologies (2D manifold).
- BUT: allow for error recovery!

Ranking Isolated Shock Curves (Triangles)

Triangle geometry:

$$
\begin{aligned}
D &= \max(d_1, d_2, d_3) \\
P &= d_1 + d_2 + d_3 \\
m &= (d_1 + d_2 - d_3)(d_3 + d_1 - d_2)(d_2 + d_3 - d_1) \\
A &= \sqrt{(P \cdot m)/16} \quad \text{(Heron's formula)} \\
C &= 4\sqrt{3} \cdot A/(d_1^2 + d_2^2 + d_3^2), \quad \text{(Compactness, Gueziec's formula, } 0<C<1)
\end{aligned}
$$



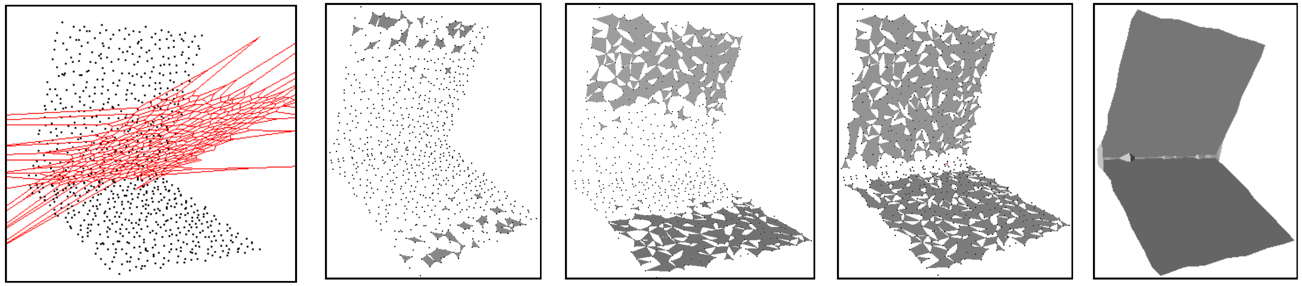
The side of smaller shock radius is more salient.

Cost: favors *small compact* triangles with large shock radius *R*.

$$
\rho_1 = \begin{cases} \frac{P}{R} \cdot \frac{1}{C^2}, & \text{if } D < d_{\max} \\ \infty, & \text{if } D \geq d_{\max} \end{cases}
$$

R: minimum shock radius

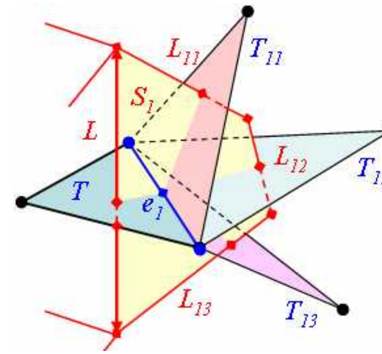
d_{max}: maximum expected triangle, estimated from d_{med}



Surface meshed from confident regions toward the sharp ridge region.

Cost Reflecting Local Context & Topology

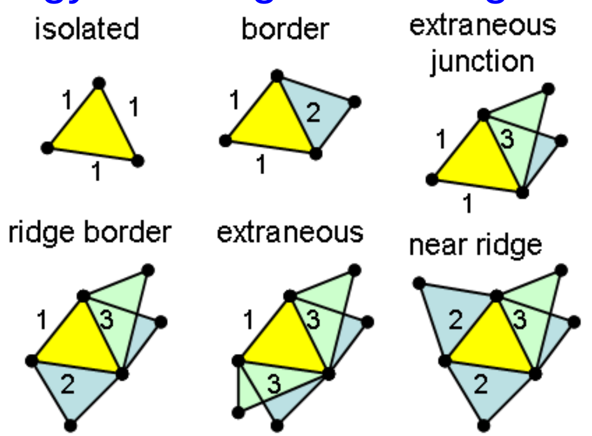
Cost to reflect smooth continuity of edge-adjacent triangles:



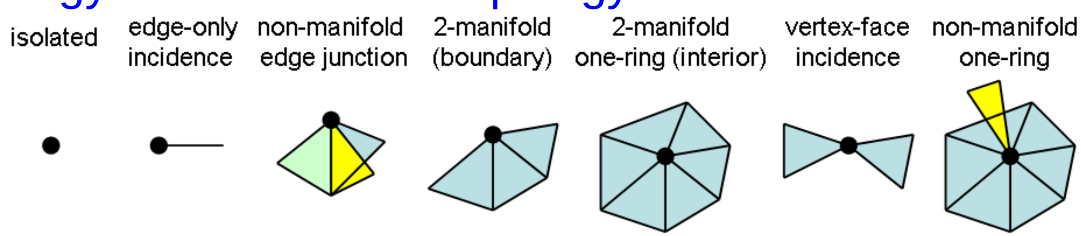
$$\rho_2 = \frac{d}{R} \cdot \frac{1}{C^2} \cdot f(\theta)\,,$$

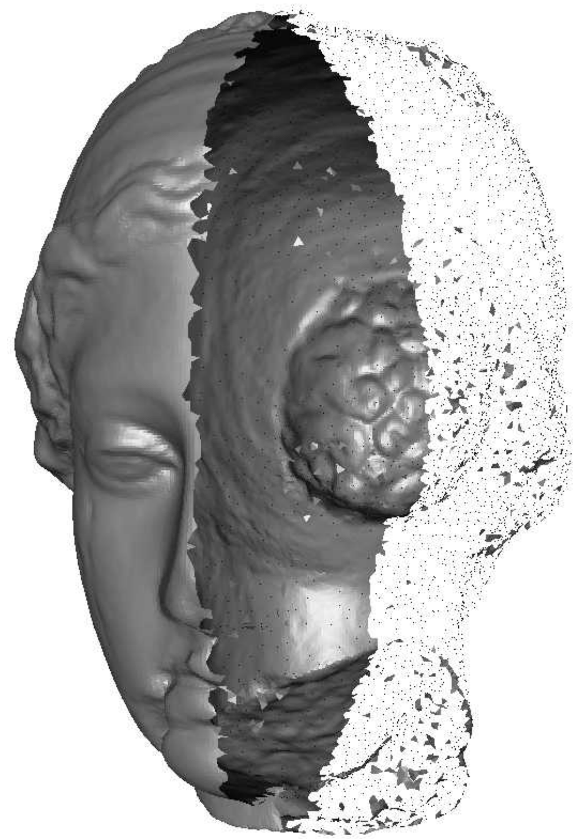
$$f(\theta) = [\exp^{\theta} - 1]^2 - 1 \quad \begin{cases} \theta = 0,\, f(\theta) = -1 \\ \theta = 40^\circ,\, f(\theta) \simeq 0 \\ \theta = 80^\circ,\, f(\theta) \simeq 8.24 \end{cases}$$

Typology of triangles sharing an edge:



Typology of mesh vertex topology





Point data courtesy of Ohtake *et al*.

Strategy in the Greedy Meshing Process

Problem: Local ambiguous decisions → errors.

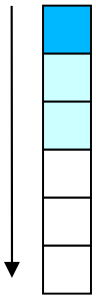
Solutions:

- Multi-pass greedy iterations

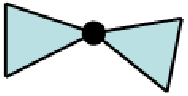
  First construct confident surface triangles without ambiguities.

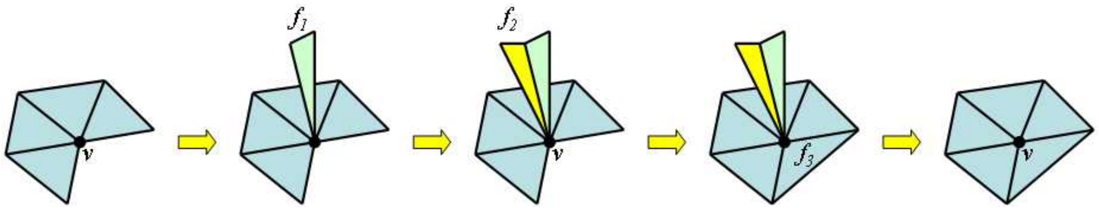
- Postpone ambiguous decisions
  - Delay related candidate Gap Transforms close in rank, until additional supportive triangles (built in vicinity) are available.
  - Delay potential topology violations.
- Error recovery
  - For each Gap Transform, re-evaluate cost of both related *neighboring (already built)* & *candidate* triangles.
  - If cost of any existing triangle exceeds top candidate, undo its Gap Transform.

Queue of ordered triangles



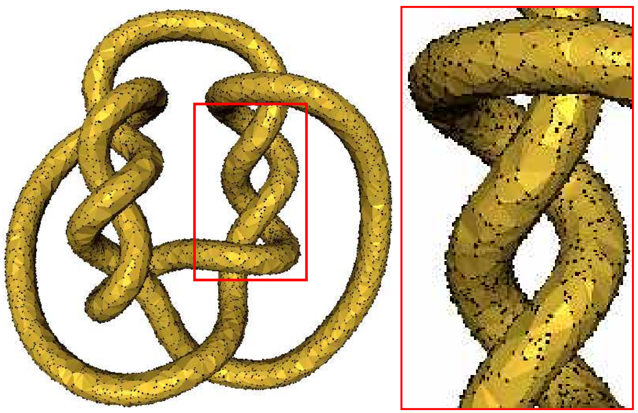
vertex-face incidence

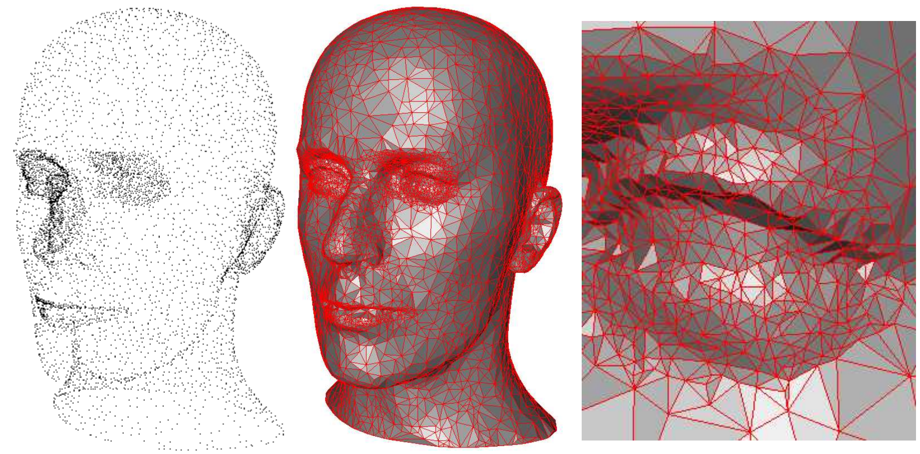




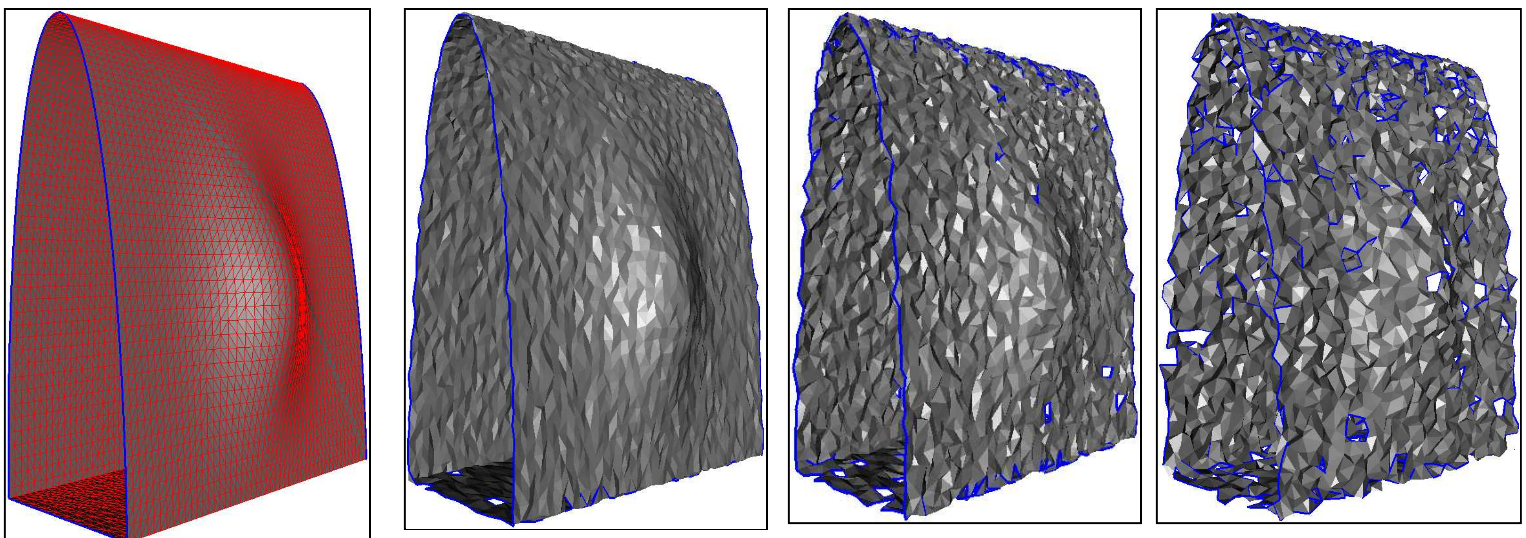
Dealing with sampling quality

Input of non-uniform and low-density sampling:





Response to additive noise:



50% 100% 150%

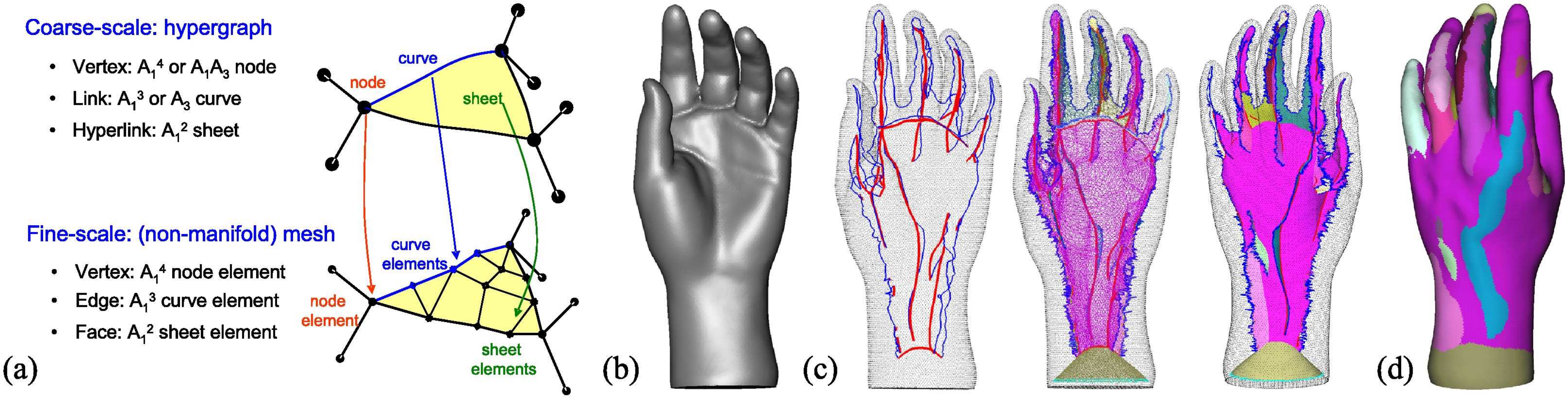
Outline

Background

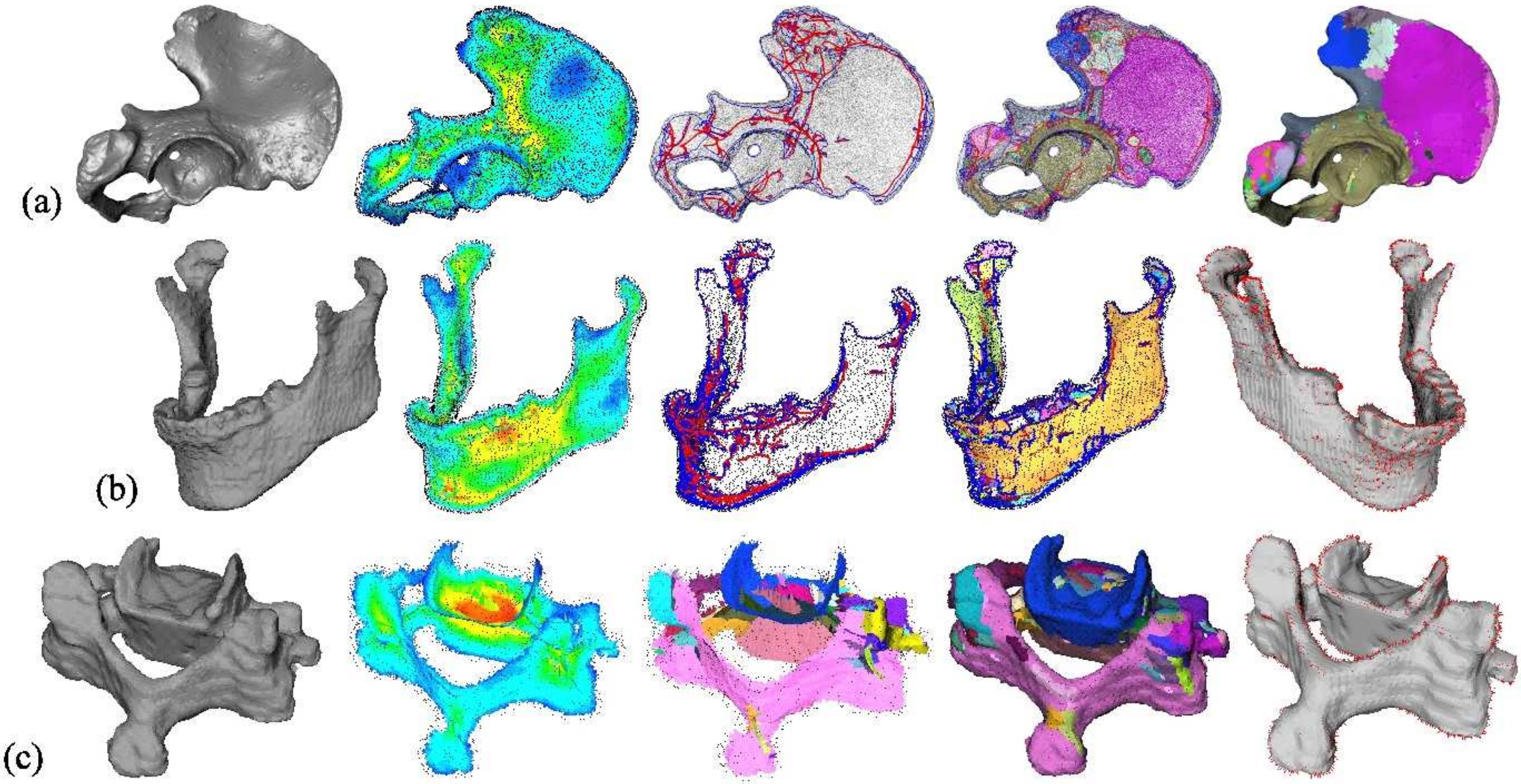
Method and some algorithmic details

Applications

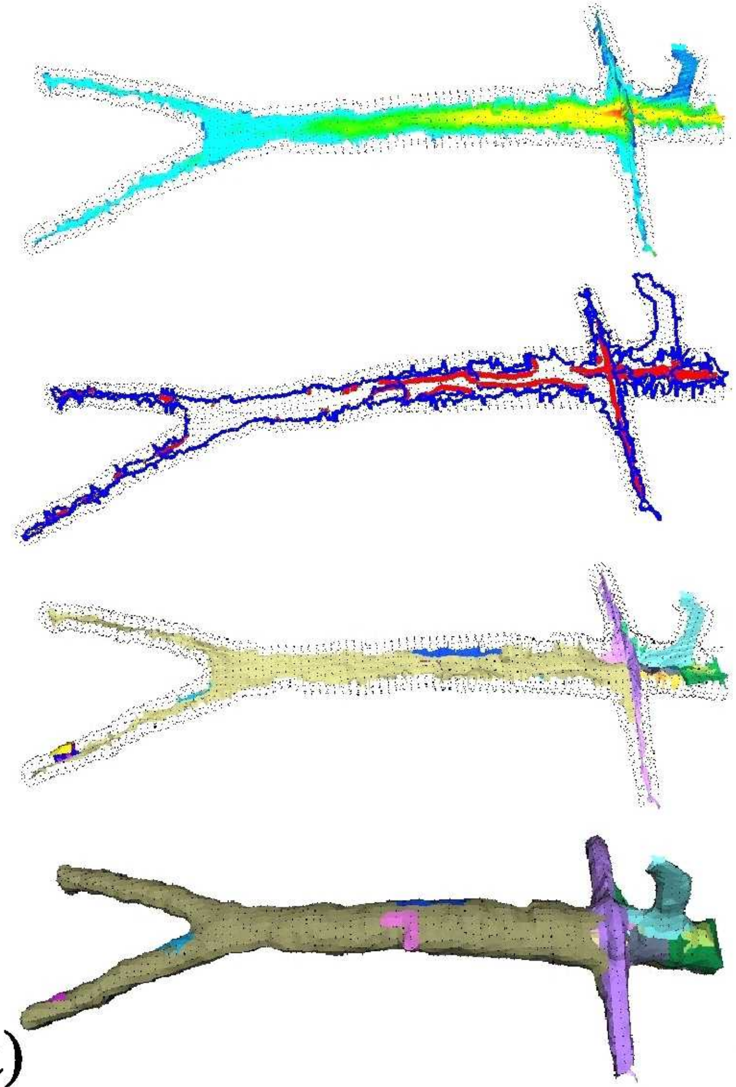
From Fine to Coarse Scales



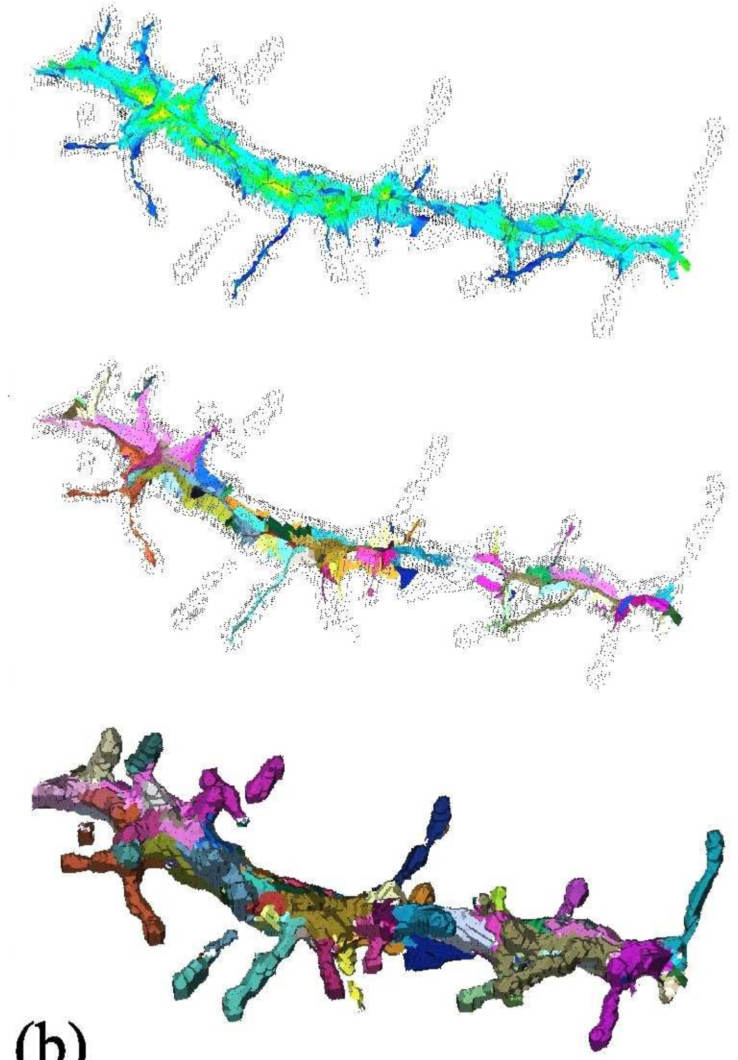
Bone shape study



3D Tubular & Branching Shapes

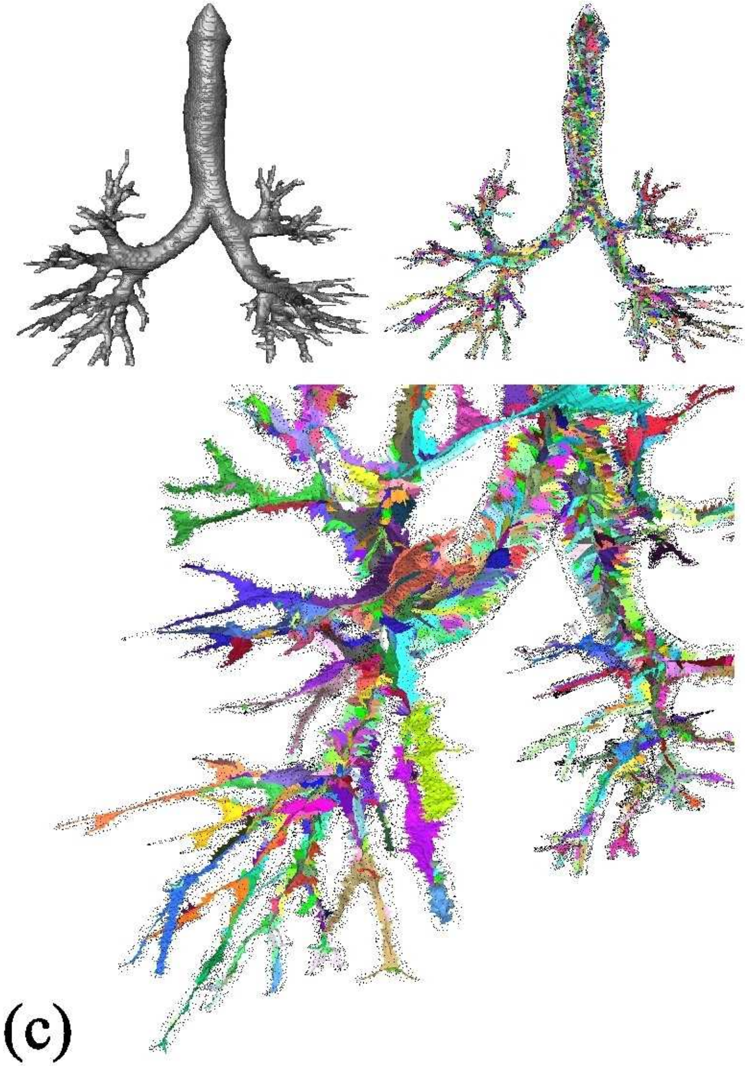

(a)

3D Tubular & Branching Shapes



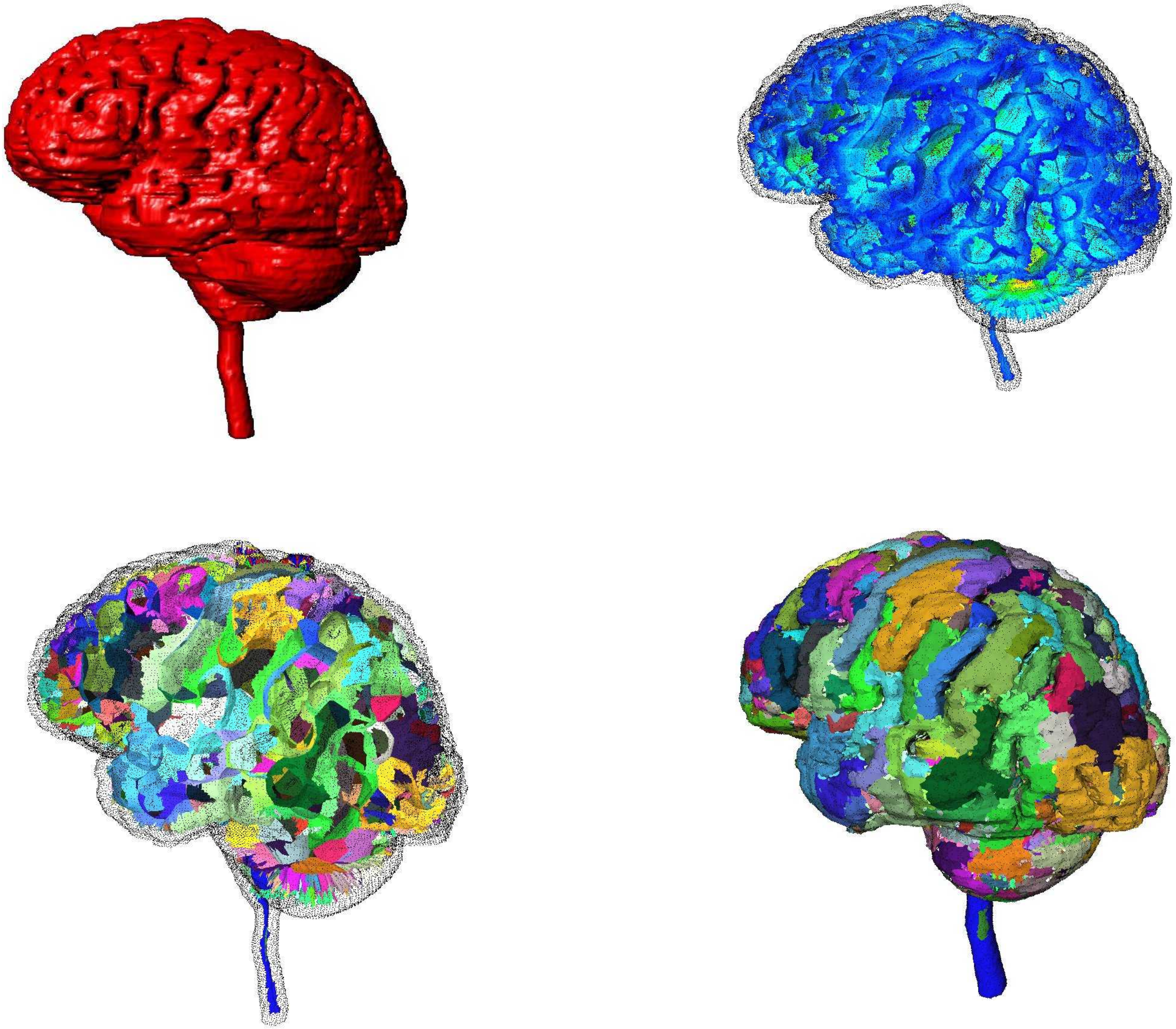
(b)

3D Tubular & Branching Shapes

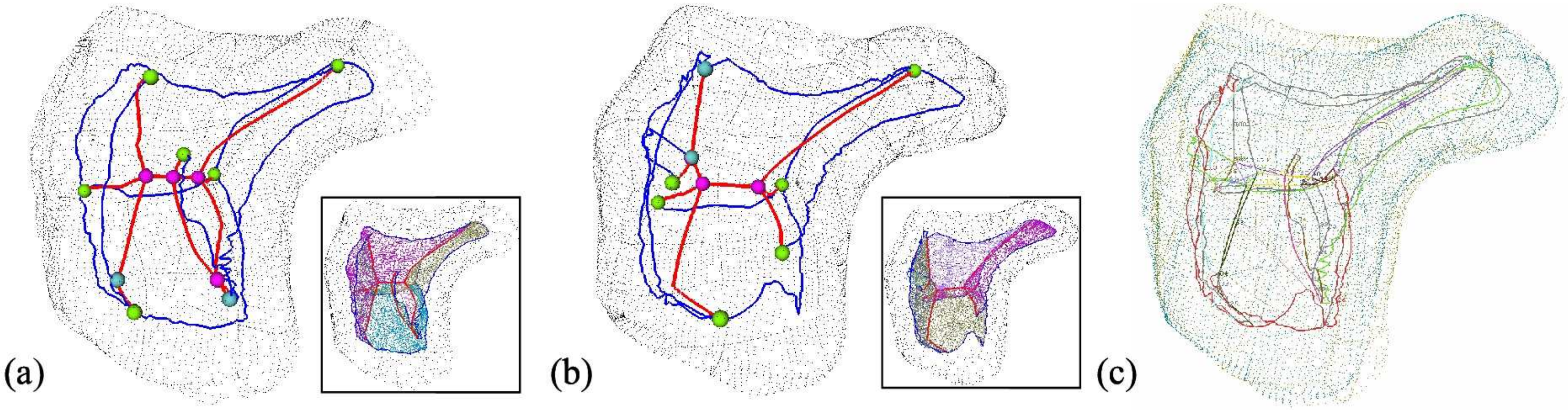


(c)

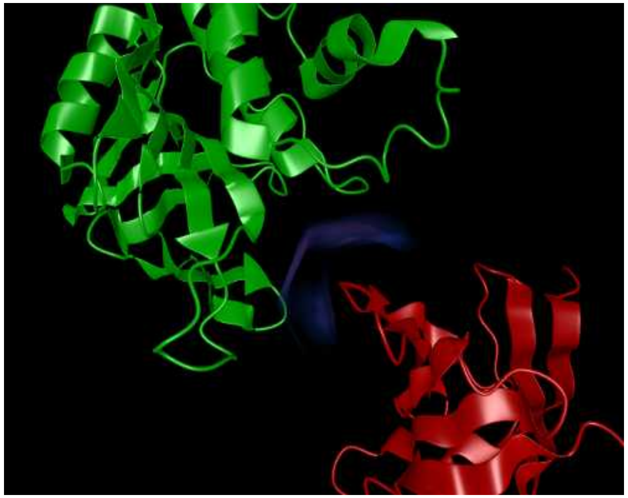
3D Convoluted Shapes: Brains

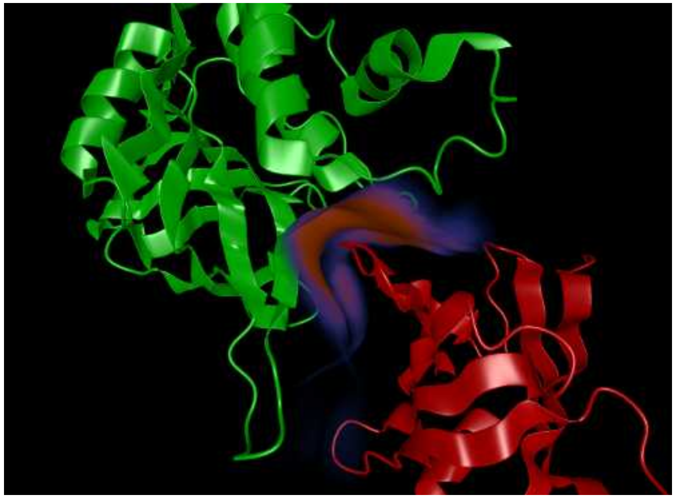


3D Shape Matching/Registration



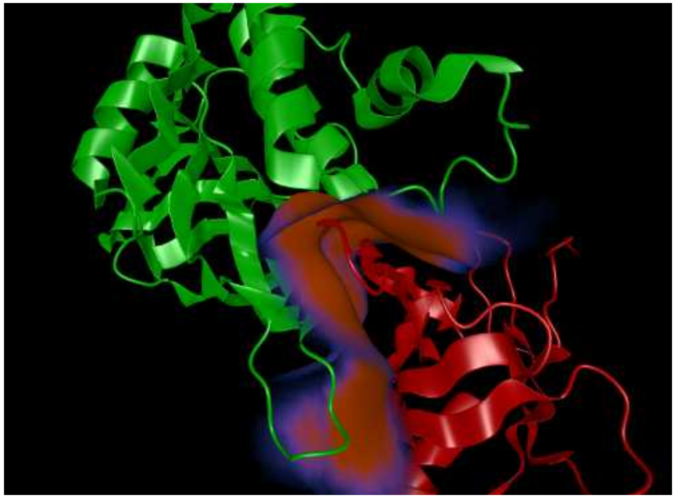
3D Shape in Molecular biochemistry





FoldSynth project: Docking

www.foldsynth.com



Outline

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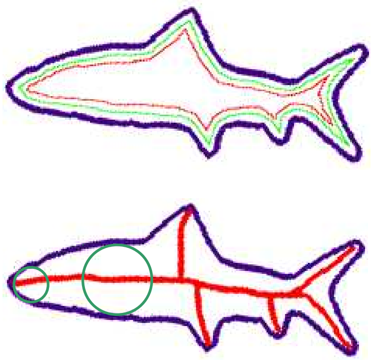
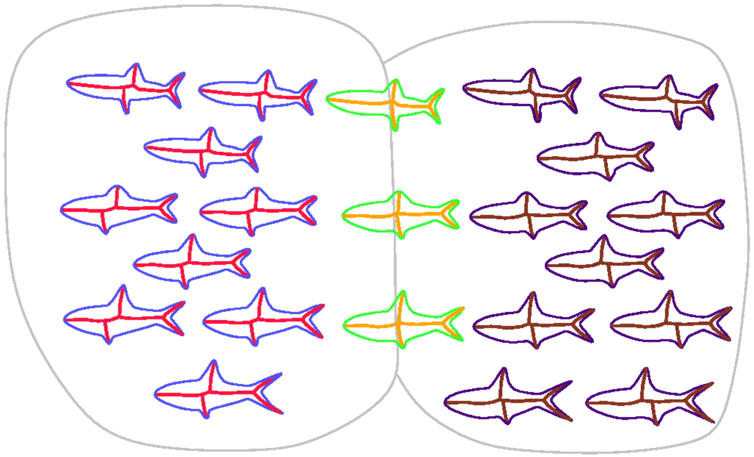
…

Conclusions

Next: 3D Shape Deformation

- [Kimia *et al.*] represent shape as a member of an equivalent class ('shape cell'), each defined as the set of shapes sharing a common shock graph (in 3D, Medial Scaffold) topology.





- Link this to **Information Models**: incorporation of human expert knowledge; e.g. in building taxonomies.
- Statistical analysis; definition of classes; distribution of features.
- Combine **exterior** with interior scaffolds.

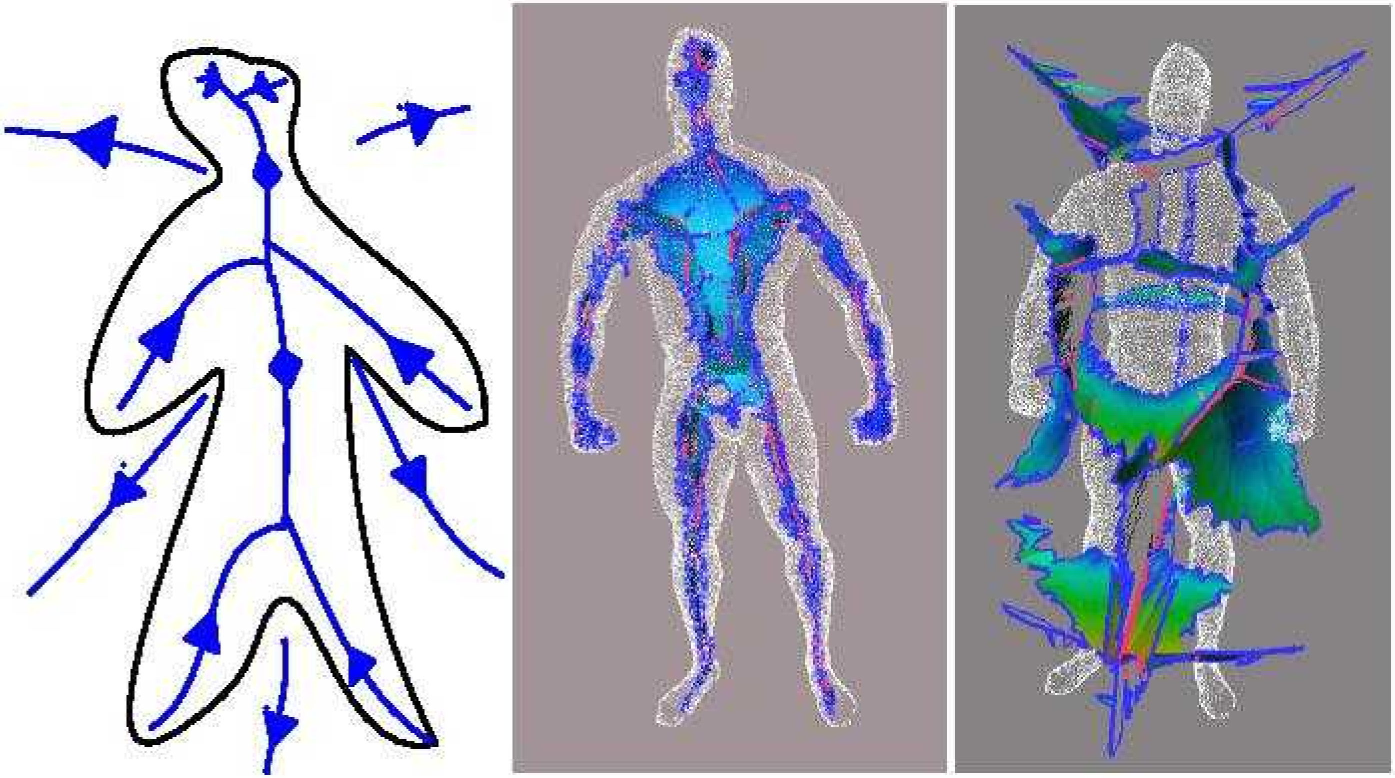


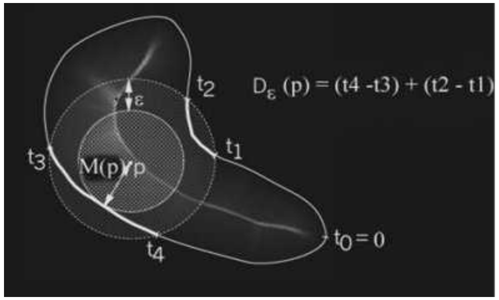
Figure 1.1: (a) Adapted from [34, Fig. 29. p. 252]: An example of an anthropomorphic figure and a sketch of its (2D) medial axis ($\mathcal{MA}$) representation. Flows indicating directions of increasing radii values are indicated by arrows. Initial flows pointing in two opposite directions are indicated by a diamond shape; these map to necks of the shape. Note how tips of branches relate to convex extrema of the boundary, for the interior part of the $\mathcal{MA}$, and concave extrema, for the exterior part. (b) and (c) A preview of the *shock scaffold*, a structure organizing the $\mathcal{MA}$ in a useful manner, for the 3D shape of a human male body scanned with a Cyberware Inc. system resulting in 21500 3D points (shown as white dots). This preview of the shock scaffold is shown in terms of medial surfaces and a network of axial curves (in pink) where these intersect. Flow along the medial surfaces, *i.e.*, how the radius function varies is indicated by color variations. (b) is the *interior* part of the computed shock scaffold, while (c) is the *exterior* (with respect to the scanned surface of the body; a recovered surface interpolant is shown in Figure 1.6.(c)).

Other open issues:

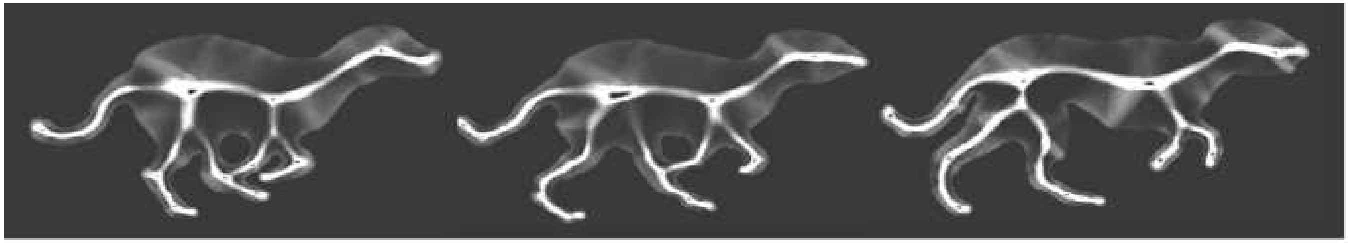
- Combine or study relations with other existing main shape representations based on propagations: Voronoi, Morse/Reeb, flow complex, 3D Curve skeletons

- Interactions between 2D and 3D inputs : visual inputs/snapshots (2D) versus 3D percepts : no trivial correspondence between 2D and 3D medial representations (including Voronoi)

Other open issues:



- Medial representations directly from intensity fields : images, video, 3D medical volumetric data : e.g. work of Kovacs et al. on robust symmetries.



- related: using a probabilistic framework or robust measures to deal with noisy inputs or sampling obtained in sausages (neighborhoods) rather than precisely on an outline/surface

- Complexity, proofs of convergences for realistic data (not too smooth).