Creative Computing II

Fourier Transforms

19th January 2009

This lab sheet covers the basic properties of the Fourier Transform, and its use in computing system responses.

1. This first part of the lab sheet is about the extraction of information for a single frequency from a signal.

   (a) Construct the *Octave* vector corresponding to the complex exponential with frequency 1Hz, at a sample rate of 100Hz, over a time of 4s.

   *exp(2*pi*i*(0:399)/100)*

   (b) Construct the *Octave* vector corresponding to a cosine wave with amplitude 1 and frequency 1Hz, at a sample rate of 100Hz, over a time of 4s.

   *cos(2*pi*(0:399)/100)*

   (c) Compute the dot product of your signals from parts 1a and 1b; note your answer.

   The answer should be approximately 200.

   (d) Repeat parts 1b and 1c with a sine wave rather than a cosine wave. Verify that you understand the relationship between the two answers.

   The answer should be 200i, though depending on the details of how you perform the multiplication you might get -200i instead. In any case, what this is saying is that the amplitude of the sinusoidal wave is the same, but the two are $\frac{pi}{2}$ (90°) out of phase with each other.

   (e) Repeat parts 1b to 1d with cosine and sine waves with frequency 2Hz (but everything else, including the frequency of the complex exponential, unchanged). Comment on your answers.

   You should get 0 for all answers. This is saying that there is no 2Hz component in a pure sinusoid of frequency 1Hz.

   (f) Repeat parts 1b to 1d with a vector representing the signal $\sin(f_0 t) + \frac{1}{3}\sin(3f_0 t) + \frac{1}{5}\sin(5f_0 t)$ over 4s, where f_0 is 1Hz.

2. This next part introduces the Fourier Transform and how to interpret the results of *Octave*'s `fft` operator.

   (a) Take the signal from part 1f, and stem plot the modulus of the vector returned from `fft` acting on that signal. Check that you understand

i. the locations of the first three peaks;
The peaks are in the 5th, 13th and 21st Octave vector elements, which correspond to the 4th, 12th and 20th frequency harmonics of the fundamental respectively. The fundamental frequency of this analysis window (4s) is 0.25Hz, so those elements correspond to 1Hz, 3Hz and 5Hz frequencies respectively, which are the component frequencies of the signal.

ii. the relative heights of the first three peaks;
The peak heights are in the ratio 15:5:3 – exactly the same as the relative amplitudes of the three sine waves in the signal.

iii. the intensity of the first peak;
The Octave function `fft` produces output that is proportional to the length of the input vector, because it is constructed from the exponential dot product.

iv. the existence of the second three peaks.
The second three peaks are at frequencies above the Nyquist frequency, or equivalently at negative frequencies (those frequencies being aliased together: see slides from week 10). For purely real signals, these peaks are always symmetrically related to those below the Nyquist frequency, but for complex signals they need not be.

(b) take the inverse Fourier Transform (using `ifft`) of the Fourier-Transformed signal, and verify that the original signal is recovered.

3. This final part introduces the basic use of the Fast Fourier Transform for computing system responses to signals; we will be making use of this in future weeks.

   (a) Compute and plot the modulus of the Fourier Transform of the unit delay system, using a window (second parameter to `fft`) of length 10;
   `fft([0 1], 10)`

   (b) Generate a random signal of length 9, and compute its Fourier Transform over a window of length 10.

   (c) Verify that the inverse Fourier Transform of the elementwise product of your answers from parts 3a and 3b produces your signal delayed by one vector element, just as the use of `conv` would have done.

Other resources:

- Stephenson, G., *Mathematical Methods for Science Students*, material on Fourier Series;
- Boas, M. L., *Mathematical Methods in the Physical Sciences*, material on Fourier Series;