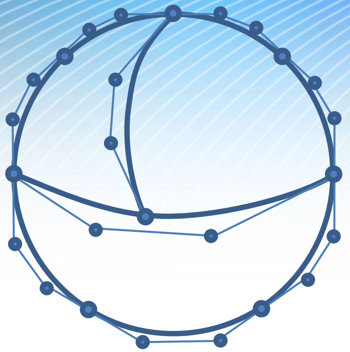
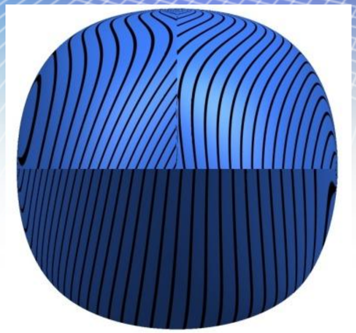
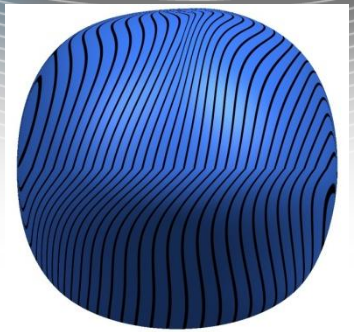
UK-Korea Focal Point Programme in Mathematics:
Geometric Modelling and Computer Graphics, Seoul, Korea

Tangent Plane Continuous Bézier Surface Interpolation with T-junction







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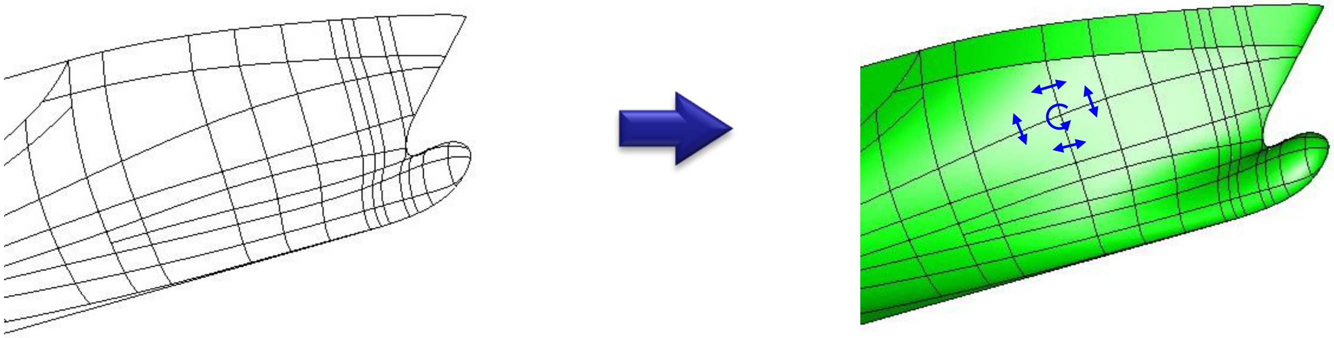
Surface Interpolation using a Boundary Curve Network

- Input
  - Boundary curve network
  - The boundary curves cannot be changed.
- Output
  - Smooth surfaces
  - The surfaces should interpolate the given boundary curves.

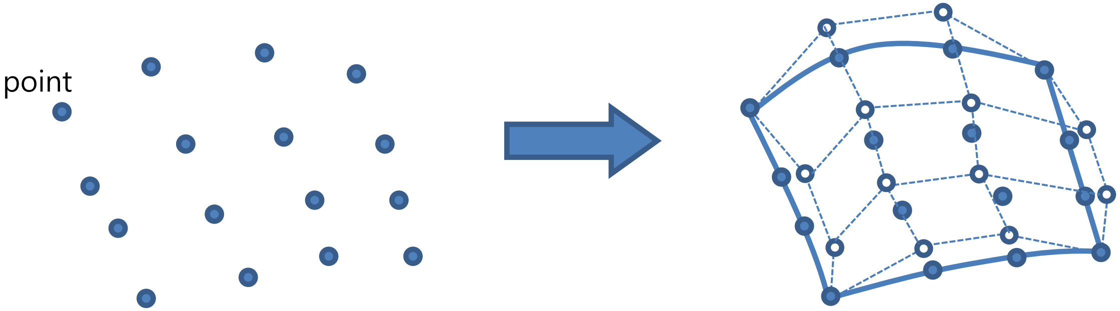
Input:
curve network from designer

Output:
smooth surface model



Surface Interpolation Method

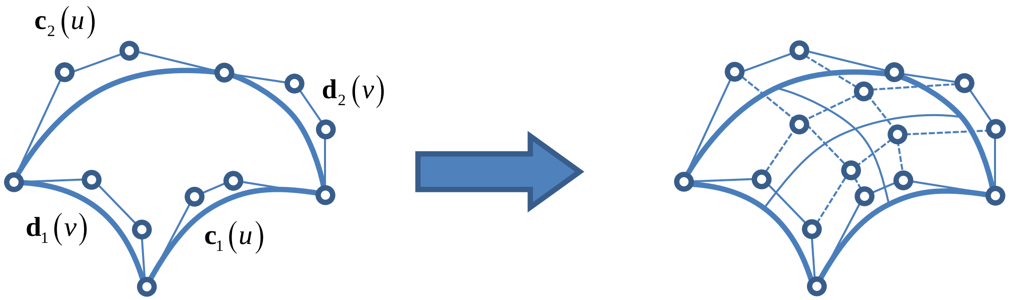
- Discrete point interpolation



input: points

output: patch

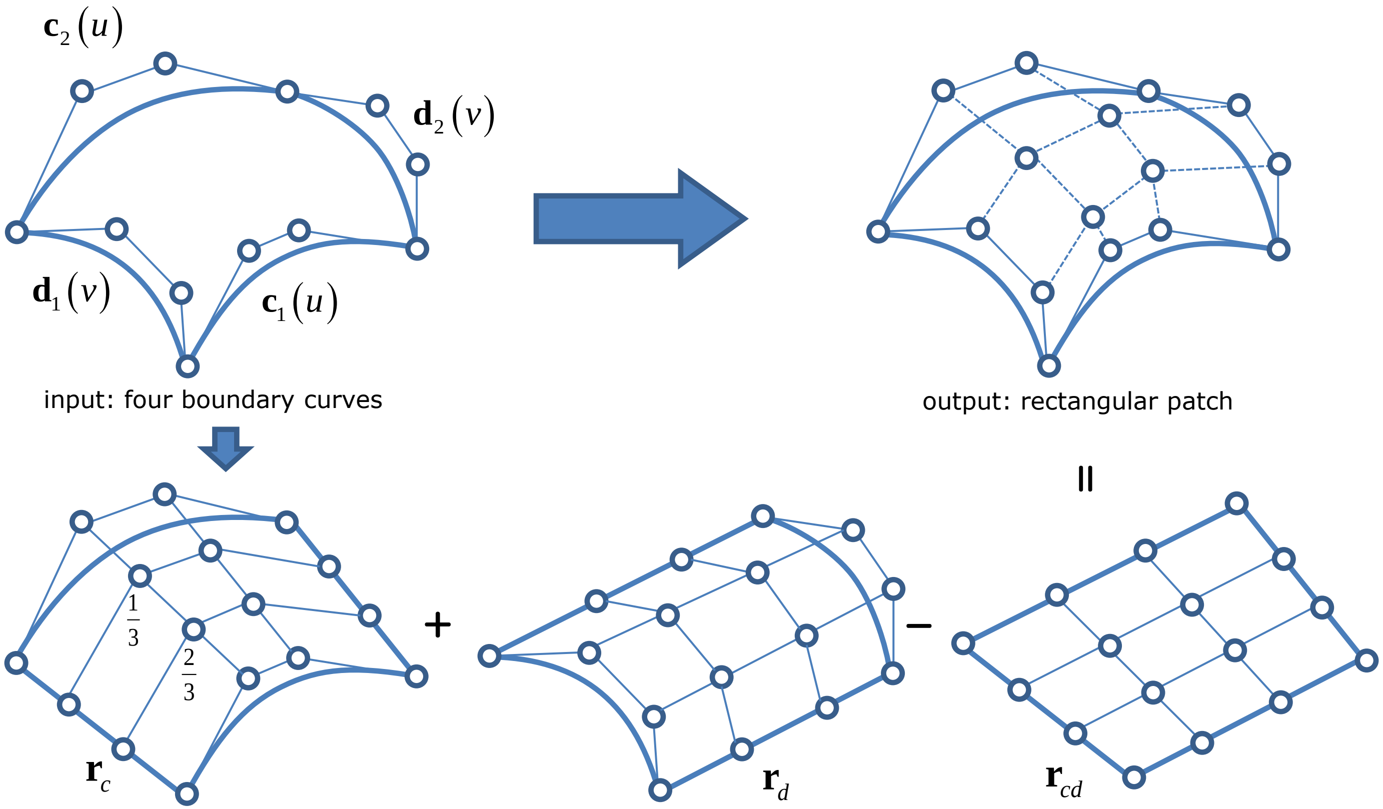
- Transfinite interpolation



input: boundary curves

output: patch

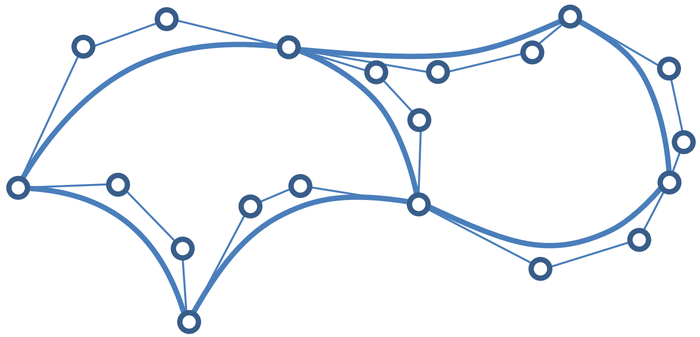
Coons Patches (1/2)



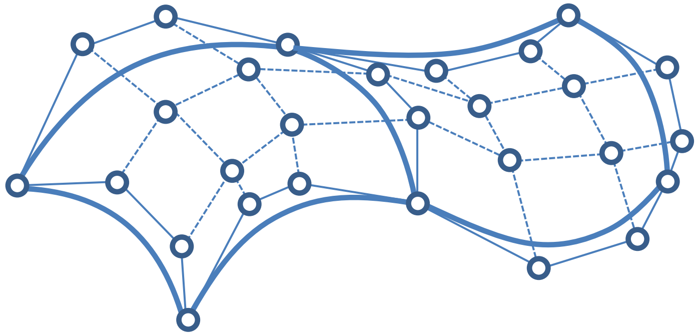
input: four boundary curves

output: rectangular patch

Coons Patches (2/2)



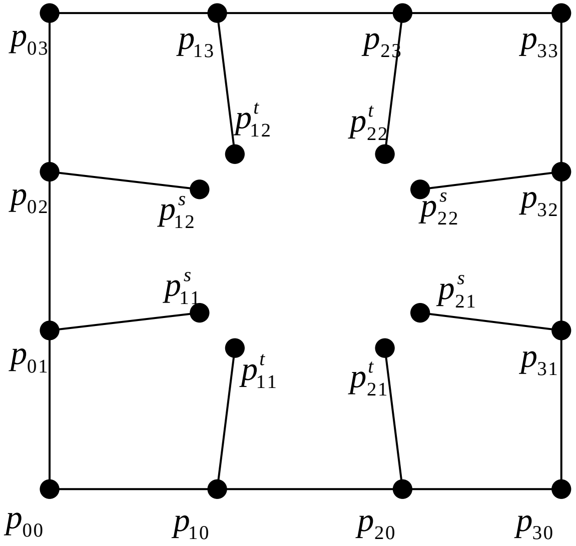
input: boundary curves



output: C^0-continuous rectangular patch
(piecewise Bézier patches)

Gregory Patch – used in DESIGNBASE by Chiyokura

- Independent control point of derivatives along edges except at corners
- Rational patches
- Different twists at corners
- Discontinuous derivative at corners
- Zero corner weight → singular problem at corners
- Higher degree of the derivative than its boundary



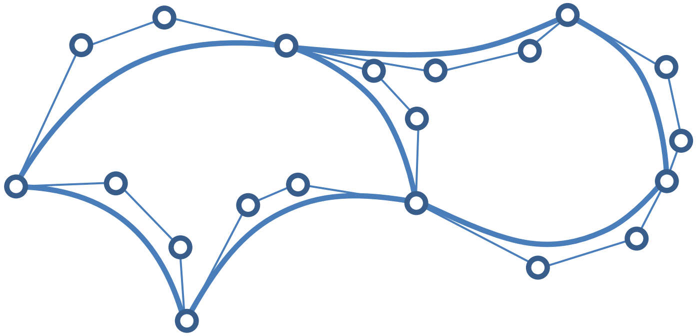
$$p_{11} = \frac{t\,p_{11}^s + s\,p_{11}^t}{s+t}$$

$$p_{21} = \frac{t\,p_{21}^s + (1-s)\,p_{21}^t}{1-s+t}$$

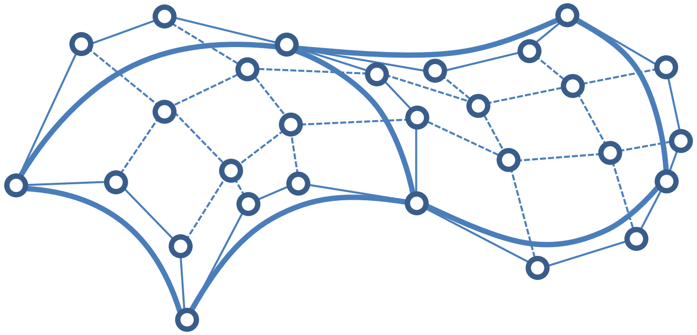
$$p_{22} = \frac{(1-t)\,p_{22}^s + (1-s)\,p_{22}^t}{2-s-t}$$

$$p_{12} = \frac{(1-t)\,p_{12}^s + s\,p_{12}^t}{1+s-t}$$

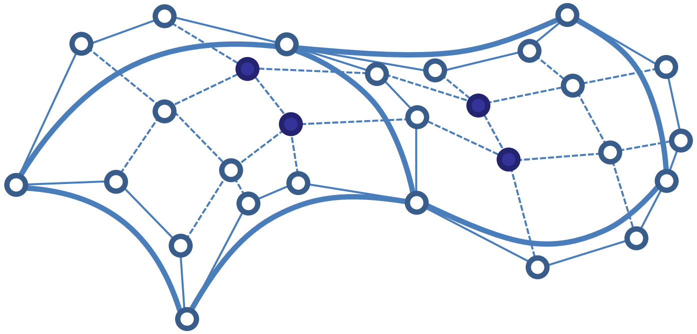
Summary of This Research



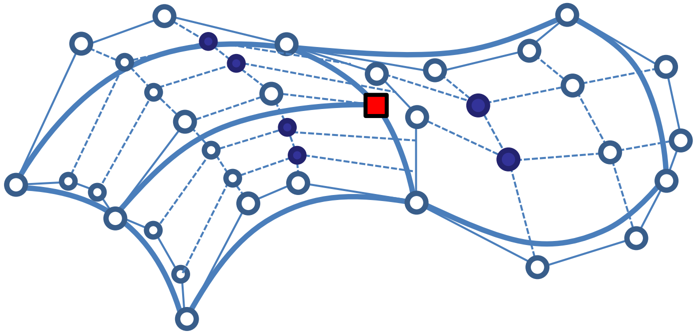
boundary Bézier curves



C^0-continuous Bézier patches
(piecewise Bézier patches)



G^1-continuous Bézier patches.
No parameters between two patches

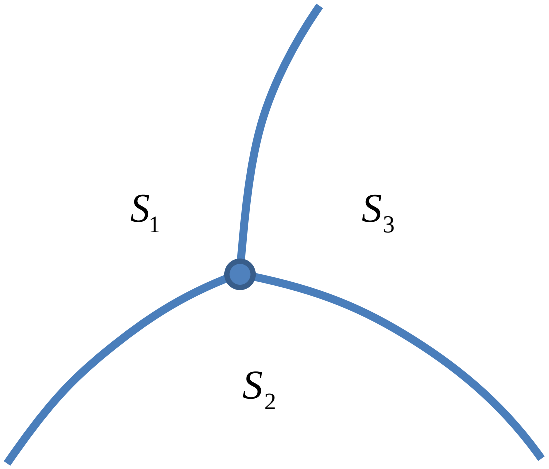


G^1-continuous Bézier patches
with a T-junction

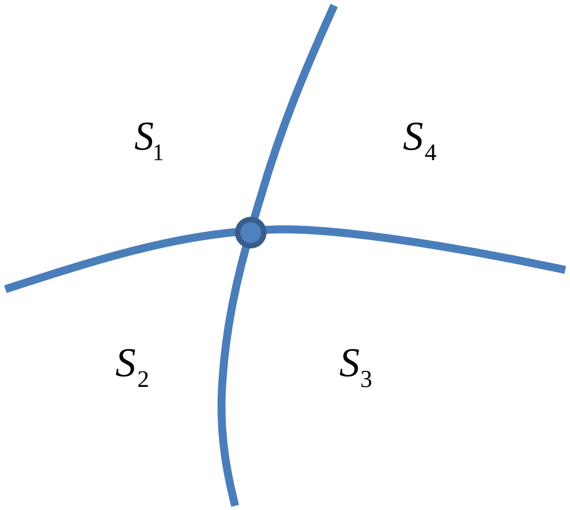
Features and Originality of This Research

Feature	Contents
Interpolation	▪ Transfinite interpolation over a curve network with a T-junction - First method to interpolate a T-junction - Interpolate all given curve network - Extension of Coons patch ▪ Feature curve is important → does not change the given boundary curves ▪ Inverse problem
Surface parameter	▪ No surface parameters by users → generating G^1 surface
Surface type	▪ Rectangular Bézier surface ▪ Polynomial basis
Input curve network	▪ A T-junction on a boundary curve ▪ A T-junction at a vertex (degenerate case) can be avoided by subdivision ▪ valence 3 and 4 ▪ 5 sided patch can be subdivided into two rectangular patches with a T-junction
Solution type (local or global)	▪ Constructive method (local method) - Solving vertex G^1 constraints and edge G^1 constraints separately
Easy to understand	▪ Explaining in terms of control points of surface

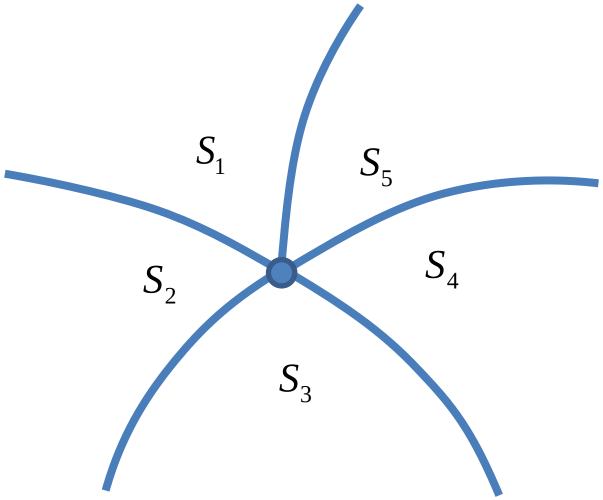
Valence



valence 3

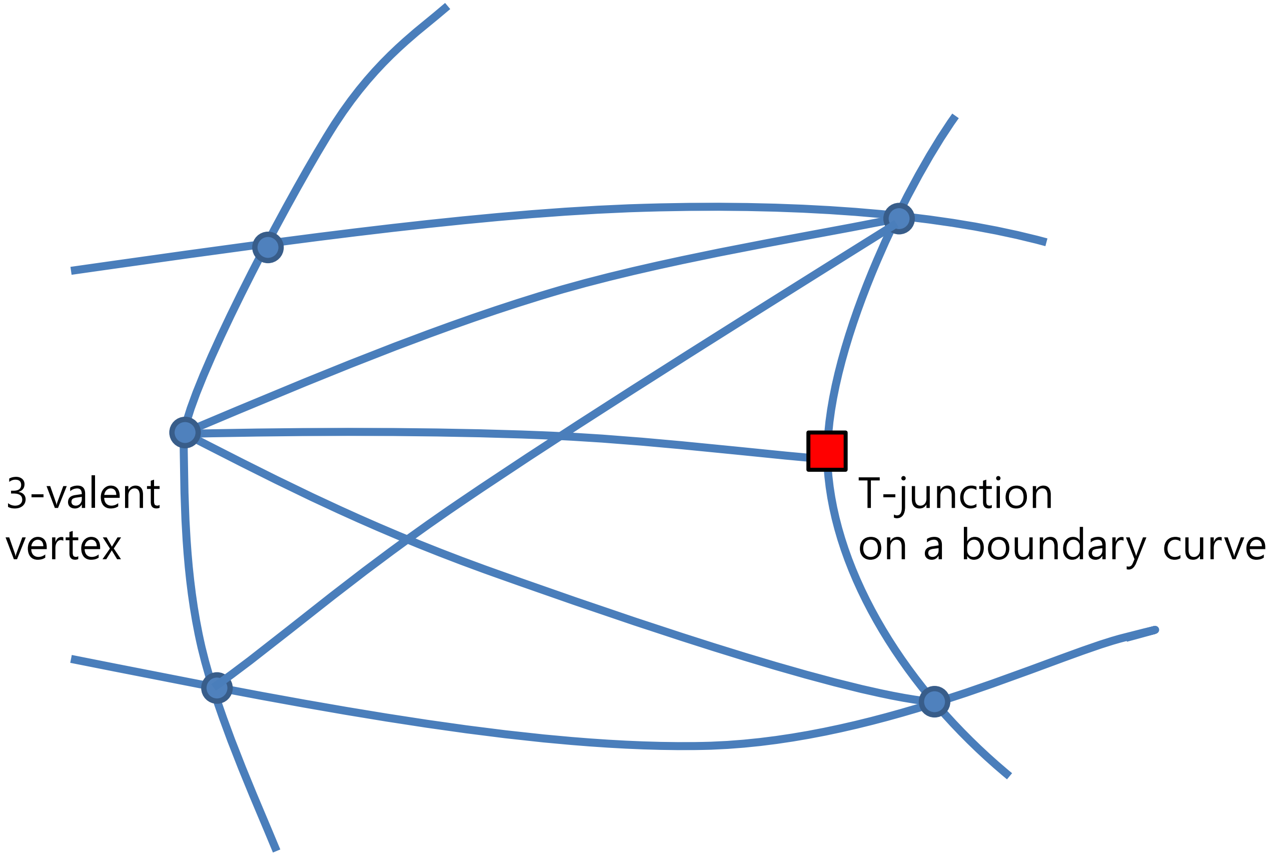


valence 4



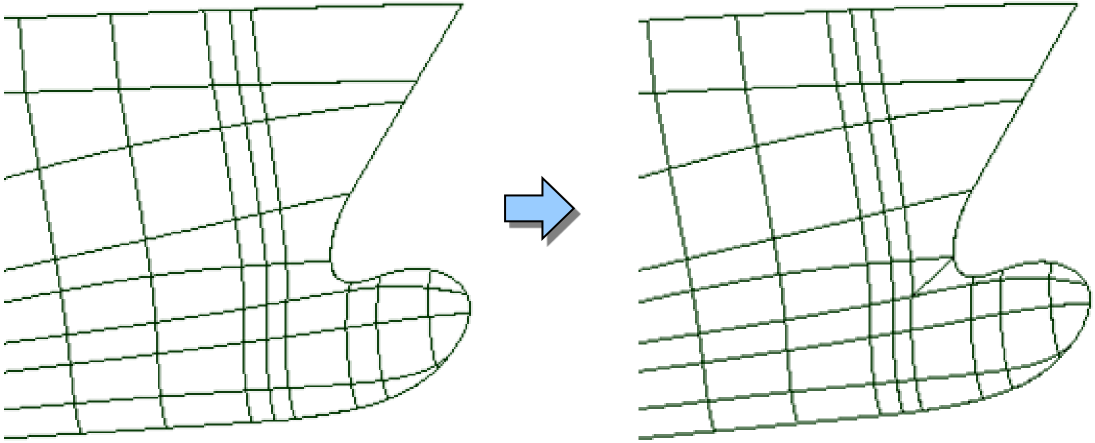
valence 5

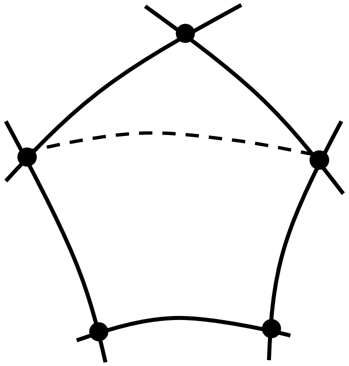
5-sided Patch (1/2)



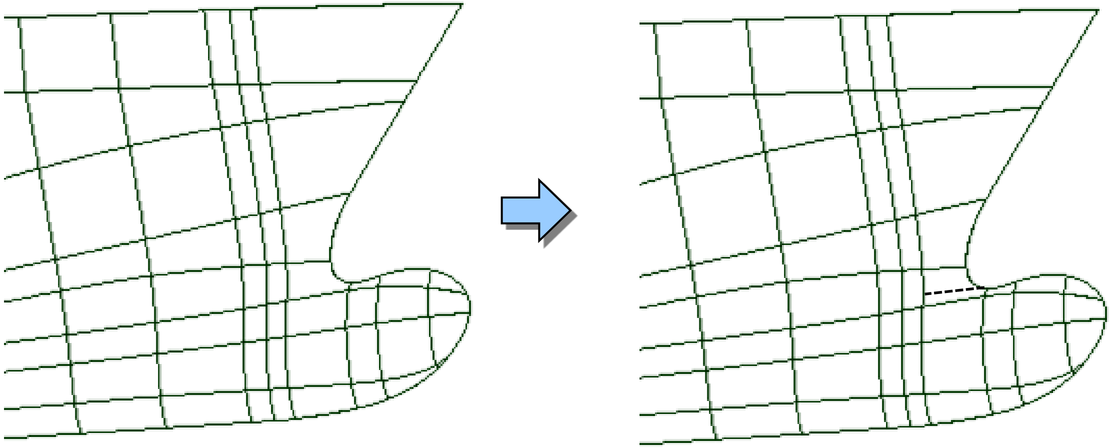
5-sided Patch (2/2)

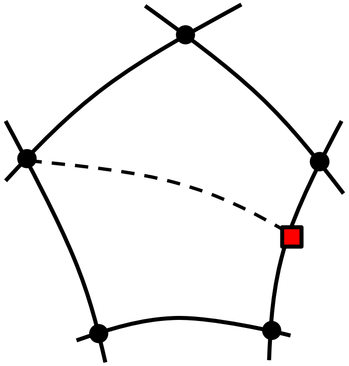
- Previous method





- Proposed method





T-junction
on a boundary
curve

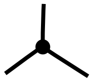
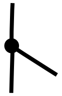
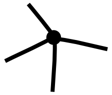
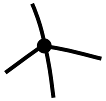
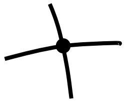
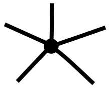
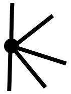
Related Works

paper	input curve network					output surfaces			solution type	keep original curve network unchanged even in singular cases
	constructing a boundary curve	face type	valence	curve type		surfaces type		continuity		
				type	degree	type	degree			
Q. Liu et al. (1994)	given	4 sided face	4[b]	Bézier	3	rectangular Bézier	4x4	G^1	global	Yes
T. Hermann (1995)	given	4 side face	n	polynomial	5	Gregory patch	5x5	G^2	local	Yes
X. Shi et al. (2004)	given	4 sided face	n	B-spline	5	rectangular B-spline surface with interior single knots	5x5	G^1	local	No
X. Che et al. (2005)	given	two NURBS surfaces	-	NURBS	n	rectangular NURBS surface	nxn	G^1	-	-
D.-Y. Cho et al. (2006)	given	3,4,5,6 sided face, T-junction at a vertex[a]	3,4[b],5	Bézier	n	triangular Bézier	n+3	G^1	local	Yes
						rectangular Bézier	(n+2)x(n+2)			
Y. Liu, S. Mann (2008)	interpolated from mesh	3 sided face	n	Bézier	4	triangular Bézier	5	ε-G^1	local	No
W.-H. Tong and T.-W. Kim (2009)	interpolated from implicit surface	3 sided face	n	C^2 approximated boundary with normal curvature		triangular Bézier	7	G^1	local	Yes
K.-L. Shi. et al. (2010)	interpolated from mesh	4 side face	n	B-spline	9	rectangular B-spline Coons	9x9	G^2	local	No
This study	given	T-junction on a boundary and at a vertex	3, 4[b], T-shape	Bézier	3	rectangular Bézier	5x5	G^1	local	Yes

[a] **5-, 6-sided faces and T-junction can be dealt with by subdivision.**

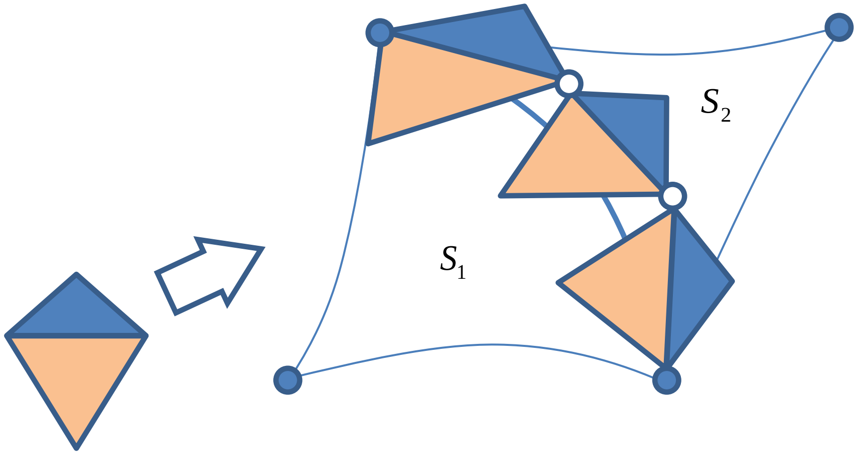
[b] **Opposing pairs of curves should meet collinearly.**

Analysis and Avoidance of Singularities in Vertex G^1 Condition*

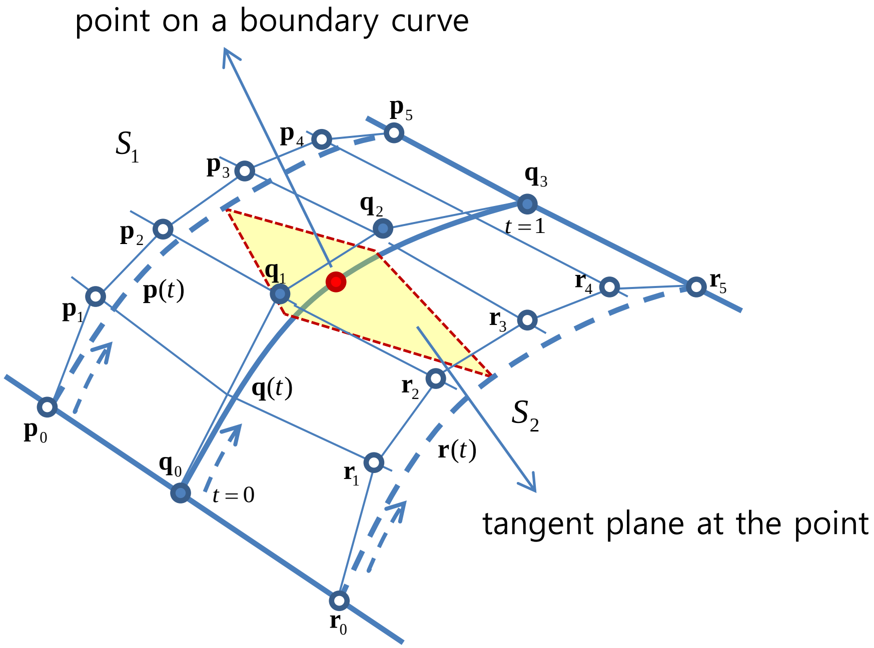
Valence	Configuration of curve network		Singularity of Vertex G^1 condition	Possible to overcome singularity?
3	 3-a	Three arbitrary curves	Non-singular	Least Squares Solution
	 3-b	Two adjacent curves	Singular	Subdivide into three rectangular patches with T-junction on a boundary
4	 4-a	Four arbitrary curves	Singular	No Solution with the curve network unchanged
	 4-b	An opposite collinear curves		No Solution with the curve network unchanged
	 4-c	Two opposite collinear curves		Least Squares Solution (N/S condition for the system to have solutions is derived)
5	 5-a	Five arbitrary curves	Non-singular	Least Squares Solution
	 5-b	An adjacent collinear curves	Singular	No Solution with the curve network unchanged

* D.-Y. Cho, K.-Y. Lee, T.-W. Kim, Interpolating G^1 Bézier surfaces over irregular curve networks for ship hull design, Computer-Aided Design Vol. 38, No. 6, pp. 641–660, 2006.

C^1 and G^1 Continuity between Two Surfaces



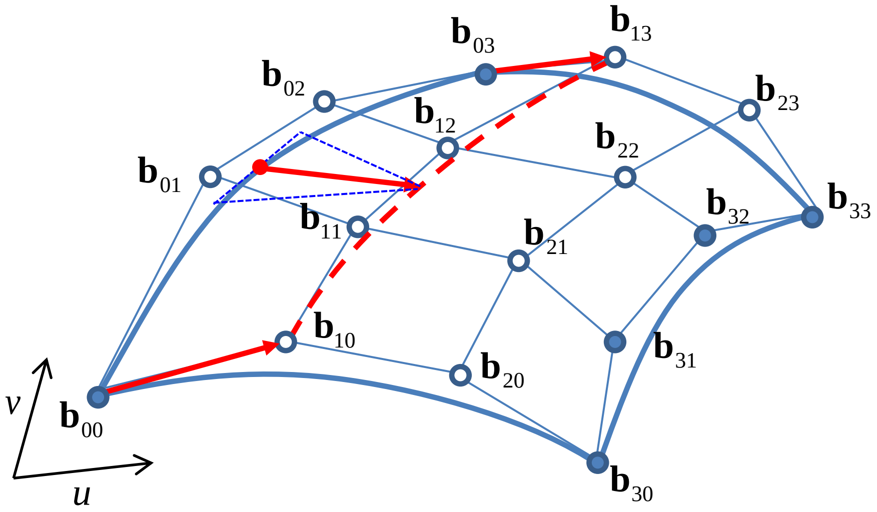
C^1-continuous surfaces


point on a boundary curve

tangent plane at the point

G^1-continuous surfaces

Cross-Derivative of Bézier Surface



Bézier surface:

$$\mathbf{b}(u,v)=\sum_{i=0}^{3}\sum_{j=0}^{3}\mathbf{b}_{ij}B_i^3(u)B_j^3(v)$$

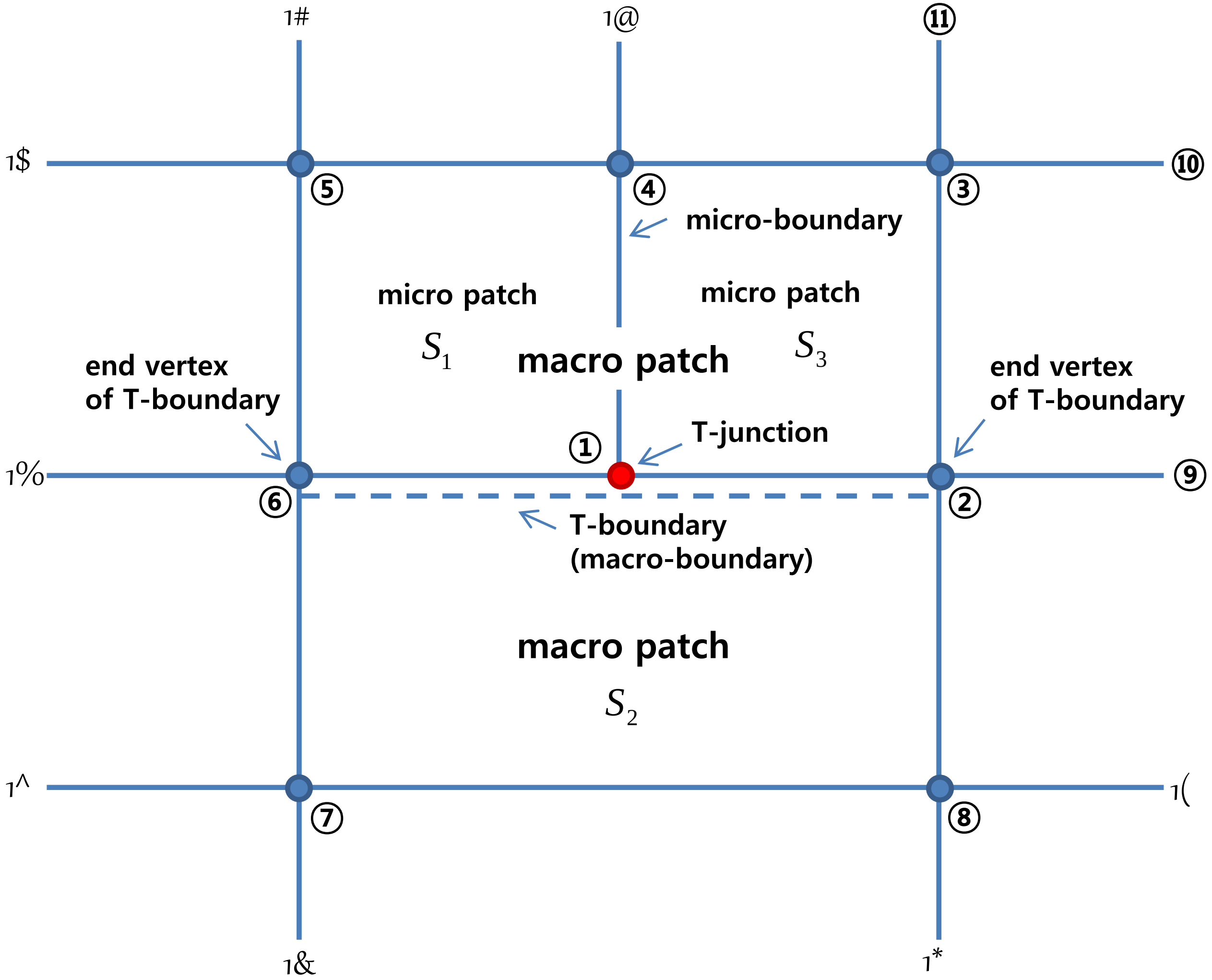
Off-boundary curve (cross-derivative curve):

$$\mathbf{r}(v)=\sum_{i=0}^{3}\mathbf{b}_{1i}B_i^3(v)$$

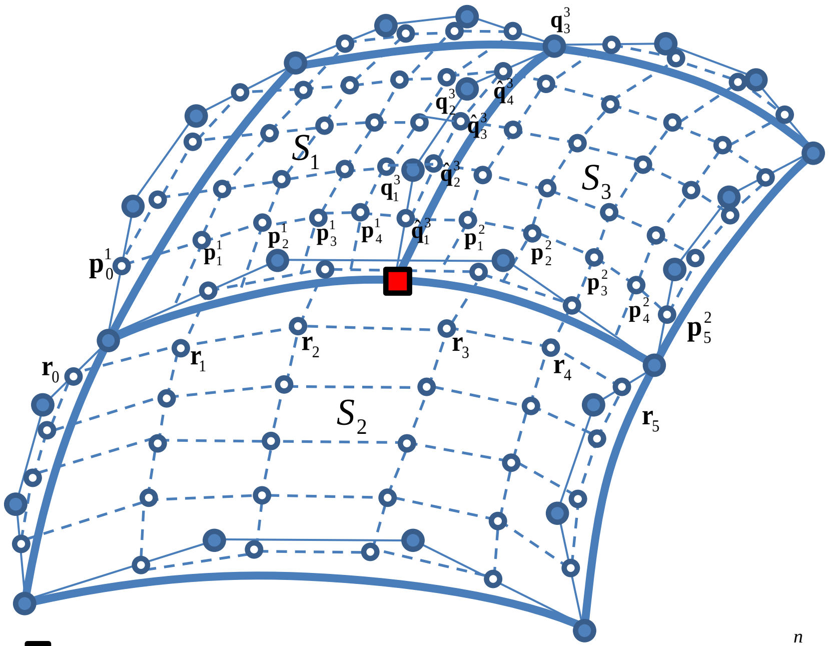
Cross-derivative vector:

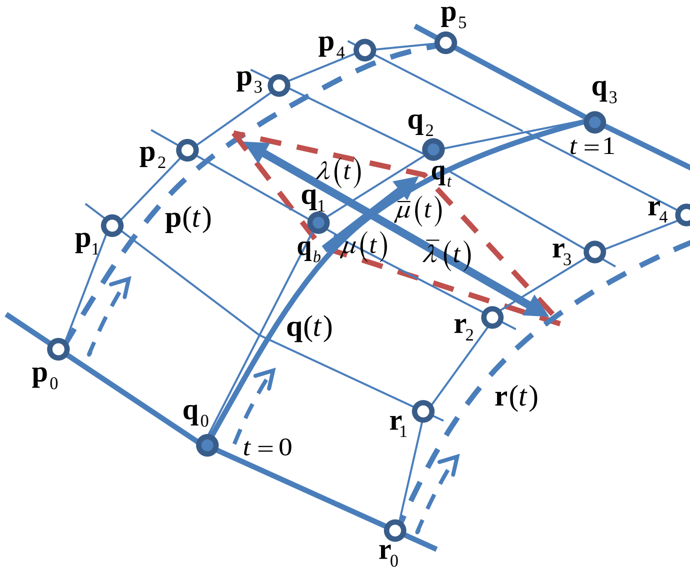
$$\mathbf{r}_v(v)=\mathbf{r}(v)-\mathbf{b}(0,v)$$

Terminology



Notation





$$\bar{\lambda}(t)=1-\lambda(t)$$
$$\bar{\mu}(t)=1-\mu(t)$$

■ : T-junction on the boundary

● : initial control points on a cubic Bézier boundary curve

○ : control points on a quintic patch

$\mathbf{p}(t)=\sum_{i=1}^{n}\mathbf{p}_i B^n(t)$: left-hand side cross-derivative (off-boundary) curve

$\mathbf{r}(t)=\sum_{i=0}^{n}\mathbf{r}_i B^n(t)$: right-hand side cross-derivative (off-boundary) curve

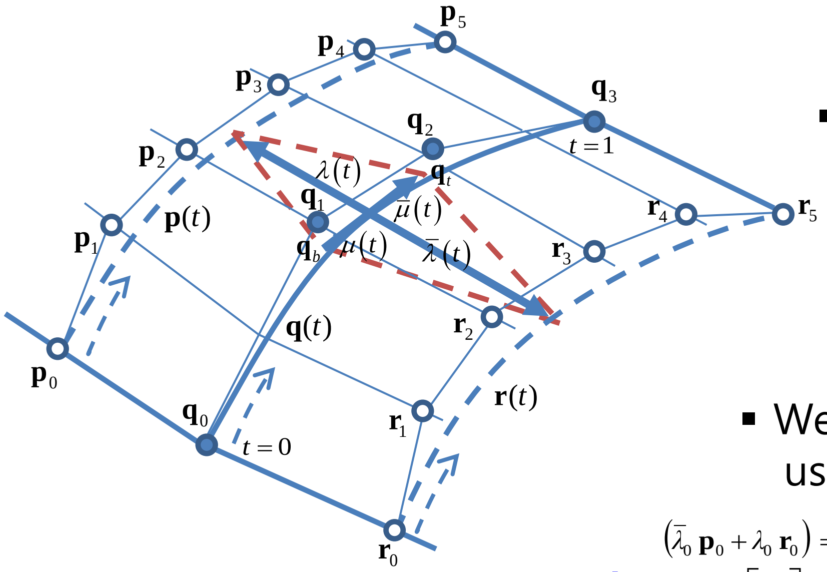
$\mathbf{q}(t)=\sum_{i=0}^{m}\mathbf{q}_i B^m(t)$: boundary curve

$\hat{\mathbf{q}}(t)=\sum_{i=0}^{l}\hat{\mathbf{q}}_i B^l(t)$: degree-elevated boundary curve

$\lambda(t)=\sum_{i=0}^{a}\lambda_i B^a(t)$: scalar weight function for cross-derivative

$\mu(t)=\sum_{i=0}^{b}\mu_i B^b(t)$: scalar weight function for tangent of common boundary

G^1 Continuity Equation between Two Patches



- 3 vectors in a plane (algebraic G^1 condition)

$$\alpha(t)\cdot\mathbf{a}(t)+\beta(t)\cdot\mathbf{b}(t)+\gamma(t)\cdot\mathbf{c}(t)=\vec{0}$$

- 4 points in a plane (geometric G^1 condition)

$$[1-\lambda(t)]\cdot\mathbf{p}(t)+\lambda(t)\cdot\mathbf{r}(t)=[1-\mu(t)]\cdot\mathbf{q}_b(t)+\mu(t)\cdot\mathbf{q}_t(t)$$

degree: 1 5 1 5 4 2 4 2

- We can write the equation using explicit expressions using Bézier control points:

$$\left(\bar{\lambda}_0\,\mathbf{p}_0+\lambda_0\,\mathbf{r}_0\right)=\left(\bar{\mu}_0\,\mathbf{q}_0+\mu_0\,\mathbf{q}_1\right), \qquad k=0$$

vertex G^1 constraint :

vertex G^1 system:

$$5\begin{bmatrix}\bar{\lambda}_0 & \lambda_0\end{bmatrix}\begin{bmatrix}\mathbf{p}_1\\ \mathbf{r}_1\end{bmatrix}=\left[-\left(\bar{\lambda}_1\mathbf{p}_0+\lambda_1\mathbf{r}_0\right)+2\cdot\left(\bar{\mu}_0\mathbf{q}_1+\mu_0\mathbf{q}_2\right)+4\cdot\left(\bar{\mu}_1\mathbf{q}_0+\mu_1\mathbf{q}_1\right)\right] \qquad k=1$$

edge G^1 constraint :

edge G^1 system:

$$10\begin{bmatrix}\bar{\lambda}_0 & \lambda_0 & 0 & 0\\ \bar{\lambda} & \lambda_1 & \bar{\lambda}_0 & \lambda_0\\ 0 & 0 & \bar{\lambda} & \lambda_1\end{bmatrix}\begin{bmatrix}\mathbf{p}_2\\ \mathbf{r}_2\\ \mathbf{p}_3\\ \mathbf{r}_3\end{bmatrix}=\begin{bmatrix}-5\cdot\left(\bar{\lambda}_1\mathbf{p}_1+\lambda_1\mathbf{r}_1\right)+\left(\bar{\mu}_0\mathbf{q}_2+\mu_0\mathbf{q}_3\right)+8\cdot\left(\bar{\mu}_1\mathbf{q}_1+\mu_1\mathbf{q}_2\right)+6\cdot\left(\bar{\mu}_2\mathbf{q}_0+\mu_2\mathbf{q}_1\right)\\ 4\cdot\left(\bar{\mu}_1\mathbf{q}_2+\mu_1\mathbf{q}_3\right)+8\cdot\left(\bar{\mu}_2\mathbf{q}_1+\mu_2\mathbf{q}_2\right)+6\cdot\left(\bar{\mu}_3\mathbf{q}_0+\mu_3\mathbf{q}_1\right)\\ -5\cdot\left(\bar{\lambda}_0\mathbf{p}_4+\lambda_0\mathbf{r}_4\right)+6\cdot\left(\bar{\mu}_2\mathbf{q}_2+\mu_2\mathbf{q}_3\right)+8\cdot\left(\bar{\mu}_3\mathbf{q}_1+\mu_3\mathbf{q}_2\right)+\left(\bar{\mu}_4\mathbf{q}_0+\mu_4\mathbf{q}_1\right)\end{bmatrix}$$

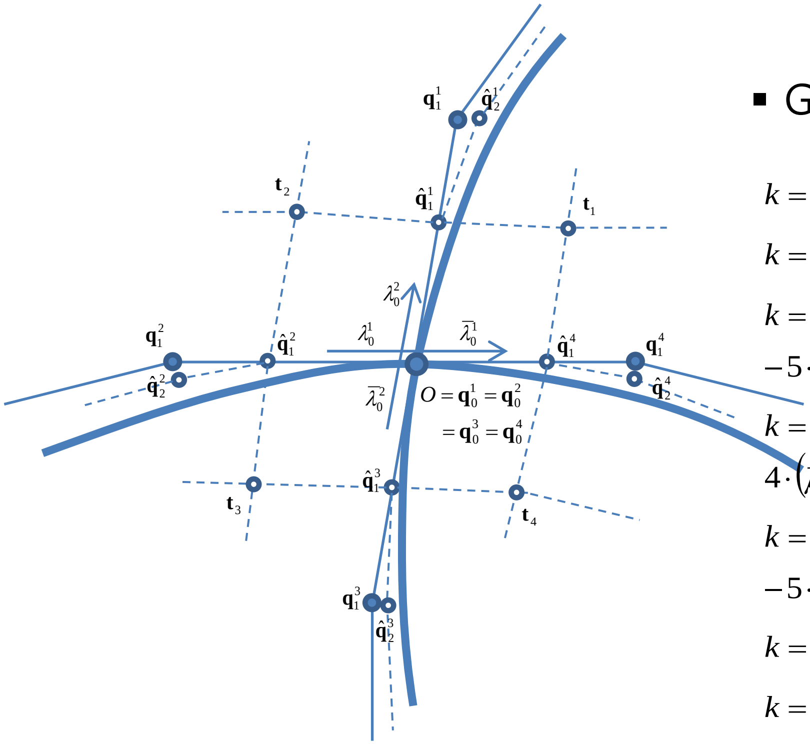
vertex G^1 constraint :

vertex G^1 system:

$$5\begin{bmatrix}\bar{\lambda}_1 & \lambda_1\end{bmatrix}\begin{bmatrix}\mathbf{p}_4\\ \mathbf{r}_4\end{bmatrix}=\left[-\left(\bar{\lambda}_0\mathbf{p}_5+\lambda_0\mathbf{r}_5\right)+4\cdot\left(\bar{\mu}_3\mathbf{q}_2+\mu_3\mathbf{q}_3\right)+2\cdot\left(\bar{\mu}_4\mathbf{q}_1+\mu_4\mathbf{q}_2\right)\right] \qquad k=6$$

where $\mathbf{p}(t)=\sum_{i=0}^{5}\mathbf{p}_iB_i^5(t),\mathbf{r}(t)=\sum_{i=0}^{5}\mathbf{r}_iB_i^5(t),\mathbf{q}(t)=\sum_{i=0}^{3}\mathbf{q}_iB_i^3(t)$

G^1 Continuity Equation among Four Patches (1/3)



● : control point of the cubic Bézier boundary curve

○ : control point of the degree-elevated boundary curve for the quintic Bézier patch

- G^1 continuity equation along the each patch

$$k=0:\left(\bar{\lambda}_0^j\,\mathbf{p}_0^j+\lambda_0^j\,\mathbf{r}_0^j\right)=\left(\bar{\mu}_0^j\,\mathbf{q}_0^j+\mu_0^j\,\mathbf{q}_1^j\right),$$

$$k=1:5\cdot\left(\bar{\lambda}_0^j\,\mathbf{p}_1^j+\lambda_0^j\,\mathbf{r}_1^j\right)=-\left(\bar{\lambda}_1^j\,\mathbf{p}_0^j+\lambda_1^j\,\mathbf{r}_0^j\right)+2\cdot\left(\bar{\mu}_0^j\,\mathbf{q}_1^j+\mu_0^j\,\mathbf{q}_2^j\right)+4\cdot\left(\bar{\mu}_1^j\,\mathbf{q}_0^j+\mu_1^j\,\mathbf{q}_1^j\right),$$

$$k=2:10\cdot\left(\bar{\lambda}_0^j\mathbf{p}_2^j+\lambda_0^j\mathbf{r}_2^j\right)=$$
$$-5\cdot\left(\bar{\lambda}_1^j\mathbf{p}_1^j+\lambda_1^j\mathbf{r}_1^j\right)+\left(\bar{\mu}_0^j\mathbf{q}_2^j+\mu_0^j\mathbf{q}_3^j\right)+8\cdot\left(\bar{\mu}_1^j\mathbf{q}_1^j+\mu_1^j\mathbf{q}_2^j\right)+6\cdot\left(\bar{\mu}_2^j\mathbf{q}_0^j+\mu_2^j\mathbf{q}_1^j\right),$$

$$k=3:10\cdot\left(\bar{\lambda}_0^j\mathbf{p}_3^j+\lambda_0^j\mathbf{r}_3^j\right)+10\cdot\left(\bar{\lambda}_1^j\mathbf{p}_2^j+\lambda_1^j\mathbf{r}_2^j\right)=$$
$$4\cdot\left(\bar{\mu}_1^j\mathbf{q}_2^j+\mu_1^j\mathbf{q}_3^j\right)+8\cdot\left(\bar{\mu}_2^j\mathbf{q}_1^j+\mu_2^j\mathbf{q}_2^j\right)+6\cdot\left(\bar{\mu}_3^j\mathbf{q}_0^j+\mu_3^j\mathbf{q}_1^j\right),$$

$$k=4:10\cdot\left(\bar{\lambda}_1^j\mathbf{p}_3^j+\lambda_1^j\mathbf{r}_3^j\right)=$$
$$-5\cdot\left(\bar{\lambda}_0^j\mathbf{p}_4^j+\lambda_0^j\,\mathbf{r}_4^j\right)+6\cdot\left(\bar{\mu}_2^j\mathbf{q}_2^j+\mu_2^j\mathbf{q}_3^j\right)+8\cdot\left(\bar{\mu}_3^j\mathbf{q}_1^j+\mu_3^j\mathbf{q}_2^j\right)+\left(\bar{\mu}_4^j\mathbf{q}_0^j+\mu_4^j\mathbf{q}_1^j\right),$$

$$k=5:5\cdot\left(\bar{\lambda}_1^j\mathbf{p}_4^j+\lambda_1^j\mathbf{r}_4^j\right)=-\left(\bar{\lambda}_0^j\mathbf{p}_5^j+\lambda_0^j\mathbf{r}_5^j\right)+4\cdot\left(\bar{\mu}_3^j\mathbf{q}_2^j+\mu_3^j\mathbf{q}_3^j\right)+2\cdot\left(\bar{\mu}_4^j\mathbf{q}_1^j+\mu_4^j\mathbf{q}_2^j\right),$$

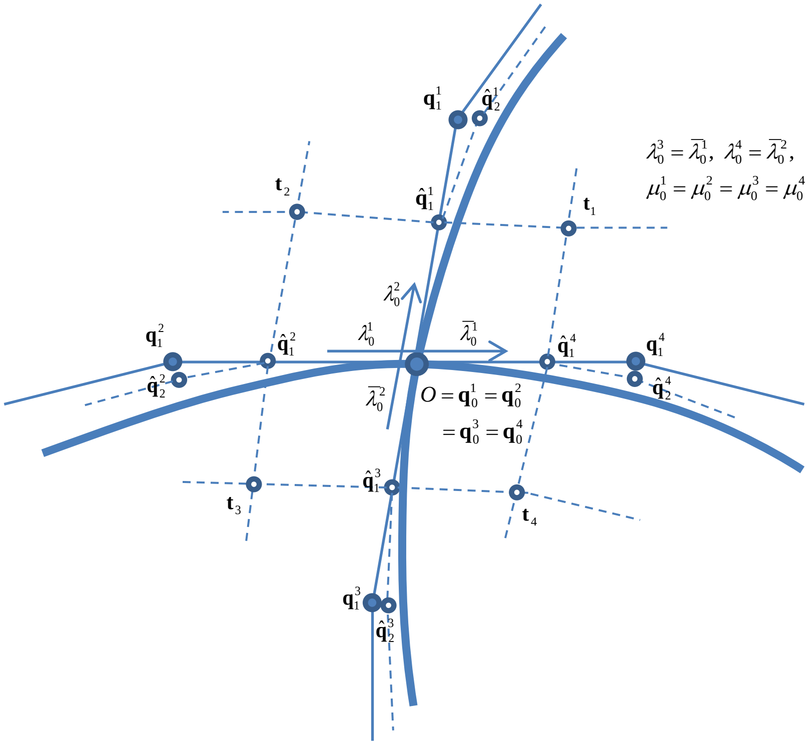
$$k=6:\left(\bar{\lambda}_1^j\mathbf{p}_5^j+\lambda_1^j\mathbf{r}_5^j\right)=\left(\bar{\mu}_4^j\mathbf{q}_2^j+\mu_4^j\mathbf{q}_3^j\right)$$

where $j=1,2,3,4.$

$$\left[1-\lambda(t)\right]\cdot\mathbf{p}(t)+\lambda(t)\cdot\mathbf{r}(t)=\left[1-\mu(t)\right]\cdot\mathbf{q}_b(t)+\mu(t)\cdot\mathbf{q}_t(t)$$

degree: 1 5 1 5 4 2 4 2

G^1 Continuity Equation among Four Patches* (2/3)



$$\lambda_0^3 = \overline{\lambda}_0^1,\ \lambda_0^4 = \overline{\lambda}_0^2,$$
$$\mu_0^1 = \mu_0^2 = \mu_0^3 = \mu_0^4 = 0.$$

● : control point of the cubic Bézier boundary curve

○ : control point of the degree-elevated boundary curve for the quintic Bézier patch

$$[1-\lambda(t)]\cdot\mathbf{p}(t)+\lambda(t)\cdot\mathbf{r}(t)=[1-\mu(t)]\cdot\mathbf{q}_b(t)+\mu(t)\cdot\mathbf{q}_t(t)$$

degree: 1 5 1 5 4 2 4 2

- vertex G^1 system

$$5\cdot\begin{pmatrix} \lambda_0^1 & \overline{\lambda}_0^1 & 0 & 0 \\ 0 & \lambda_0^2 & \overline{\lambda}_0^2 & 0 \\ 0 & 0 & \lambda_0^3 & \overline{\lambda}_0^3 \\ \overline{\lambda}_0^4 & 0 & 0 & \lambda_0^4 \end{pmatrix}\begin{pmatrix} \mathbf{t}_1 \\ \mathbf{t}_2 \\ \mathbf{t}_3 \\ \mathbf{t}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{rhs}_1^1(\mu_1^1) \\ \mathbf{rhs}_1^2(\mu_1^2) \\ \mathbf{rhs}_1^3(\mu_1^3) \\ \mathbf{rhs}_1^4(\mu_1^4) \end{pmatrix}$$

where

$$\mathbf{rhs}_j = -\left(\overline{\lambda}_1^j\hat{\mathbf{q}}_1^{j-1} + \lambda_1^j\hat{\mathbf{q}}_1^{j+1}\right) + \frac{5}{3}\left(2+4\mu_1^j\right)\left(\hat{\mathbf{q}}_1^j - O\right) + 6O + 2\mu_0^j\left(\mathbf{q}_2^j - \mathbf{q}_1^j\right),$$

$$\det = \prod_{j=1}^{4}\overline{\lambda}_0^j - \prod_{j=1}^{4}\lambda_0^j = 0,$$

$$j = 1,2,3,4.$$

This system can be solved with

$$\mu_1^1 - \mu_1^3 = \frac{3}{20}\left(\frac{\overline{\lambda}_0^1\lambda_1^4}{\overline{\lambda}_0^2} + \frac{\lambda_0^1\overline{\lambda}_1^2}{\overline{\lambda}_0^2} - \frac{\overline{\lambda}_0^1\overline{\lambda}_1^4}{\lambda_0^2} - \frac{\lambda_0^1\lambda_1^2}{\lambda_0^2}\right),$$

$$\mu_1^2 - \mu_1^4 = \frac{3}{20}\left(\frac{\overline{\lambda}_0^2\lambda_1^1}{\lambda_0^1} + \frac{\lambda_0^2\overline{\lambda}_1^3}{\lambda_0^1} - \frac{\overline{\lambda}_0^2\overline{\lambda}_1^1}{\overline{\lambda}_0^1} - \frac{\lambda_0^2\lambda_1^3}{\overline{\lambda}_0^1}\right).$$

- edge G^1 system

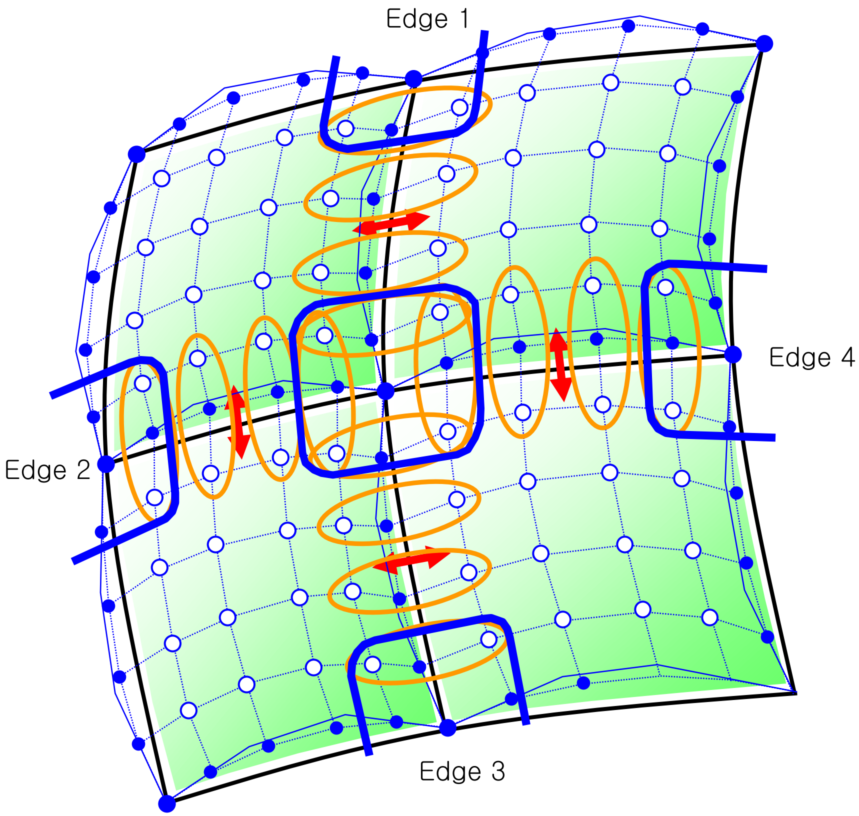
$$10\cdot\begin{pmatrix} \overline{\lambda}_0^j & \lambda_0^j & 0 & 0 \\ \overline{\lambda}_1^j & \lambda_1^j & \overline{\lambda}_0^j & \lambda_0^j \\ 0 & 0 & \overline{\lambda}_1^j & \lambda_1^j \end{pmatrix}\begin{pmatrix} \mathrm{p}_2^j \\ \mathrm{r}_2^j \\ \mathrm{p}_3^j \\ \mathrm{r}_3^j \end{pmatrix} = \begin{pmatrix} \mathbf{rhs}_2^j(\mu_2^j) \\ \mathbf{rhs}_3^j(\mu_2^j) \\ \mathbf{rhs}_4^j(\mu_2^j) \end{pmatrix}$$

if $\lambda_0^j = \lambda_1^j$, this system will be singular.
we can solve this system with $\mu_2^j = -\frac{1}{6}\left(\mu_0^j - 4\mu_1^j - 4\mu_3^j + \mu_4^j\right).$

* D.-Y. Cho, K.-Y. Lee, T.-W. Kim, Interpolating G^1 Bézier surfaces over irregular curve networks for ship hull design, Computer-Aided Design Vol. 38, No. 6, pp. 641–660, 2006.

G^1 Continuity Equation among Four Patches (3/3)

- Constructive method (local scheme)



○ ○ ○ ○

Four G^1 conditions are coupled at corner vertex → Vertex G^1 condition

$$5 \cdot \begin{pmatrix} \lambda_0^1 & \bar{\lambda}_0^1 & 0 & 0 \\ 0 & \lambda_0^2 & \bar{\lambda}_0^2 & 0 \\ 0 & 0 & \lambda_0^3 & \bar{\lambda}_0^3 \\ \bar{\lambda}_0^4 & 0 & 0 & \lambda_0^4 \end{pmatrix} \begin{pmatrix} \mathrm{t}_1 \\ \mathrm{t}_2 \\ \mathrm{t}_3 \\ \mathrm{t}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{rhs}_1^1(\mu_1^1) \\ \mathbf{rhs}_1^2(\mu_1^2) \\ \mathbf{rhs}_1^3(\mu_1^3) \\ \mathbf{rhs}_1^4(\mu_1^4) \end{pmatrix}$$

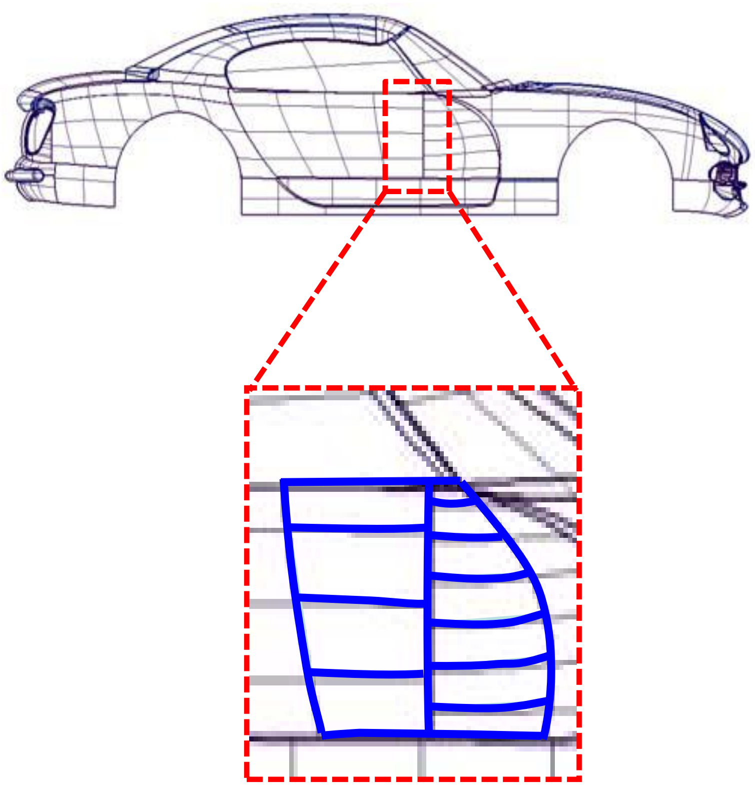
○ ○

G^1 condition of the edge can be solved independently → Edge G^1 condition

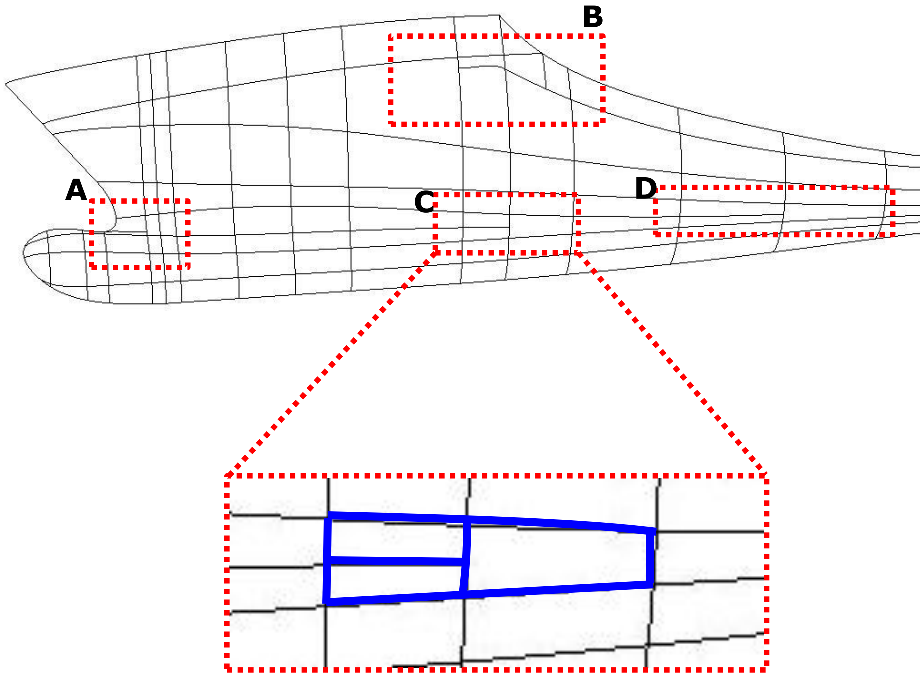
$$10 \cdot \begin{pmatrix} \bar{\lambda}_0^j & \lambda_0^j & 0 & 0 \\ \bar{\lambda}_1^j & \lambda_1^j & \bar{\lambda}_0^j & \lambda_0^j \\ 0 & 0 & \bar{\lambda}_1^j & \lambda_1^j \end{pmatrix} \begin{pmatrix} \mathrm{p}_2^j \\ \mathrm{r}_2^j \\ \mathrm{p}_3^j \\ \mathrm{r}_3^j \end{pmatrix} = \begin{pmatrix} \mathbf{rhs}_2^j(\mu_2^j) \\ \mathbf{rhs}_3^j(\mu_2^j) \\ \mathbf{rhs}_4^j(\mu_2^j) \end{pmatrix}$$

➡ Solve using Least Squares Method

What is a T-junction?

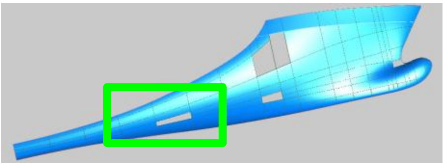


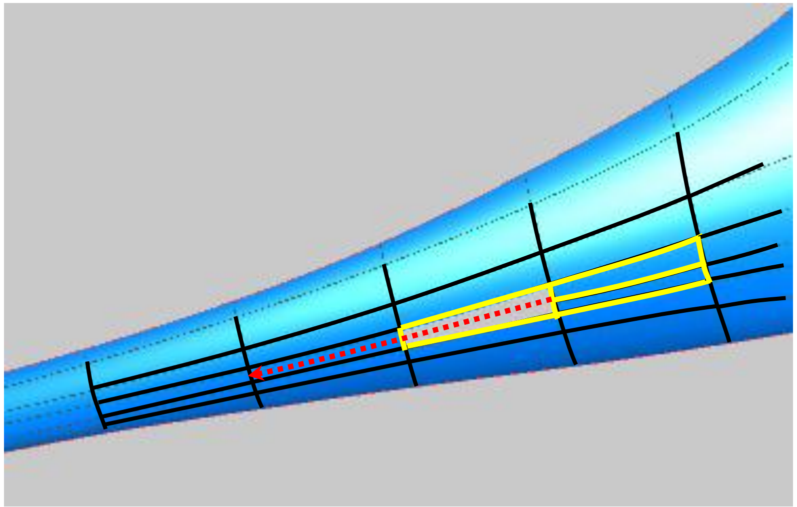
T-junction on a car body curve network



T-junction on a ship hull curve network

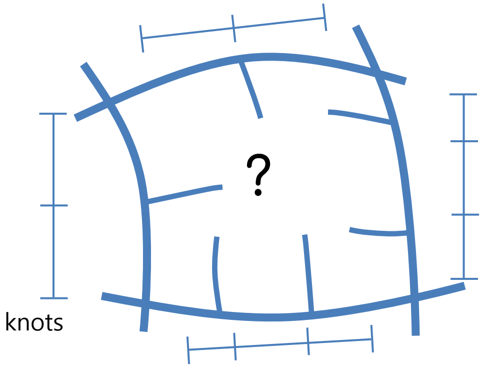
Why Does the T-junction Appear? (1/3)





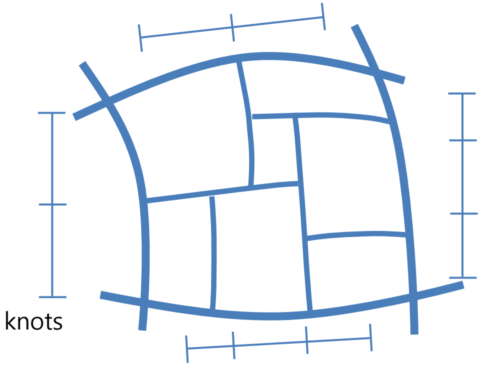
- We should stop extending the boundary curve at the narrow area of the boundary curves.
- The more boundary information is not necessary in the narrow area for representing the model.
- We need an interpolation method for T-junction when the boundary curve stopped at T-junction.

Why Does the T-junction Appear? (2/3)



B-spline boundary curves

Exact boundary

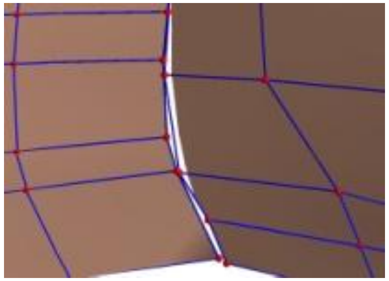


Surfaces with same boundary curves

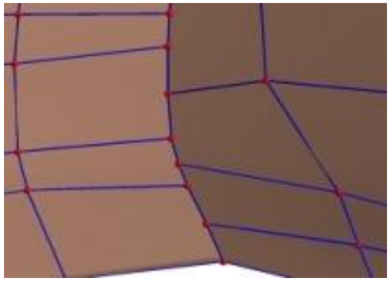
- There will be T-junctions when we interpolate the boundary curves without changing.

- It is possible to generate one surface with changing boundary curves.

Why Does the T-junction Appear? (3/3)



Gap with B-splines patch



No gap with T-splines*

- The gabs among the surfaces cause many errors in CAD/Graphics systems.
- The numerical errors from the gabs are decrease when we allow the T-junction because the surface model with T-junction can be watertight.
- T-splines is the one of the solutions for T-junction.

  → not popular in current CAD system

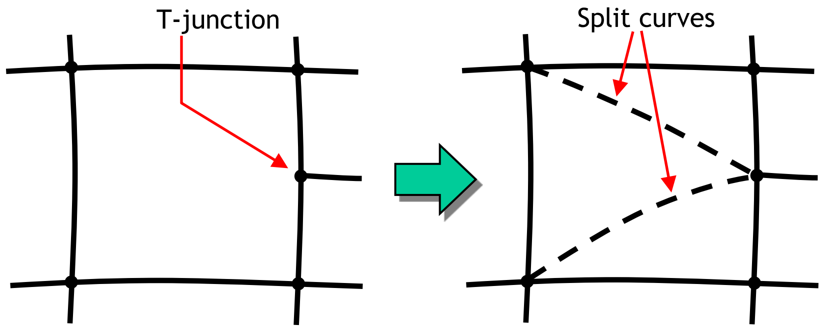
  → not inverse problem

* T. W. Sederberg, et. al., "T-splines and T-NURCCs," SIGGRAPH 03

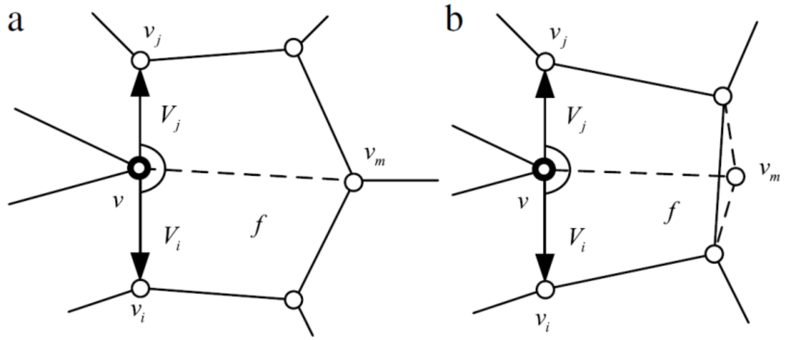
Previous Method for T-junction

- Subdivision method
  - subdivide the region to avoid the T-junction
  - triangles can appear → change triangle into rectangle
  - split curves are added

1.

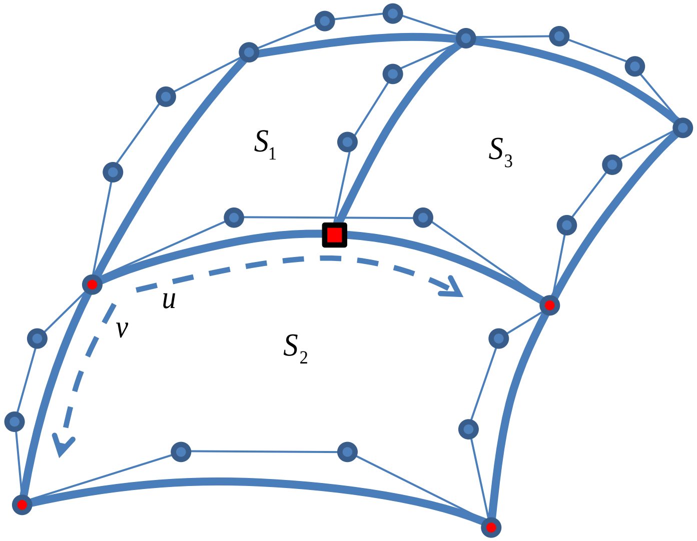


2.

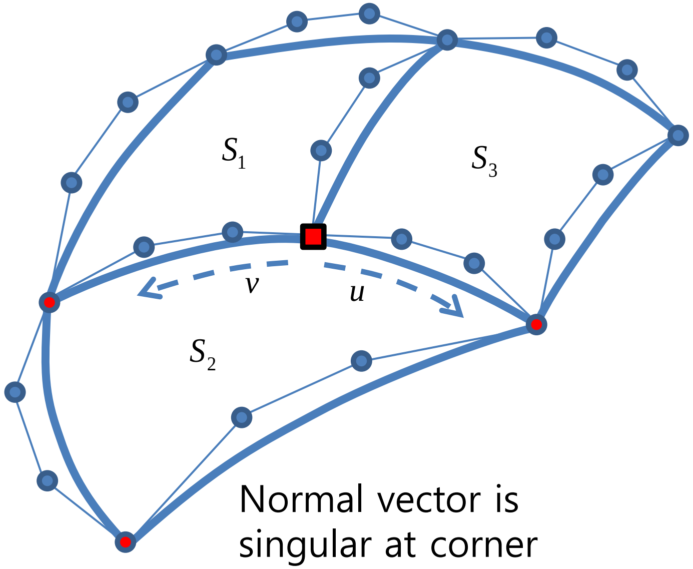


1. D.-Y. Cho, K.-Y. Lee, T.-W. Kim, Interpolating G^1 Bézier surfaces over irregular curve networks for ship hull design, Computer-Aided Design Vol. 38, No. 6, pp. 641–660, 2006.
2. K.-L. Shi, S. Zhang, H. Zhang, J.-H. Yong, J.-G. Sun, G^2 B-spline interpolation to a closed mesh, Computer-Aided Design, Vol. 43, No. 2, pp. 145–160, 2011.

Two Types of T-junction



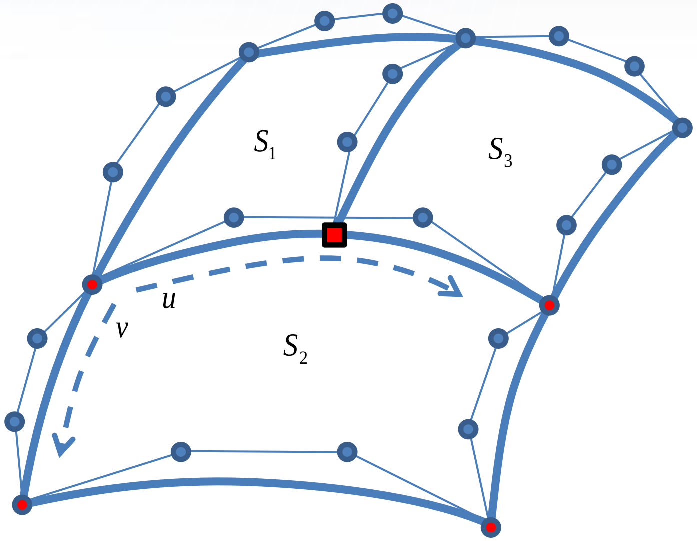
T-junction on a boundary



Normal vector is singular at corner

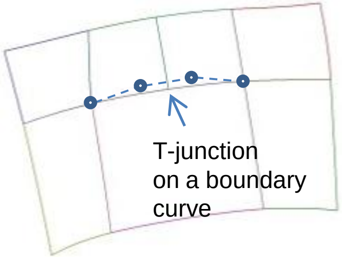
T-junction at a vertex

Constructing G^1 Bézier surfaces with a T-junction on a boundary curve

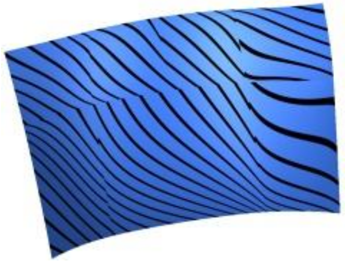


Outline of the Algorithm

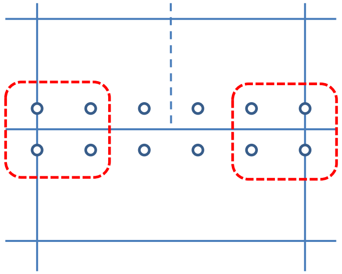
Boundary curve network with a T-junction



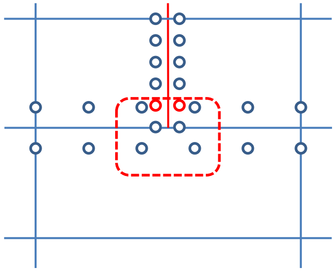
1. Generation of initial Bézier surfaces using the Coons patch method



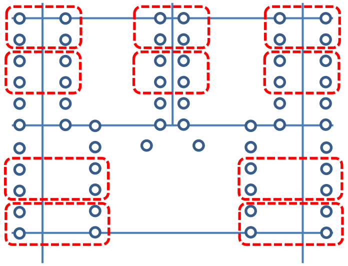
2. Solving vertex G^1 constraint around the T-junction without the T-junction



3. Solving edge G^1 constraints on the T-junction with subdividing vertex G^1 constraint

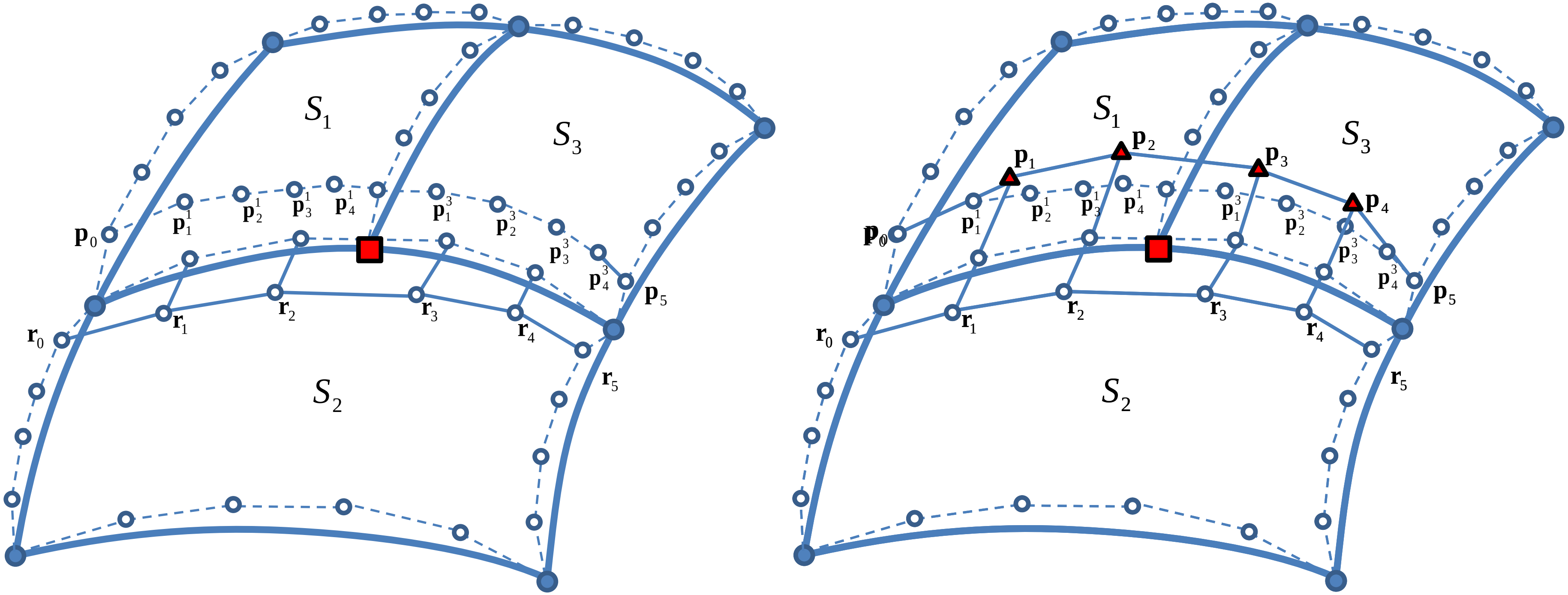


4. Solving the vertex and edge G^1 constraints for the other boundaries



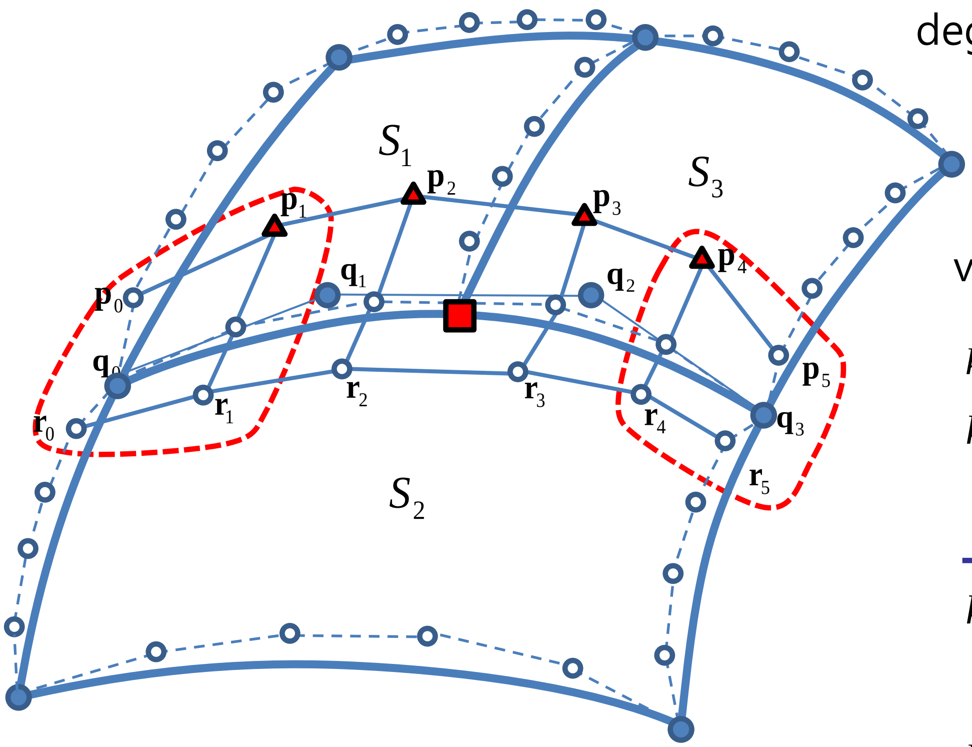
5. G^1 surface with a T-junction

Auxiliary Cross Derivative Curve of Two Patches



▲ : auxiliary cross derivative
control point for the S_1 and S_3

Solving Vertex G^1 Constraint without the T-junction



G^1 constraint between two patches :

$$[1-\lambda(t)]\cdot\mathbf{p}(t)+\lambda(t)\cdot\mathbf{r}(t)=[1-\mu(t)]\cdot\mathbf{q}_b(t)+\mu(t)\cdot\mathbf{q}_t(t)$$

degree: 1 5 1 5 4 2 4 2

vertex G^1 constraints:

$$k=0:\ \left(\bar{\lambda}_0\,\mathbf{p}_0+\lambda_0\,\mathbf{r}_0\right)=\left(\bar{\mu}_0\,\mathbf{q}_0+\mu_0\,\mathbf{q}_1\right),$$

$$k=1:\ 5\cdot\left(\bar{\lambda}_0\mathbf{p}_1+\lambda_0\mathbf{r}_1\right)=$$

$$-\left(\bar{\lambda}_1\mathbf{p}_0+\lambda_1\mathbf{r}_0\right)+2\cdot\left(\bar{\mu}_0\mathbf{q}_1+\mu_0\mathbf{q}_2\right)+4\cdot\left(\bar{\mu}_1\mathbf{q}_0+\mu_1\mathbf{q}_1\right),$$

$$k=5:\ 5\cdot\left(\bar{\lambda}_1\mathbf{p}_4+\lambda_1\mathbf{r}_4\right)=$$

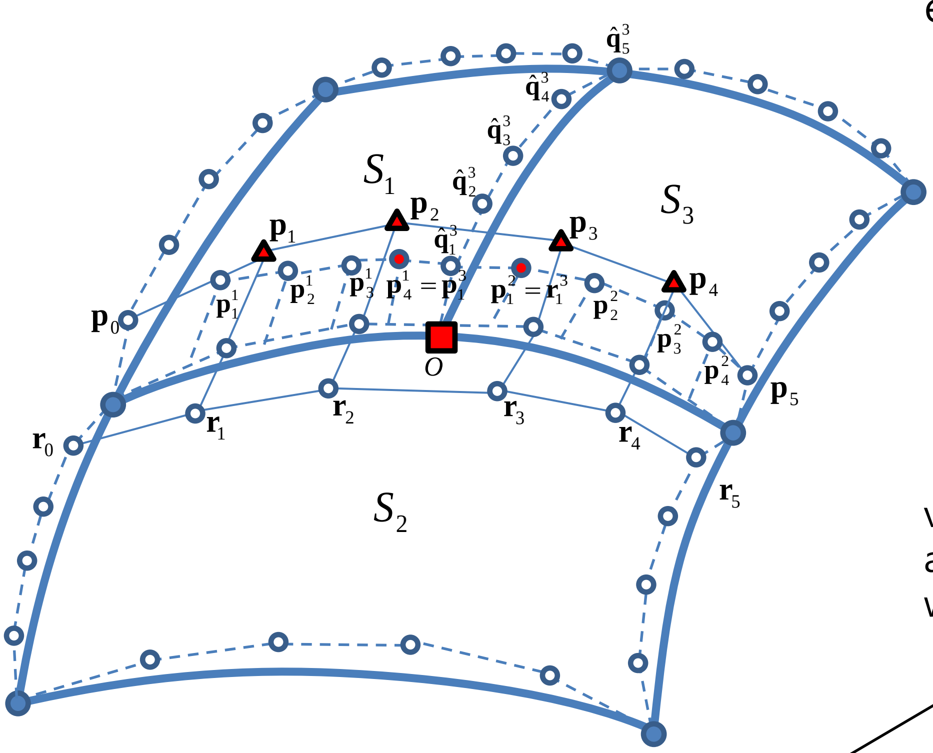
$$-\left(\bar{\lambda}_0\mathbf{p}_5+\lambda_0\mathbf{r}_5\right)+4\cdot\left(\bar{\mu}_3\mathbf{q}_2+\mu_3\mathbf{q}_3\right)+2\cdot\left(\bar{\mu}_4\mathbf{q}_1+\mu_4\mathbf{q}_2\right),$$

$$k=6:\ \left(\bar{\lambda}_1\mathbf{p}_5+\lambda_1\mathbf{r}_5\right)=\left(\bar{\mu}_4\mathbf{q}_2+\mu_4\mathbf{q}_3\right),$$

$\mathbf{p}_1,\mathbf{r}_1,\mathbf{p}_4,\mathbf{r}_4$: vertex enclosure control points

$\mathbf{p}_2,\mathbf{r}_2,\mathbf{p}_3,\mathbf{r}_3$: edge enclosure control points

Solving Edge G¹ Constraint Around the T-junction (1/2)



edge G¹ constraints:

$$k=2:\ 10\left(\bar{\lambda}_0\mathbf{p}_2+\lambda_0\mathbf{r}_2\right)=-5\left(\bar{\lambda}_1\mathbf{p}_1+\lambda_1\mathbf{r}_1\right)+\left(\bar{\mu}_0\mathbf{q}_2+\mu_0\mathbf{q}_3\right)$$
$$+8\left(\bar{\mu}_1\mathbf{q}_1+\mu_1\mathbf{q}_2\right)+6\left(\bar{\mu}_2\mathbf{q}_0+\mu_2\mathbf{q}_1\right),$$

$$k=3:\ 10\left(\bar{\lambda}_0\mathbf{p}_3+\lambda_0\mathbf{r}_3\right)+10\left(\bar{\lambda}_1\mathbf{p}_2+\lambda_1\mathbf{r}_2\right)$$
$$=4\left(\bar{\mu}_1\mathbf{q}_2+\mu_1\mathbf{q}_3\right)+8\left(\bar{\mu}_2\mathbf{q}_1+\mu_2\mathbf{q}_2\right)+6\left(\bar{\mu}_3\mathbf{q}_0+\mu_3\mathbf{q}_1\right),$$

$$k=4:\ 10\left(\bar{\lambda}_1\mathbf{p}_3+\lambda_1\mathbf{r}_3\right)=-5\left(\bar{\lambda}_0\mathbf{p}_4+\lambda_0\mathbf{r}_4\right)+6\left(\bar{\mu}_2\mathbf{q}_2+\mu_2\mathbf{q}_3\right)$$
$$+8\left(\bar{\mu}_3\mathbf{q}_1+\mu_3\mathbf{q}_2\right)+\left(\bar{\mu}_4\mathbf{q}_0+\mu_4\mathbf{q}_1\right).$$

+

vertex G¹ constraints
along the edge between S_1 and S_3
with quadratic λ :

$$k=1:\ 5\left(\bar{\lambda}_0^3\mathbf{p}_1^3+\lambda_0^3\mathbf{r}_1^3\right)\ =$$
$$-2\left(\bar{\lambda}_1^3\mathbf{p}_0^3+\lambda_1^3\mathbf{r}_0^3\right)+2\left(\bar{\mu}_0^3\mathbf{q}_1^3+\mu_0^3\mathbf{q}_2^3\right)+5\left(\bar{\mu}_1^3\mathbf{q}_0^3+\mu_1^3\mathbf{q}_1^3\right)$$

+

subdivision constraint :

$$\mathbf{p}_1^3B_0^1(c)+\mathbf{r}_1^3B_1^1(c)=\hat{\mathbf{q}}_1^3,$$

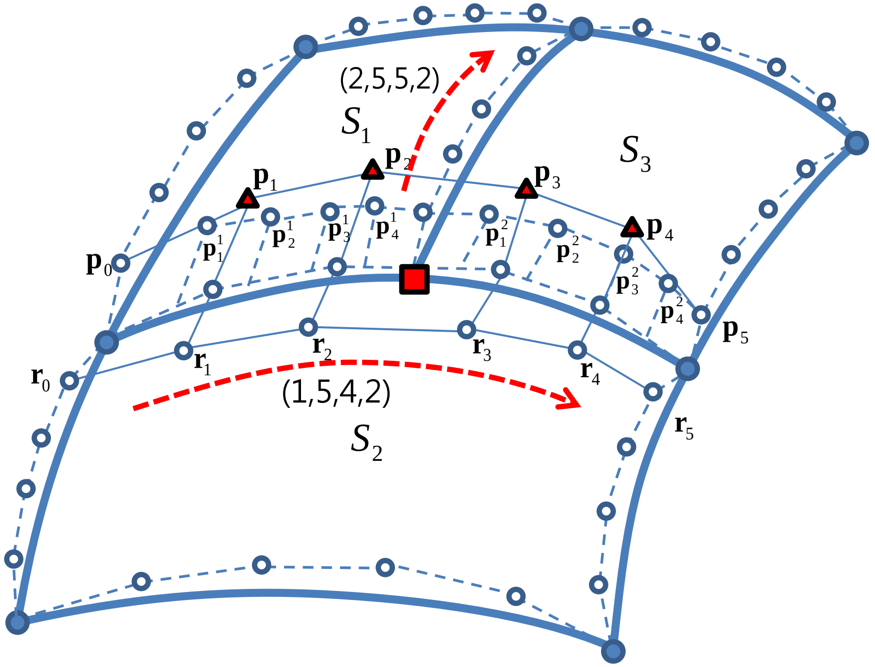
and we know

$$\mathbf{p}_1^3=\mathbf{p}_0B_0^4(c)+\mathbf{p}_1B_1^4(c)+\mathbf{p}_2B_2^4(c)+\mathbf{p}_3B_3^4(c)+\mathbf{p}_4B_4^4(c),$$
$$\mathbf{r}_1^3=\mathbf{p}_1B_0^4(c)+\mathbf{p}_2B_1^4(c)+\mathbf{p}_3B_2^4(c)+\mathbf{p}_4B_3^4(c)+\mathbf{p}_5B_4^4(c).$$

These constraints can be one constraint with

$$\bar{\lambda}_0^3=B_0^1(c),\quad \mu_1^3=\frac{1}{5}.$$

Solving Edge G^1 Constraint Around the T-junction (2/2)



▲ : Auxiliary cross-derivative control point for the S_1 and S_3

$$\bar{\lambda}(t)\cdot\mathbf{p}(t)+\lambda(t)\cdot\mathbf{r}(t)=\bar{\mu}(t)\cdot\mathbf{q}_b(t)+\mu(t)\cdot\mathbf{q}_t(t)$$

* (a,b,c,d)=degree of (λ, surface, μ, boundary)

- Edge G^1 system with a vertex G^1 system for the T-junction

$$\begin{bmatrix} 10\bar{\lambda}_0 & 10\lambda_0 & 0 & 0 \\ 10\bar{\lambda}_1 & 10\lambda_1 & 10\bar{\lambda} & 10\lambda_0 \\ 0 & 0 & 10\bar{\lambda} & 10\lambda_1 \\ \hdashline 5B_2^5(c) & 0 & 5B_3^5(c) & 0 \end{bmatrix}\begin{bmatrix} \mathbf{p}_2 \\ \mathbf{r}_2 \\ \mathbf{p}_3 \\ \hdashline \mathbf{r}_3 \end{bmatrix}=\begin{bmatrix} \mathbf{rhs}_1(\mu_2) \\ \mathbf{rhs}_2(\mu_2) \\ \mathbf{rhs}_3(\mu_2) \\ \hdashline \mathbf{rhs}_4 \end{bmatrix}$$

where

$$\mathbf{rhs}_1(\mu_2)=-5\left(\bar{\lambda}_1\mathbf{p}_1+\lambda_1\mathbf{r}_1\right)+\left(\bar{\mu}_0\mathbf{q}_2+\mu_0\mathbf{q}_3\right)+8\left(\bar{\mu}_1\mathbf{q}_1+\mu_1\mathbf{q}_2\right)+6\left(\bar{\mu}_2\mathbf{q}_0+\mu_2\mathbf{q}_1\right),$$

$$\mathbf{rhs}_2(\mu_2)=4\left(\bar{\mu}_1\mathbf{q}_2+\mu_1\mathbf{q}_3\right)+12\left(\bar{\mu}_2\mathbf{q}_1+\mu_2\mathbf{q}_2\right)+4\left(\bar{\mu}_3\mathbf{q}_0+\mu_3\mathbf{q}_1\right),$$

$$\mathbf{rhs}_3(\mu_2)=-5\left(\bar{\lambda}_0\mathbf{p}_4+\lambda_0\mathbf{r}_4\right)+6\left(\bar{\mu}_2\mathbf{q}_2+\mu_2\mathbf{q}_3\right)+8\left(\bar{\mu}_3\mathbf{q}_1+\mu_3\mathbf{q}_2\right)+\left(\bar{\mu}_4\mathbf{q}_0+\mu_4\mathbf{q}_1\right),$$

$$\mathbf{rhs}_4=2\left(\bar{\mu}_0^3\mathbf{q}_1^3+\mu_0^3\mathbf{q}_2^3\right)+5\left(\bar{\mu}_1^3\mathbf{q}_0^3+\mu_1^3\mathbf{q}_1^3\right)-2\left(\bar{\lambda}_1^3\mathbf{p}_0^3+\lambda_1^3\mathbf{r}_0^3\right)$$
$$-5\left(\bar{\lambda}_0^3\left(\mathbf{p}_0B_0^4(c)+\mathbf{p}_1B_1^4(c)+\mathbf{p}_4B_4^4(c)\right)+\lambda_0^3\left(\mathbf{p}_1B_0^4(c)+\mathbf{p}_4B_3^4(c)+\mathbf{p}_5B_4^4(c)\right)\right),$$

and

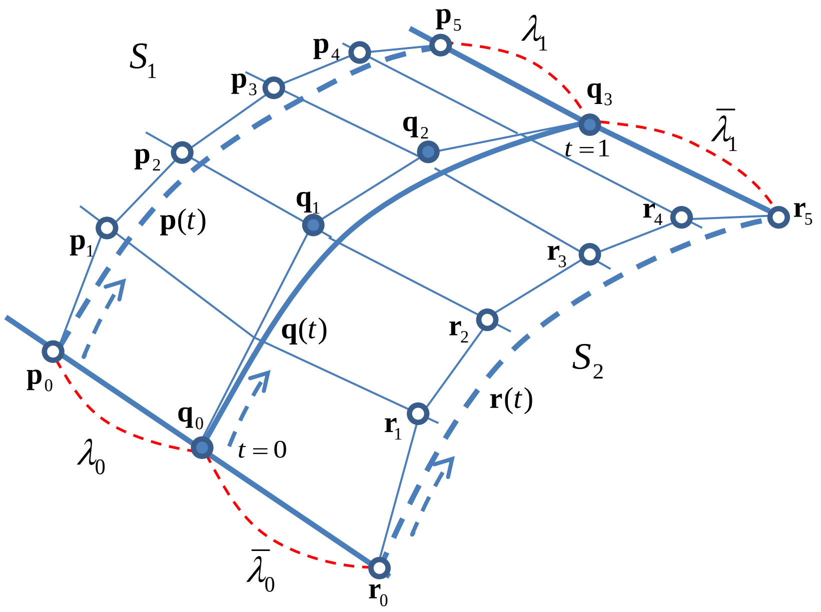
$$\mu_1^3=\frac{1}{5},\lambda_1^3=\lambda_0^3$$

$$rank(A)=\begin{cases} 4 & \text{if } \lambda_0\neq\lambda_1 \\ 3 & \text{if } \lambda_0=\lambda_1 \end{cases}$$

If $\lambda_0=\lambda_1$ then the solution exists if the following equality holds :

$$\mu_2=\frac{1}{6}\left(-\mu_0+4\mu_1+4\mu_3-\mu_4\right).$$

Scalar Weight Function: Linear λ



- In general, $\lambda_0 \neq \lambda_1$.
- If S_1 and S_2 are one patch, $\lambda_0 = \lambda_1$ from tensor product surface property.
- We cannot generate G¹ surface

$$\lambda(t) = \lambda_0 B_0^1(t) + \lambda_1 B_1^1(t)$$

$$[1-\lambda(t)]\cdot\mathbf{p}(t) + \lambda(t)\cdot\mathbf{r}(t) = [1-\mu(t)]\cdot\mathbf{q}_b(t) + \mu(t)\cdot\mathbf{q}_t(t)$$

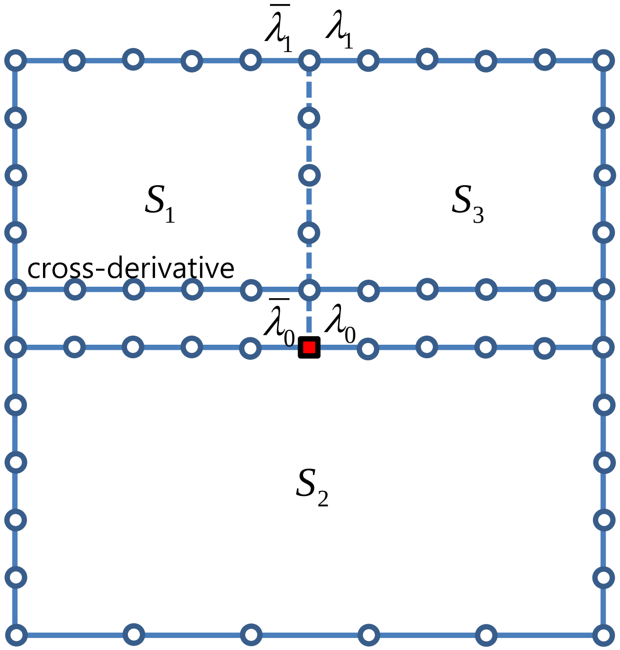
1 5 2 5 4 2 4 2

Scalar Weight Function: Quadratic λ

- If we use linear scalar weight function λ, it does not satisfy the subdivision of cross-derivative curve.
- We need a weight function which is more than quadratic.
- We use quadratic λ

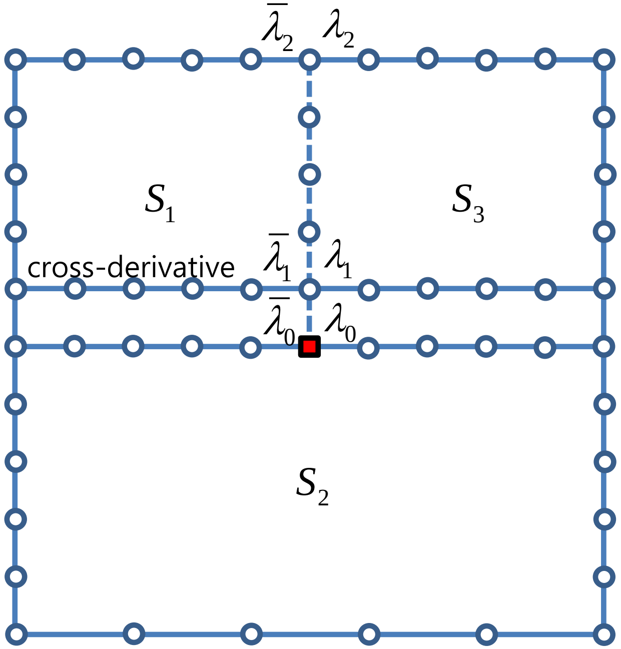
$$[1-\lambda(t)]\cdot\mathbf{p}(t)+\lambda(t)\cdot\mathbf{r}(t)=[1-\mu(t)]\cdot\mathbf{q}_b(t)+\mu(t)\cdot\mathbf{q}_t(t)$$

2 5 2 5 5 2 5 2



Linear weight function

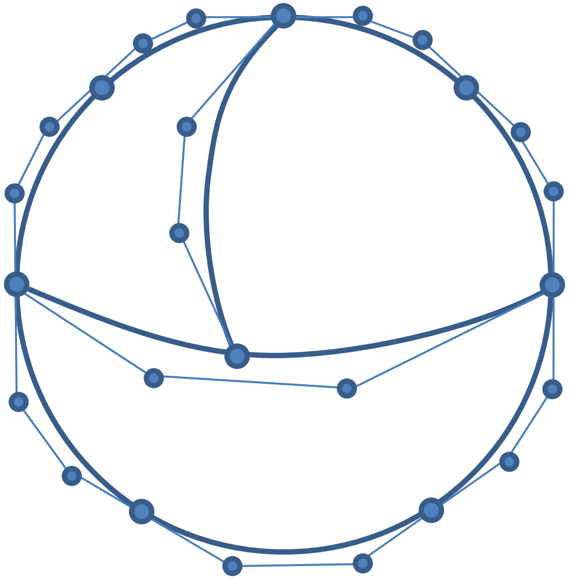
$\lambda_0=\lambda_1$, usually the given boundary curve is not satisfied with this condition.



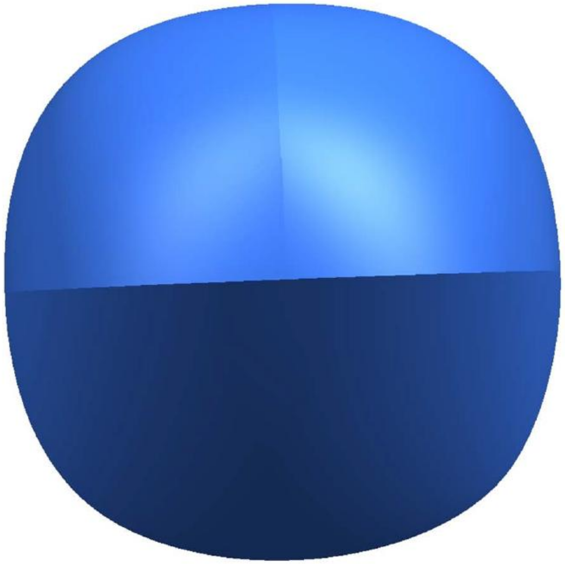
Quadratic weight function

$\lambda_0=\lambda_1$, and $\lambda_1\neq\lambda_2$

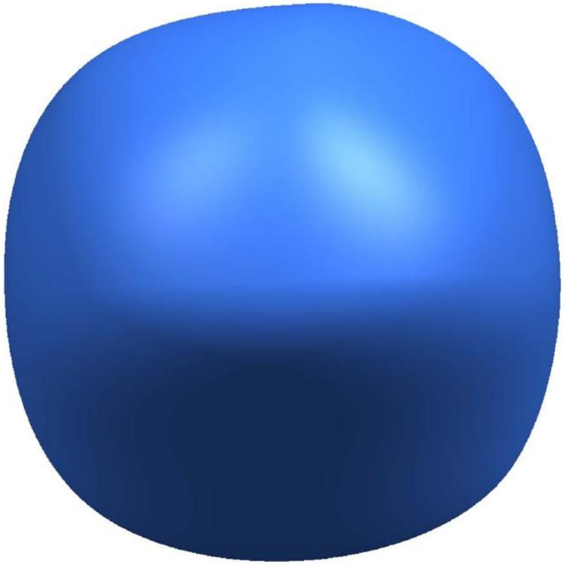
Result: Semi-sphere Shape



Example of a T-junction on a boundary curve network

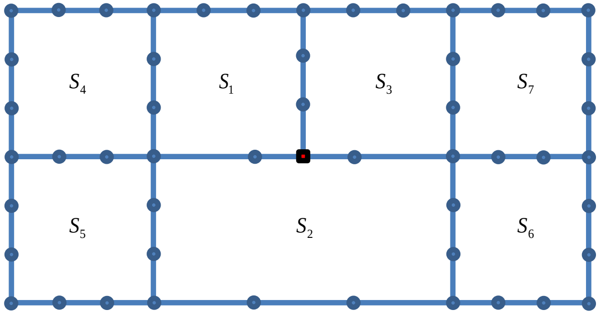


Lines of reflection from a C^0 surface constructed from Coons patches

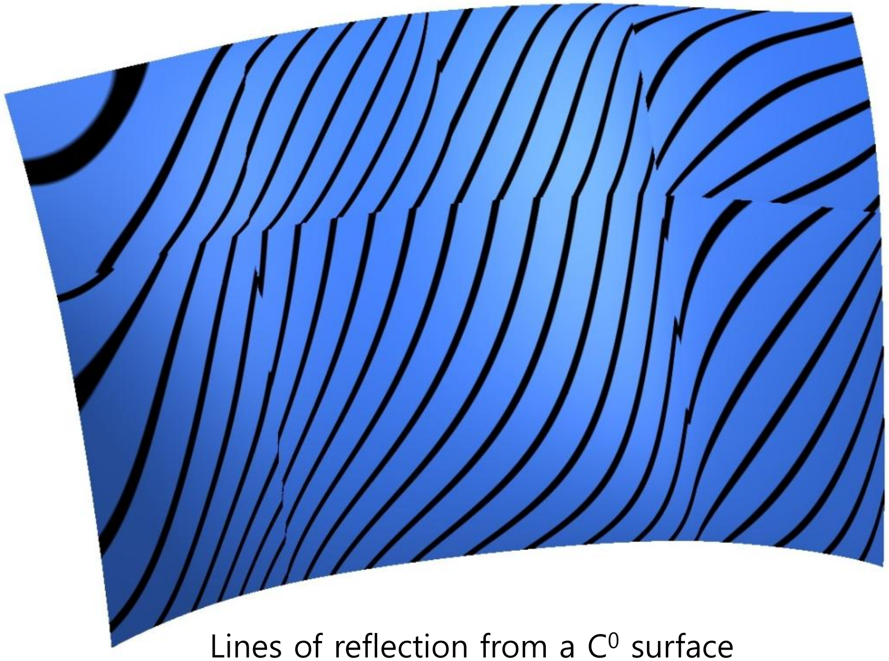


Lines of reflection from a G^1 surface constructed from our algorithm

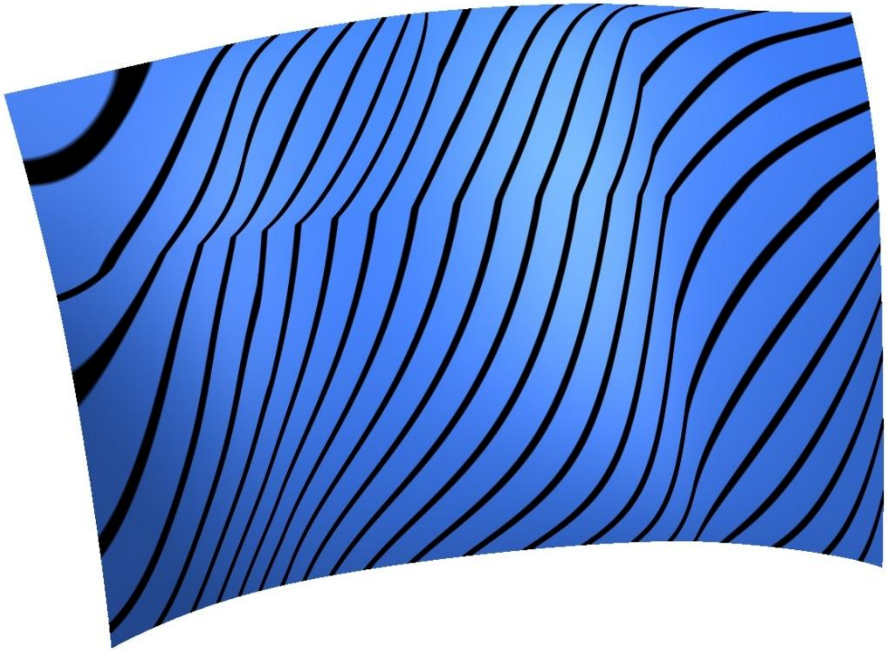
Result: T-junction with Surrounding Patches



Example of a T-junction on a boundary with the patches at the sides

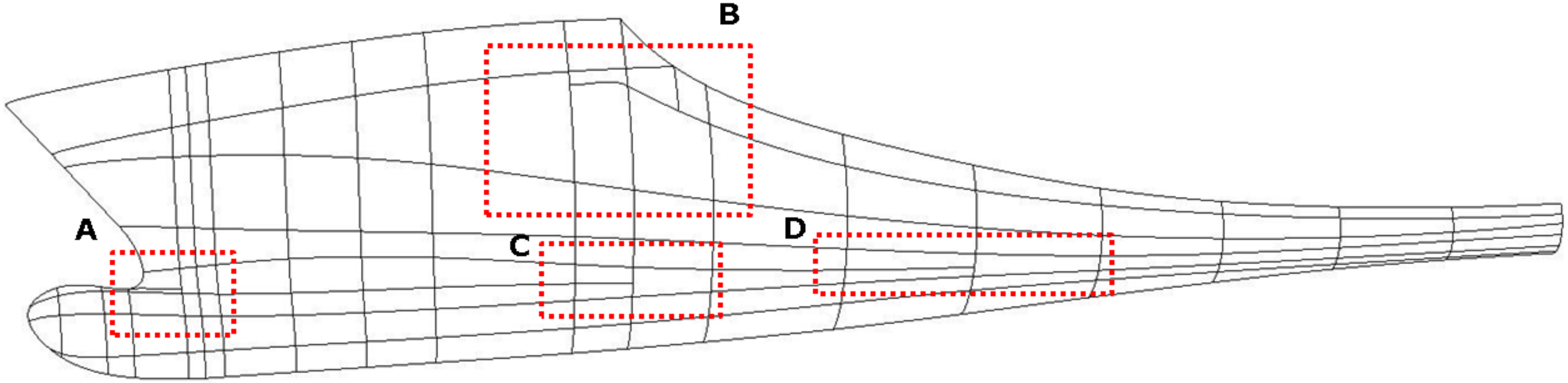


Lines of reflection from a C^0 surface constructed from Coons patches



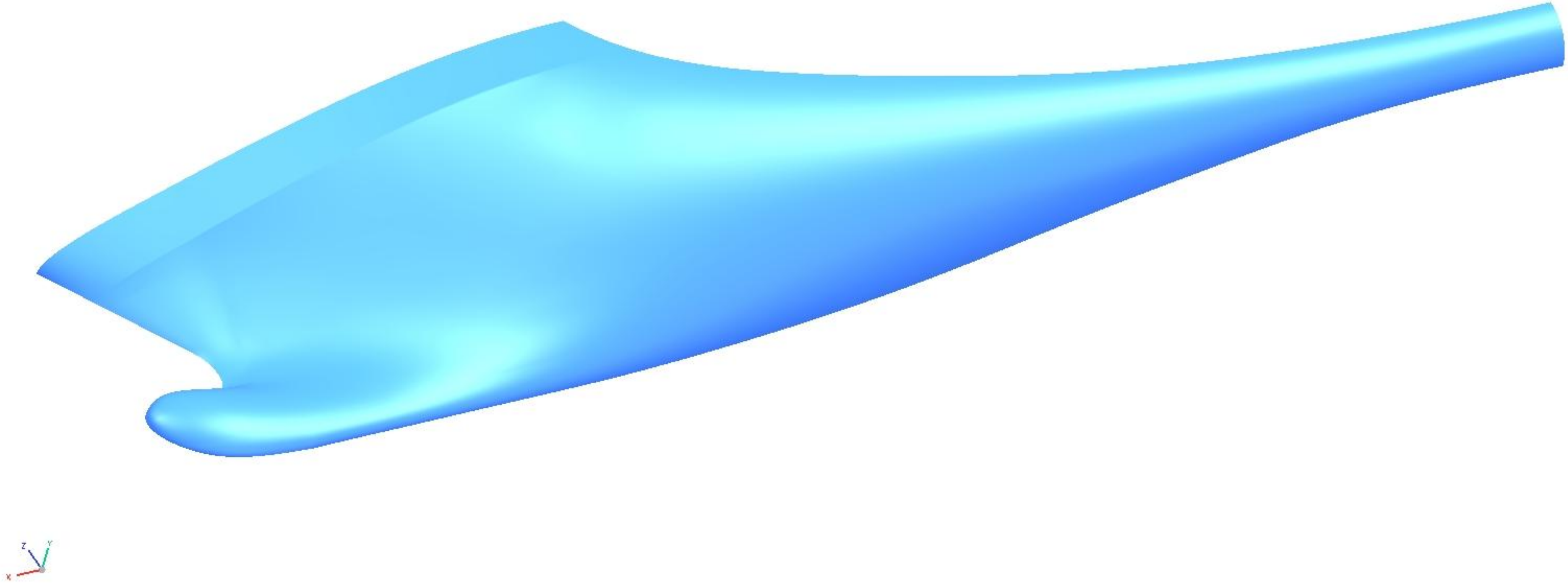
Lines of reflection from a G^1 surface constructed from our algorithm

Result: Ship Hull Wireframe Model



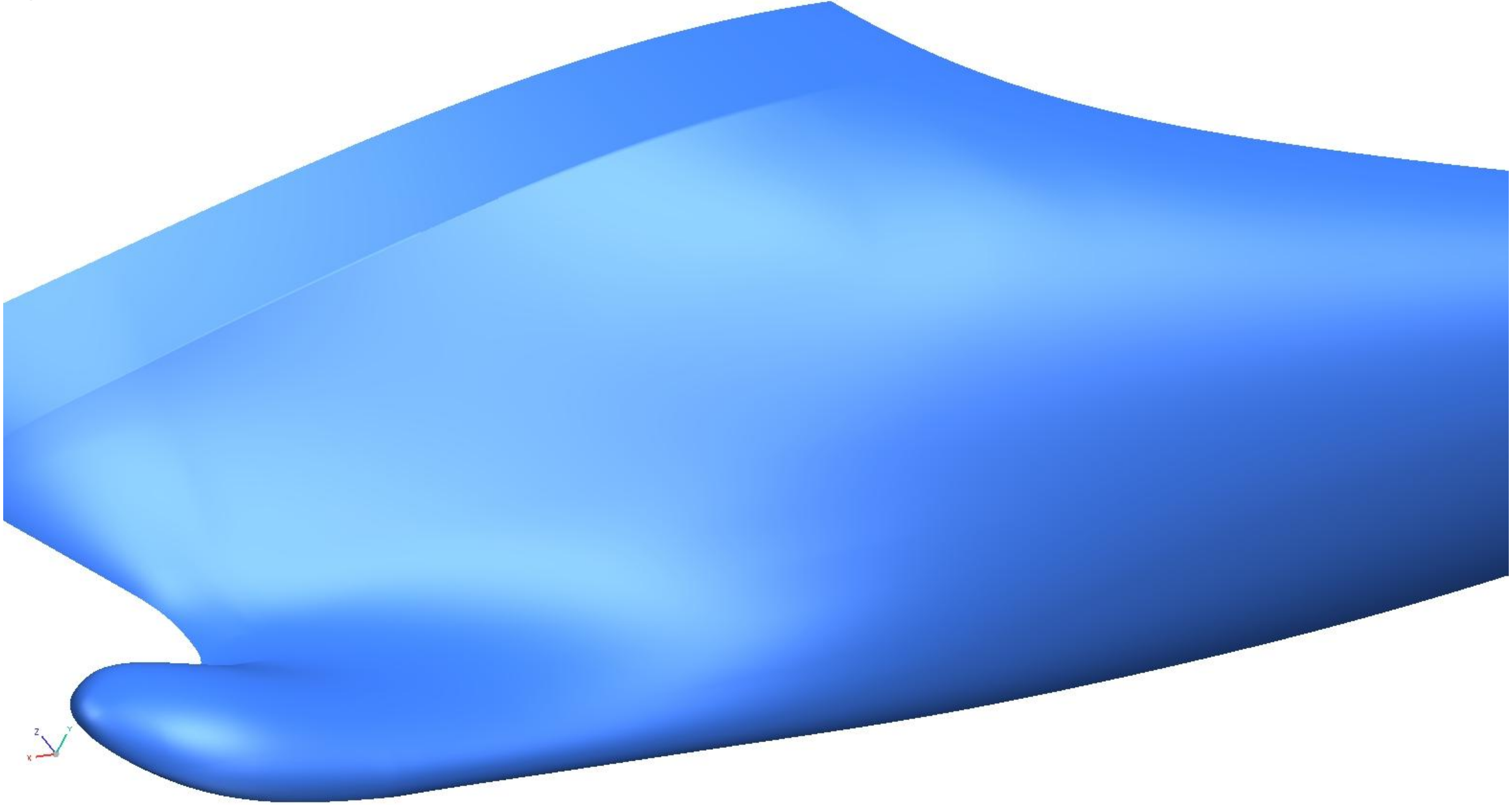
Shaded Ship Hull Surface Example (1/3)

G^1



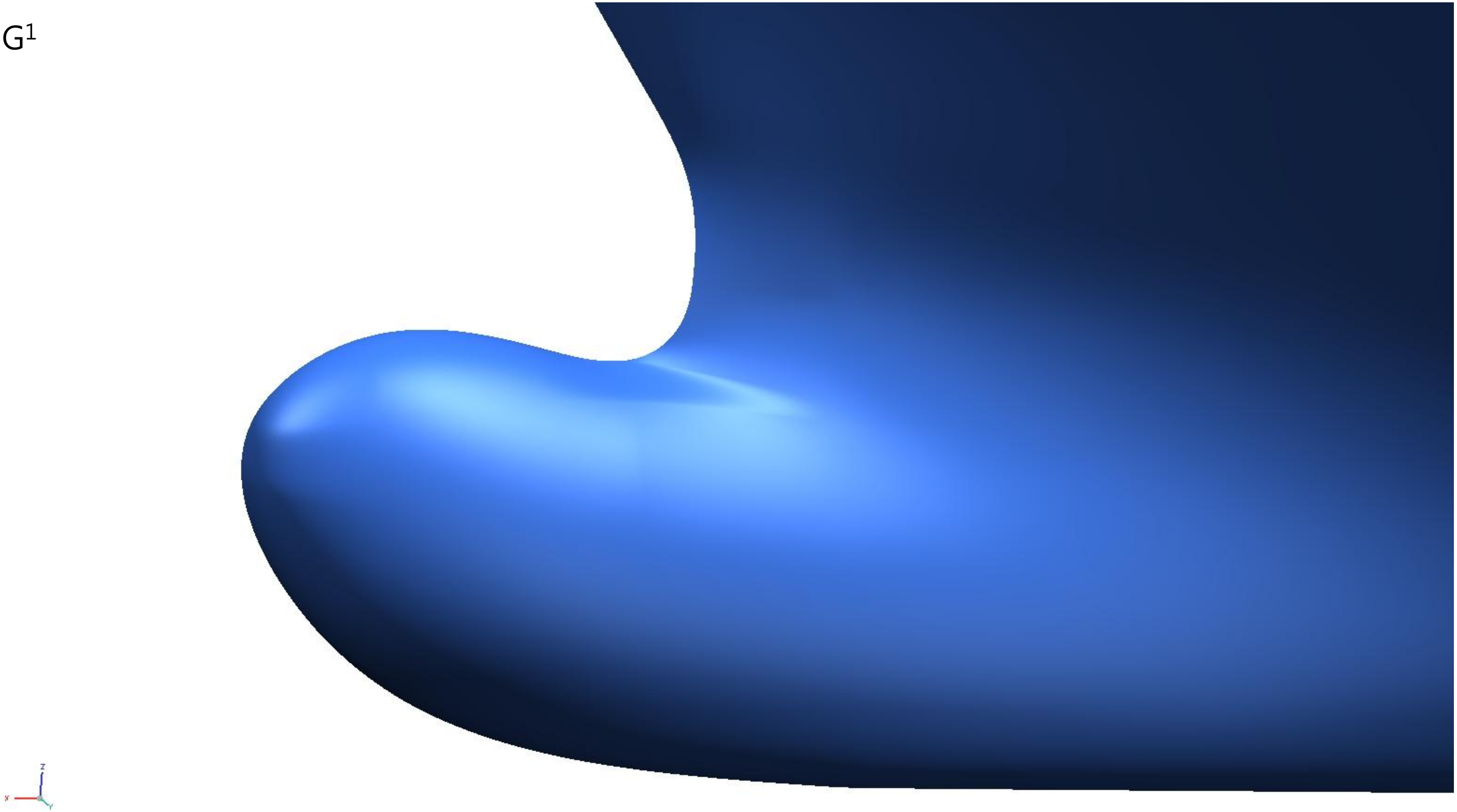
Shaded Ship Hull Surface Example (2/3)

G^1

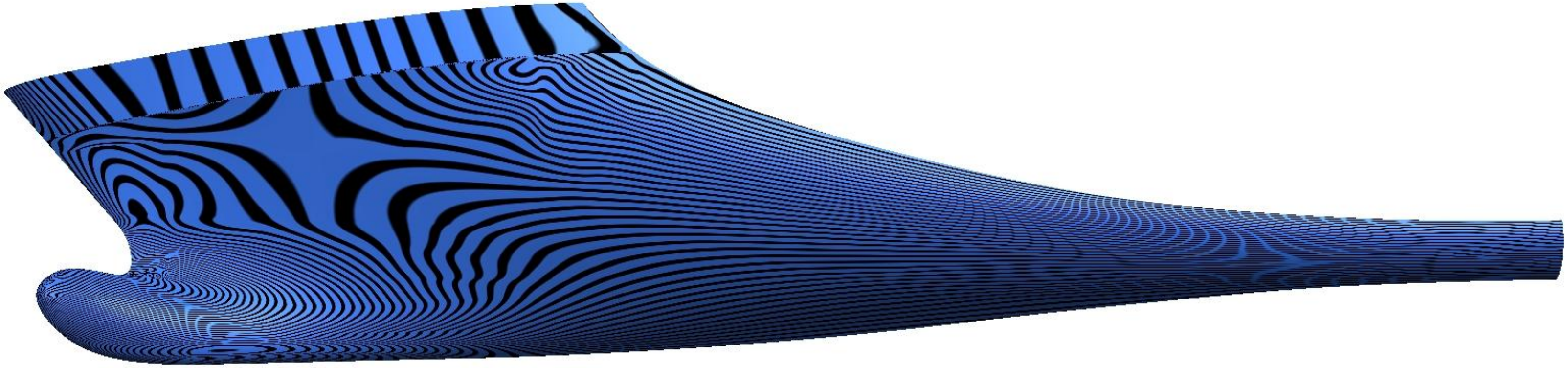


Shaded Ship Hull Surface Example (3/3)

G^1



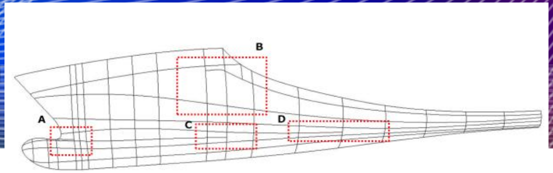
G^1 Ship Hull Surface Example (1/5)

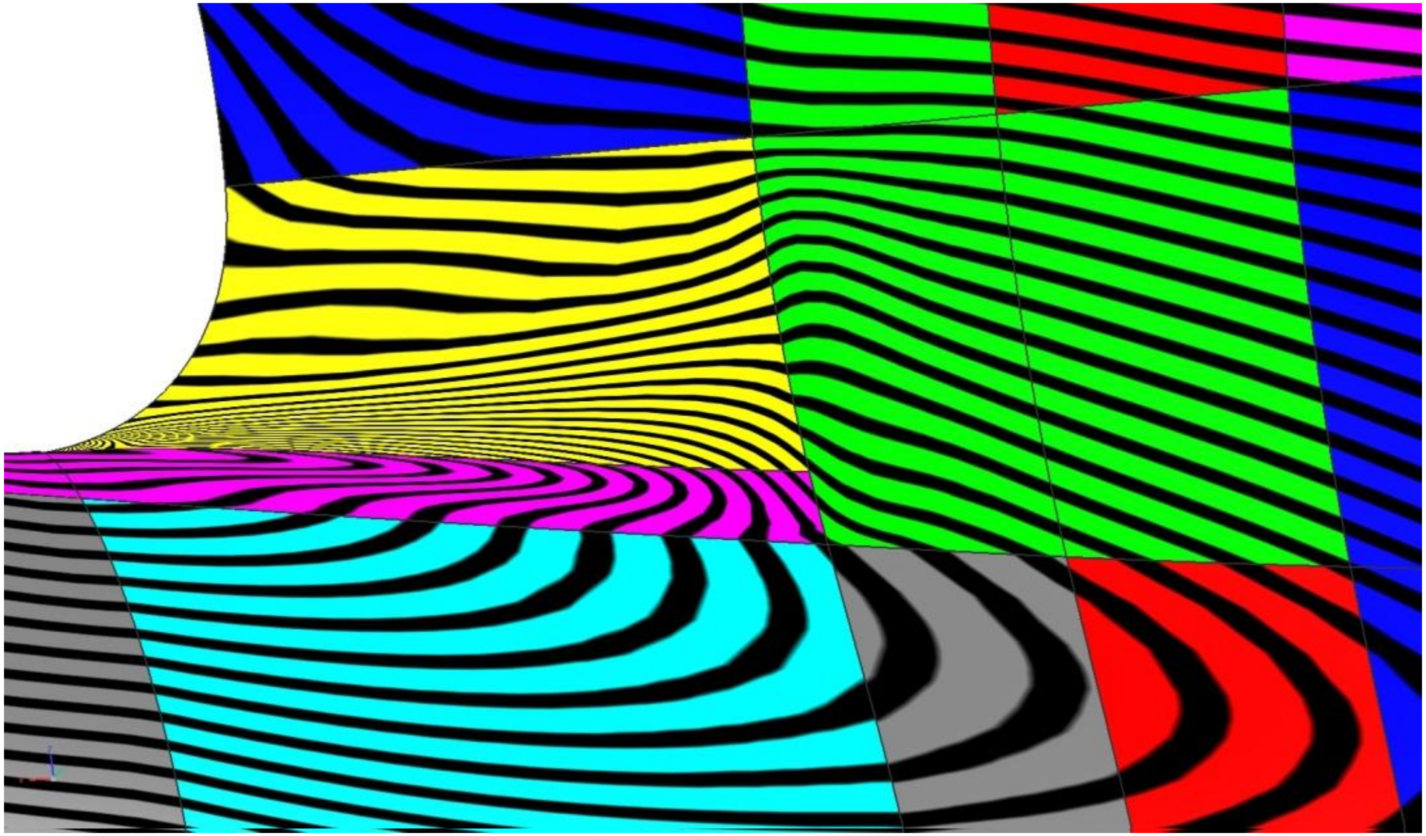


C^0 Coons patches

G^1 surface

G^1 Ship Hull Surface Example (2/5)

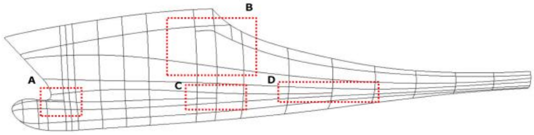




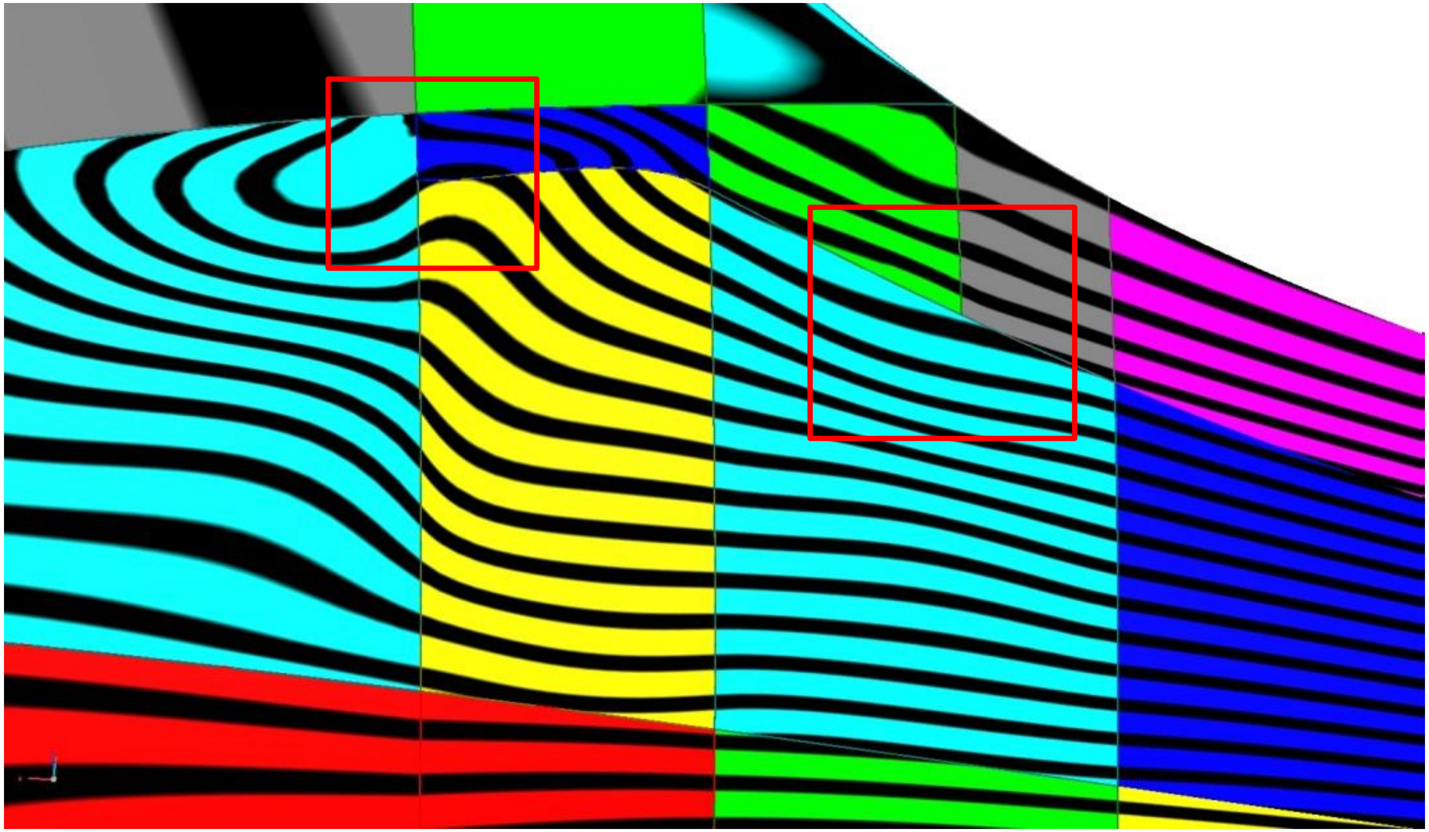
C^0 Coons patches

G^1 surface

G^1 Ship Hull Surface Example (3/5)



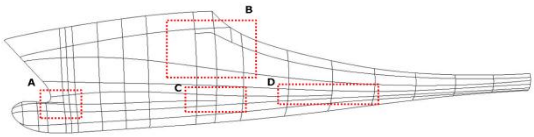
B.



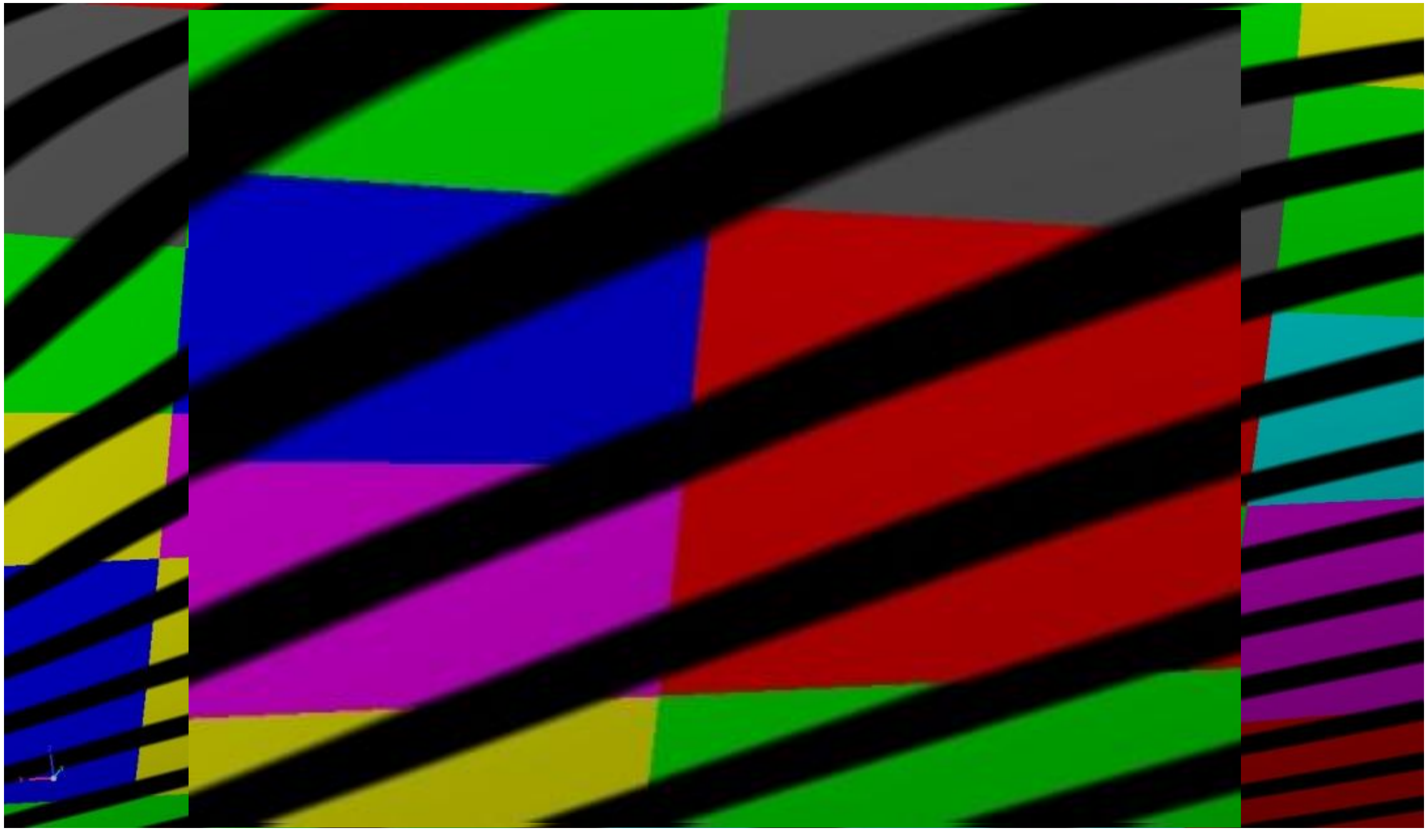
C^0 Coons patches

G^1-surface

G^1 Ship Hull Surface Example (4/5)



C.

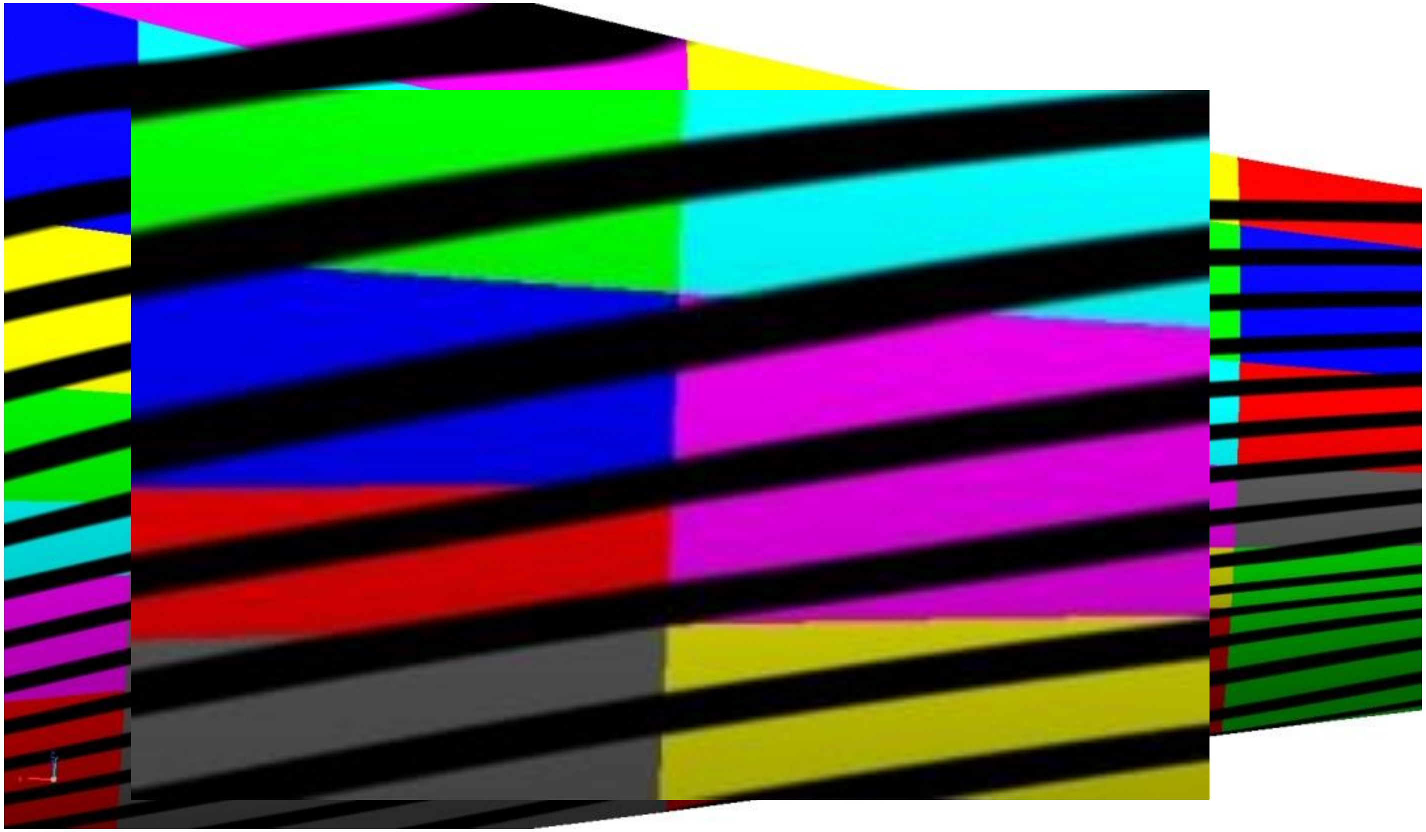


C^0 Coons surface

G^1 patches

G^1 Ship Hull Surface Example (5/5)

D.

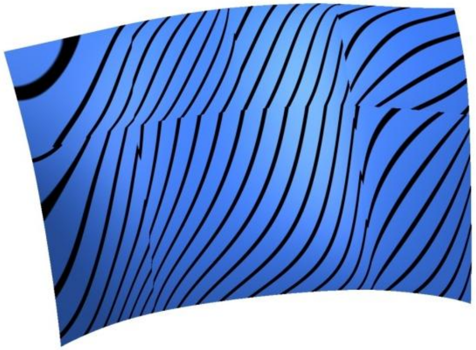


C^0 Coons patches

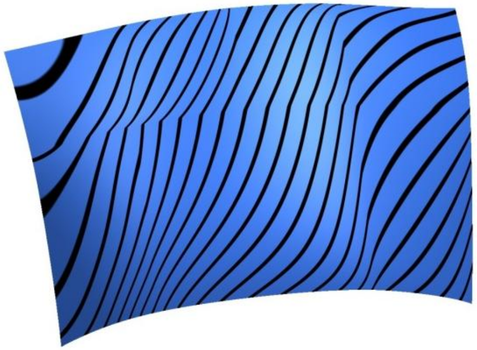
G^1 surface

Verifying the G^1 Continuity

- Reflection lines

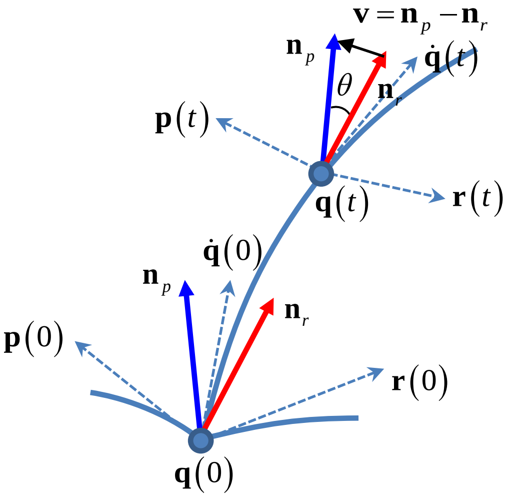


Lines of reflection from a C^0 surface constructed from Coons patches



Lines of reflection from a G^1 surface constructed from our algorithm

- Angle between two patches along the common boundary

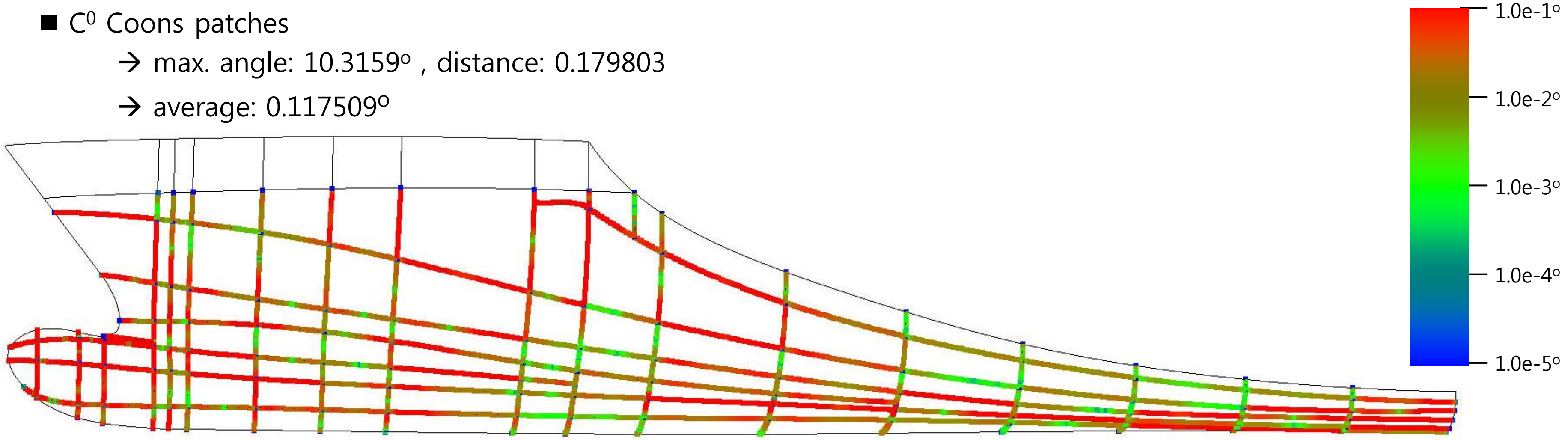


$\theta = 0^\circ$ or

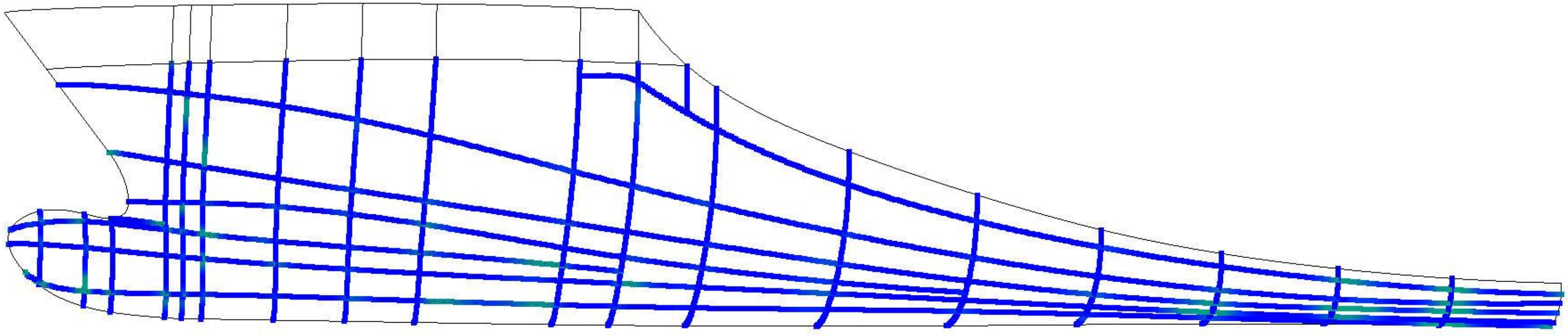
$$\|\mathbf{v}\| = \|\mathbf{n}_p - \mathbf{n}_r\| = 0$$

Verifying the G^1 Continuity using Angle

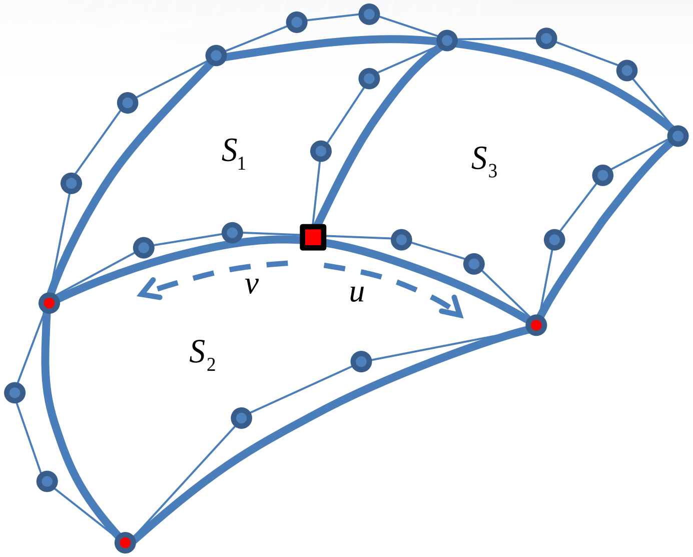
- C^0 Coons patches
  - → max. angle: 10.3159° , distance: 0.179803
  - → average: 0.117509°



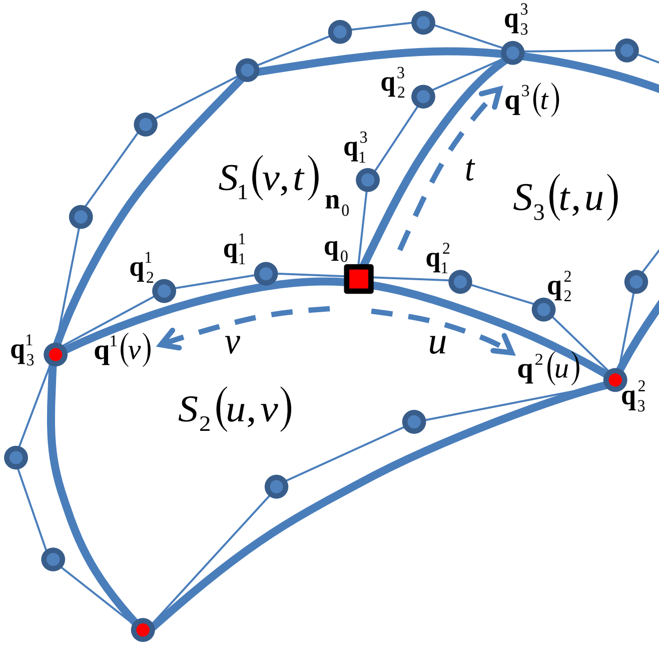
- Final G^1 Bézier patches
  - → max. angle: 0.000334°, distance: 5.82808e-6
  - → average: 2.12021e-5°



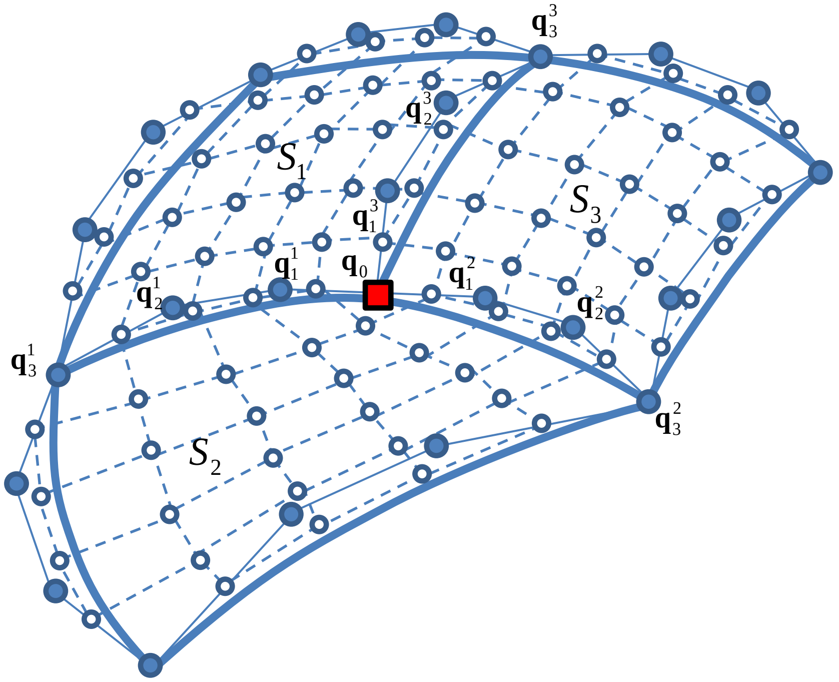
Constructing G^1 Bézier surfaces with a T-junction at a vertex



Given Problem

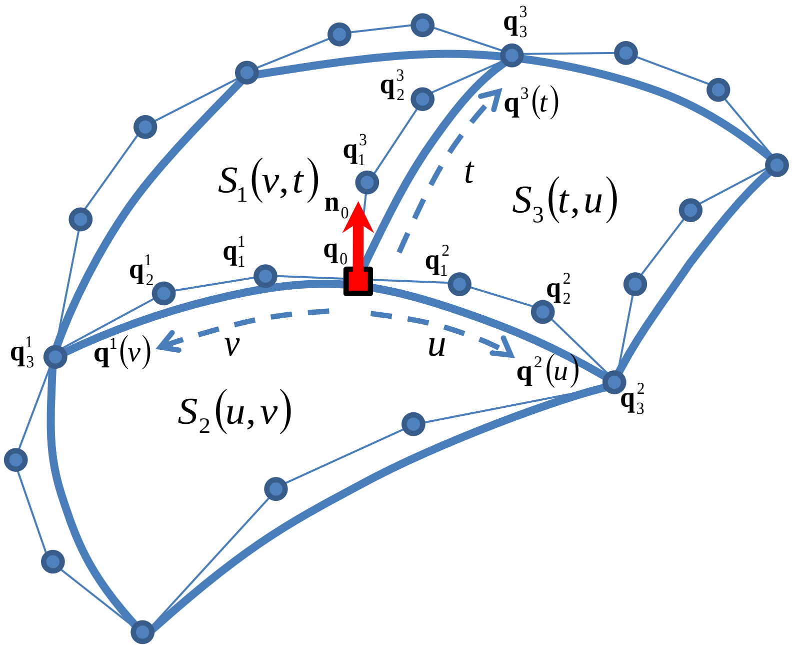


boundary curve network
with a T-junction



control points of the Bézier surfaces

Constraint for G^1 Surfaces at a T-junction



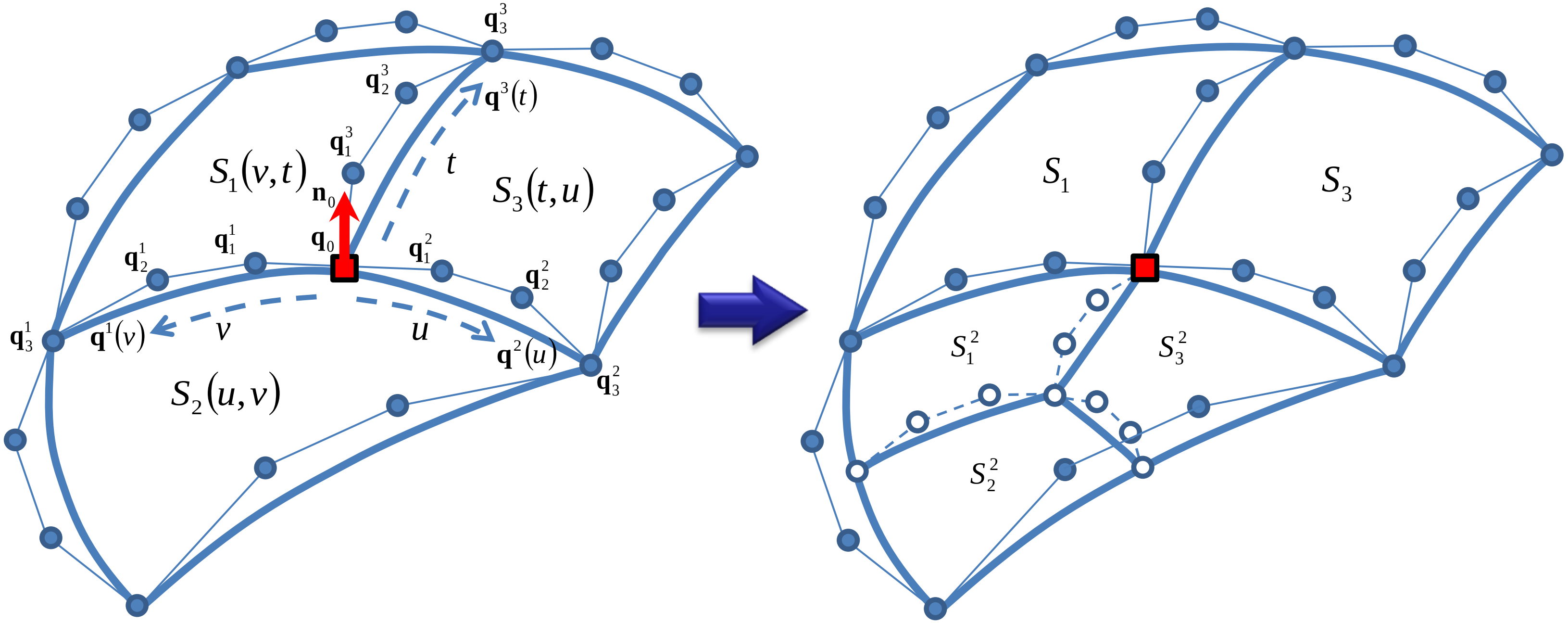
- Theorem. *If there are G^1 continuous Bézier surface with a T-junction at a 3-valent vertex, then* $\left\langle \left(\mathbf{q}_2^1 - \mathbf{q}_1^1\right), \mathbf{n}_0 \right\rangle = k^2 \left\langle \left(\mathbf{q}_2^2 - \mathbf{q}_1^2\right), \mathbf{n}_0 \right\rangle$.

- We can prove the theorem using the twist compatibility property from the boundary curves.

- In general, given boundary curves does not meet the condition in theorem.

- So we suggest a subdivision method with T-junctions.

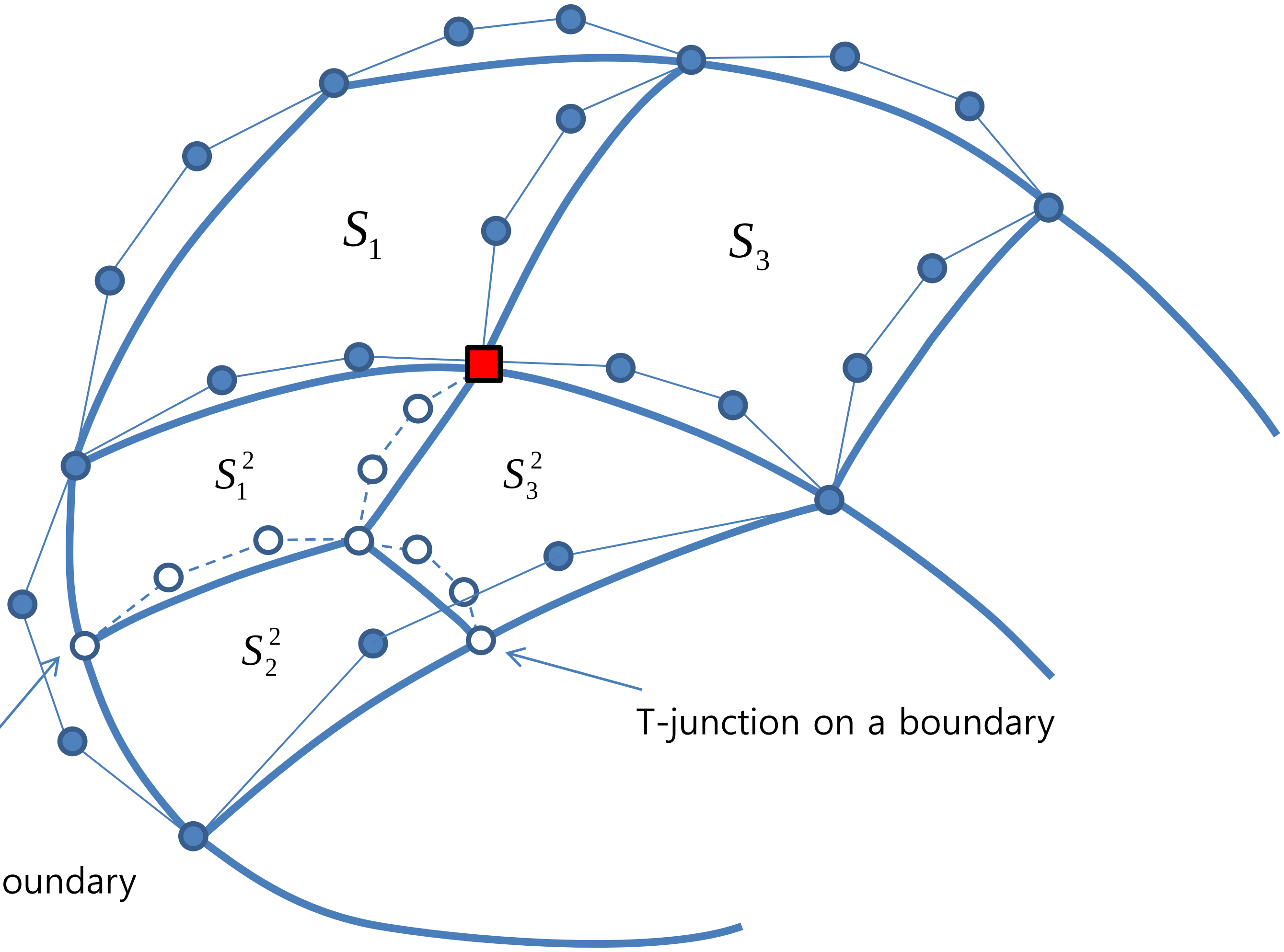
Subdivision Method for a T-junction



boundary curve network
with a T-junction

subdivision for degenerated surface

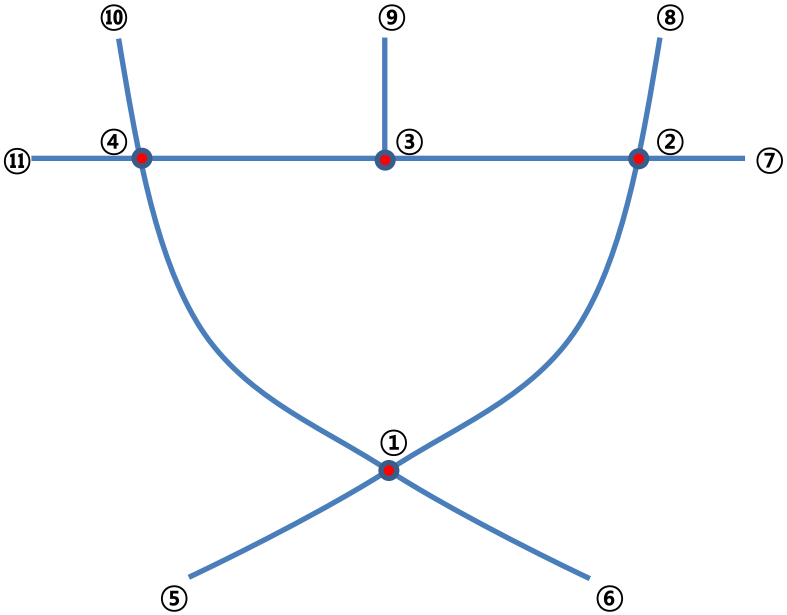
T-junction at Corner → Two T-junction on each boundary
- Subdivided with three rectangles

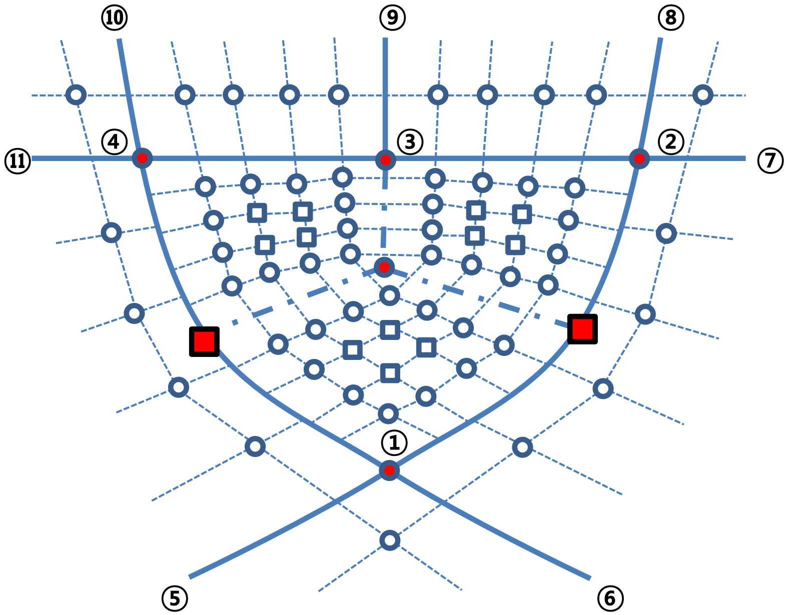


T-junction on a boundary

T-junction on a boundary

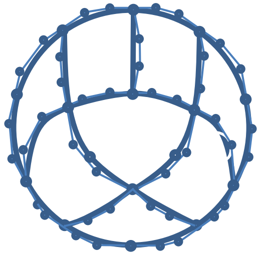
Input Boundary Curves and Output Surfaces



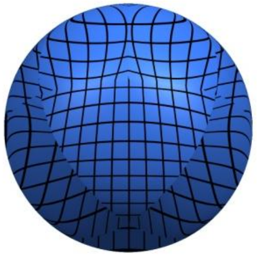


Outline of the Algorithm : G^1 Surfaces Interpolation Method using Subdivision

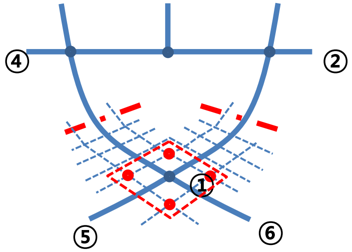
Boundary curve Network with a T-junction at a vertex



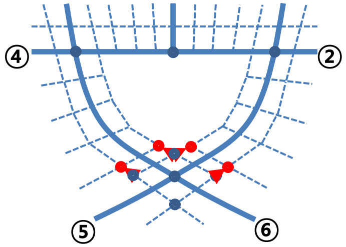
1. Generation of initial Bézier surfaces using the Coons patch method



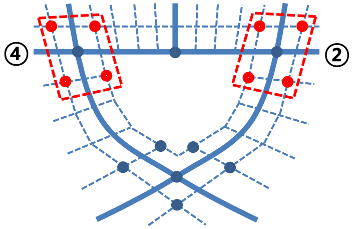
2. Determining four vertex enclosure control points



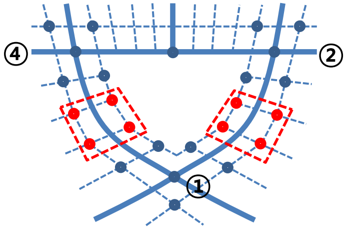
3. Determining vertex enclosure control points for big patches



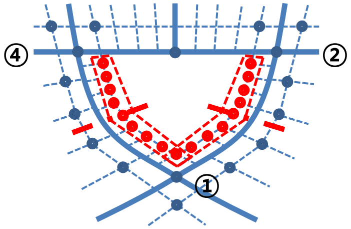
4. Determining vertex enclosure control points for the side vertices



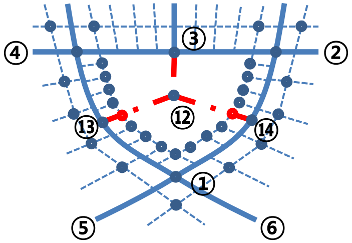
5. Determining edge enclosure control points for the big boundaries



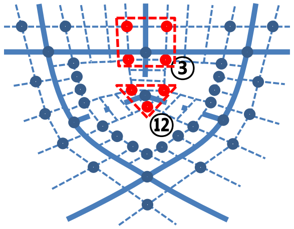
6. Subdividing the off-boundary curves



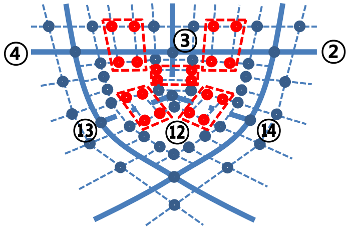
7. Generation of subdividing boundary curves



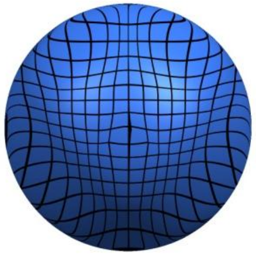
8. Determining vertex enclosure control points for the new vertex and the T-junction



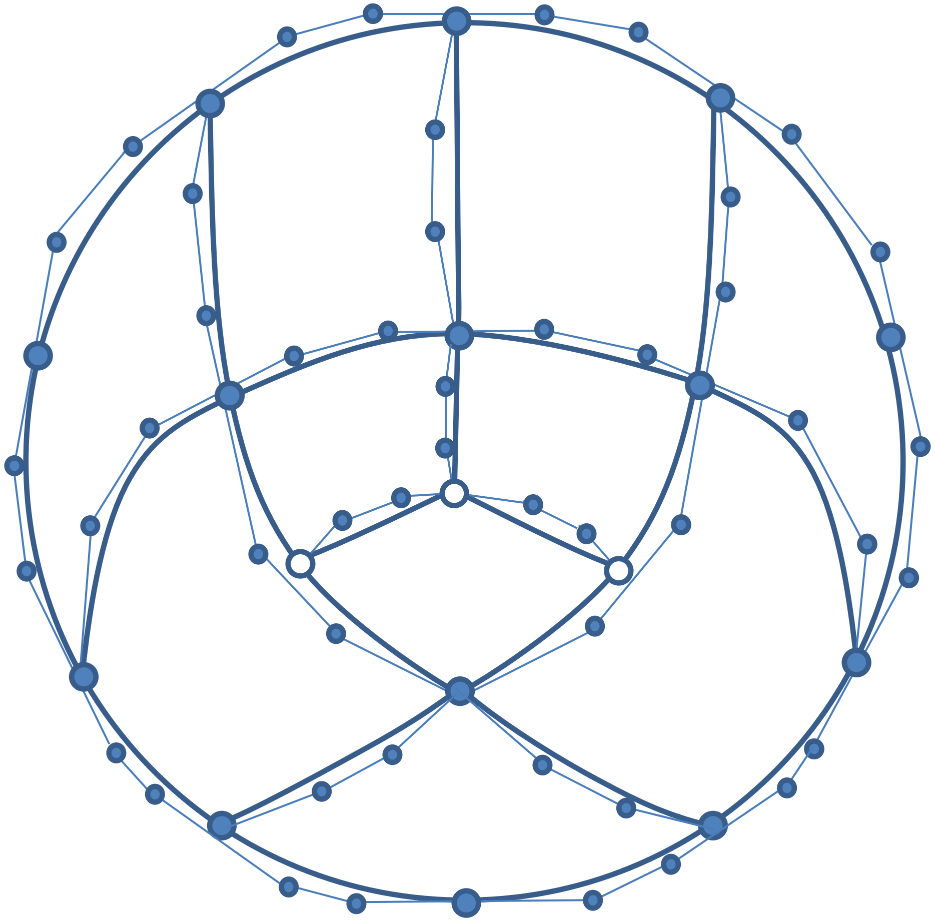
9. Determining edge enclosure control points for the remained edges



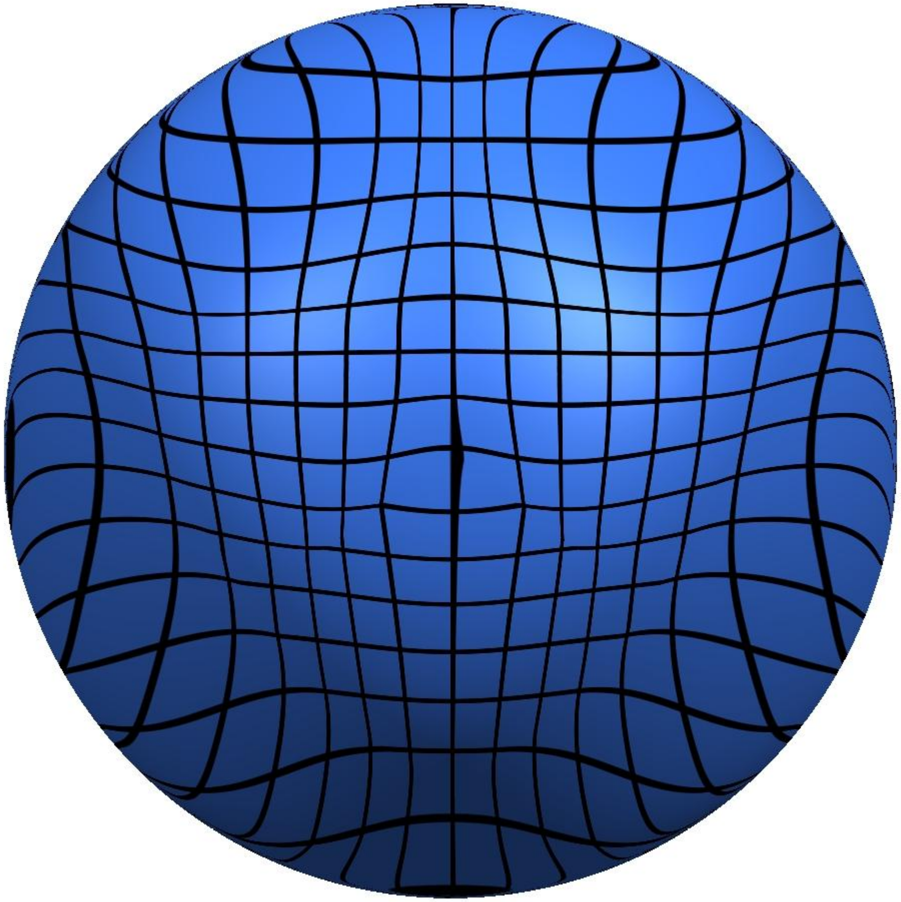
G^1 tensor-product Bézier surfaces



Result



Result



G^0 initial Bézier surfaces using the Coons patch

Conclusion and Future Works

- Conclusion
  - G^1 surface generating method for a T-junction on the boundary is presented.
    - The auxiliary cross-derivative curve is proposed.
    - The boundary curve network is unchanged.
    - Subdivision is not necessary.
  - G^1 surface generating method for a T-junction at a vertex is presented.
    - The constraint for the G^1 surface at a T-junction is proposed.
    - The subdivision method is proposed for the degenerate patch.
    - The subdivided patches make a T-junction on a boundary curve.
  - These are the first methods to construct G^1 surface from the boundary curve network with T-junction.
- Future works
  - Interpolation with many T-junctions on a boundary curve.
  - Transfinite interpolation of B-spline boundary curve network.
  - Generating the G^2 surfaces with a T-junction.

Thank you

Q&A