

Creative Computing II

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Autumn 2009, Tuesdays, 10:00–15:00
Winter 2010, Tuesdays, 13:30–17:30

Course Information

Administrative matters

- ▶ Course website:
`http://doc.gold.ac.uk/~mas01cr/teaching/cc227/`
- ▶ My e-mail address: `c.rhodes@gold.ac.uk`
- ▶ Feedback and Consultation hours:
 - ▶ Thursday, 14:00–16:00, room 2.06, BPB
 - ▶ Use phone for entry to BPB 2nd floor

Course Outline

Syllabus

- ▶ Visual perception;
- ▶ Animation;
- ▶ Sound, hearing and music;
- ▶ Signals;
- ▶ Audio and image filtering;
- ▶ Multimedia information retrieval;

Course Outline

Syllabus

- ▶ **Visual perception:** cones, rods and the eye; optical illusions; colour vision; colour spaces and profiles; motion perception and Gestalt psychology.
- ▶ **Animation:** approaches to animation; perception in video and film; making animations; visualisation.
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- ▶ **Signals:** the nature of signals; special signals; audio signals and sampling; frequency, amplitude and phase; the Fourier representation.
- ▶ **Systems:** linearity and time-invariance; impulse responses and convolution; spectral analysis; convolution by spectrum multiplication.
- ▶ **Audio and image filtering:** EQ; filter design; subtractive synthesis; echo and reverberation; resampling; image representation; two-dimensional convolution and image effects.
- ▶ **Multimedia information retrieval:** retrieval, fingerprinting and similarity; features and distance measures; systems for multimedia information retrieval.

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Systems

Convolution

The convolution operation is notated

$$y(t) = (h * x)(t)$$

In discrete time we define the operation as

$$y[n] = (h * x)[n] = \sum_{k=-\infty}^{\infty} h[k] \times x[n - k]$$

Systems

Convolution

Properties:

- ▶ commutative: $(a * b) = (b * a)$;
- ▶ associative: $(a * (b * c)) = ((a * b) * c)$;
- ▶ distributive over addition: $(a * (b + c)) = (a * b + a * c)$

Octave: `conv` function

- ▶ care required in interpreting output (time origin);
- ▶ `length(conv(a,b)) = length(a) + length(b) - 1`

Systems

Convolution

Why convolution?

- ▶ implementation of LTI systems!

The output y of a system H for an input signal x is the convolution of the input and the impulse response of the system.

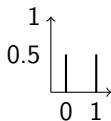
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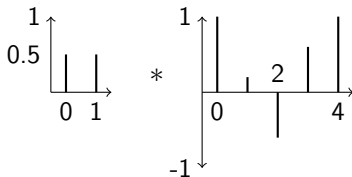
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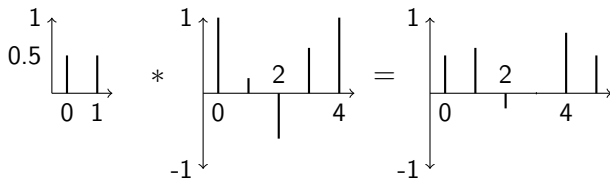
Systems

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Systems

The Unit Delay System, Again

Let

$$h[k] = \begin{cases} 1 & k = 1 \\ 0 & \text{otherwise} \end{cases}$$

The convolution of h with an input signal x :

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

is equal to $x[n-1]$. Therefore $h[\tau]$ represents the unit delay system.

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Octave:

- ▶ $\mathbf{h} = [0 \ 1]$
- ▶ $[0 \ 1]$ is also the representation of the impulse response of the unit delay system.
- ▶ NB: vector index 1 corresponds to discrete time 0.

Systems

Scaled Delay Systems

General transformation built from scaling and time delay:

- ▶ delay by k time units;
- ▶ scale by a factor s .

System notated as $sT_k\{\}$.

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Octave:

- ▶ `[zeros(1,k) s]`
- ▶ exercise: check that convolving this kernel with a signal outputs the appropriate scaled delayed signal.

Systems

Convolution Efficiency

Consider a linear, time-invariant system H with Finite Impulse Response

- ▶ Impulse Response of length L (in samples);
- ▶ All other values 0

Implementation of that system with direct convolution for an input signal X of length N :

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{L-1} h[k]x[n-k]$$

Computational Complexity:

- ▶ $O(NL)$
- ▶ ... but we can do better ...

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Computational Complexity:

- ▶ $O(NL)$
- ▶ ... but we can do better ...
- ▶ ... with some more technical machinery.

Numbers

The Natural Numbers

The *Natural Numbers* are the (strictly) positive integers.

$\overset{\bullet}{1}$ $\overset{\bullet}{2}$ $\overset{\bullet}{3}$ $\overset{\bullet}{4}$ $\overset{\bullet}{5}$ $\overset{\bullet}{6}$ $\overset{\bullet}{7}$ $\overset{\bullet}{8}$...

- ▶ First recorded use (of tallies) c. 35000 BC;
- ▶ Place-value systems:
 - ▶ Babylonians (base 60, c. 3400 BC);
 - ▶ Egyptians (base 10, c. 3100 BC).
- ▶ The set of natural numbers is sometimes denoted \mathbb{N}

Numbers

The Number Zero

Zero as a number in an of itself:

- ▶ What number, added to 4, gives 4?
 - ▶ Solve $4 + x = 4$ for x .
- ▶ What do I have left if I subtract 7 from 7?
 - ▶ $7 - 7 = ?$

0 1 2 3 4 5 6 7 8 ...

- ▶ NB: not the same as the 'no tens' in a place-value system;
- ▶ Historical:
 - ▶ India, China in 4th Century BC;
 - ▶ Greece: Ptolemy 130AD
 - ▶ Persia: Abū ʿAbdallāh Muḥammad ibn Mūsā al-Khwārizmī (c.780–c.850)
- ▶ The natural numbers and zero denoted by \mathbb{N}_0 .

Numbers

Negative Numbers

- ▶ Solve $4x + 20 = 0$ for x .

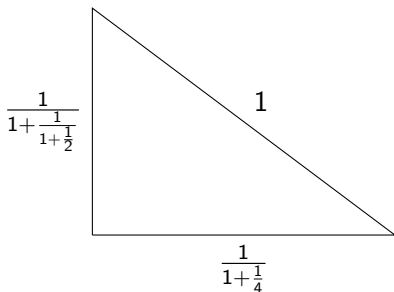
... $\overset{\bullet}{-4}$ $\overset{\bullet}{-3}$ $\overset{\bullet}{-2}$ $\overset{\bullet}{-1}$ $\overset{\bullet}{0}$ $\overset{\bullet}{1}$ $\overset{\bullet}{2}$ $\overset{\bullet}{3}$ $\overset{\bullet}{4}$...

- ▶ Historical:
 - ▶ China, 100BC;
 - ▶ Greece, 3rd C. AD (Diophantus, c.210–c.290);
 - ▶ Europe, 12th C. AD (Fibonacci, c.1170–c.1250)
- ▶ Uses:
 - ▶ Initially, as a calculation aid;
 - ▶ Given independent meaning in accounts as debts or losses.
- ▶ the set of integers is denoted \mathbb{Z} .

Numbers

Rationals

Rational numbers are those which can be expressed as fractions of two integers.

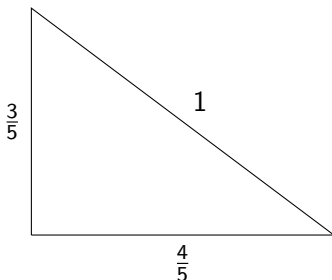


- ▶ Used since antiquity; recognized as numbers in Greek times;
- ▶ Connection to musical tuning (Pythagoras);
- ▶ The set of rationals is denoted \mathbb{Q} .

Numbers

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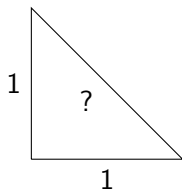
Numbers

Irrationals

System of numbers \mathbb{Q} is *closed*:

- ▶ for linear equations;
- ▶ with the exception of division by zero.

Trouble in numerical paradise (I):



- ▶ Proof attributed to the Pythagorean school;
- ▶ The set of real numbers is denoted \mathbb{R} .

Numbers

Quadratic equations

Examples of quadratic equations:

- ▶ equilateral right-angled triangle: solve $x^2 - 2 = 0$ for x ;
- ▶ general quadratic: solve $ax^2 + bx + c = 0$ for x .

Solution:

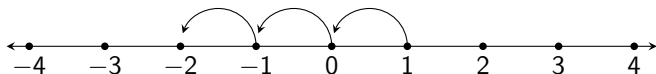
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- ▶ What if $b^2 - 4ac$ (the *discriminant*) is negative?
 - ▶ no solutions?
- ▶ History:
 - ▶ First known solution: Babylonian, c.2000 BC;
 - ▶ Geometrical interpretation: India, 8th C. BC; Euclid (Greece), 3rd C. BC;
 - ▶ First recorded general solution: Abraham bar Ḥiyya ha-Nasi (Catalonia, 12th C. AD)

Numbers

The Number Line

Just like at primary school: $1 - 3 = ?$



Numbers

Cubic equations

General Cubic:

- ▶ Solve $ax^3 + bx^2 + cx + d = 0$ for x .

Trouble in numerical paradise (II):

- ▶ Omar Khayyàm (Persia, 1048–1131)
 - ▶ Equation can have one or three real-number solutions;
- ▶ Gerolamo Cardano (Italy, 1501–1576)
 - ▶ Publishes method for solving cubic equation;
 - ▶ Method sometimes involves taking the square root of a negative number;
- ▶ Rafael Bombelli (Italy, 1526–1572)
 - ▶ publishes rules for dealing with square roots of negative numbers;

Numbers

Imaginary Numbers

Imaginary numbers (contrast with real numbers):

- ▶ Basic building block: $i = \sqrt{-1}$
- ▶ General imaginary number: $a \times i$ for $a \in \mathbb{R}$

Rules (after Bombelli):

- ▶ $a \times bi = abi$
- ▶ $-a \times bi = -abi$
- ▶ $a \times -bi = -abi$
- ▶ $-a \times -bi = abi$
- ▶ $ai \times bi = -ab$
- ▶ $ai \times -bi = ab$
- ▶ $-ai \times bi = ab$
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Or: i behaves just like another number, but $i \times i = -1$

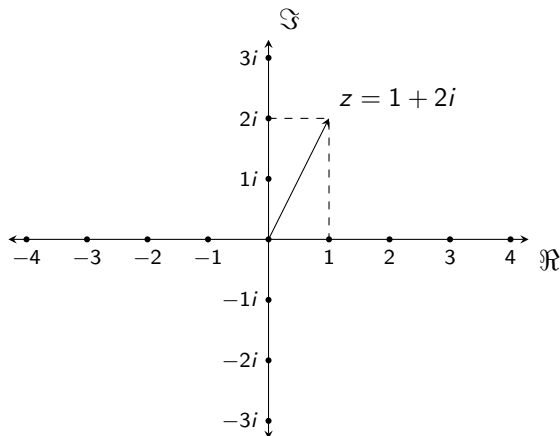
Numbers

Complex Numbers

Complex numbers:

- ▶ sum of a real and an imaginary part
- ▶ $z = a + ib$

Geometrical interpretation (Argand plane):



Numbers

Complex Numbers

Arithmetic Rules:

- ▶ Add and subtract parts:

- ▶ $(a + ib) + (c + id) = (a + c) + i(b + d)$

- ▶ $(a + ib) - (c + id) = (a - c) + i(b - d)$

- ▶ Multiply out:

- ▶ $(a + ib) \times (c + id) = ac + ibc + iad + iibd = (ac - bd) + i(bc + ad)$

- ▶ Divide using complex conjugate:

- ▶ $(a + ib) \div (c + id)$

- ▶ $= ((a + ib)(c - id)) \div ((c + id)(c - id))$

- ▶ $= ((a + ib)(c - id)) \div (c^2 + d^2)$

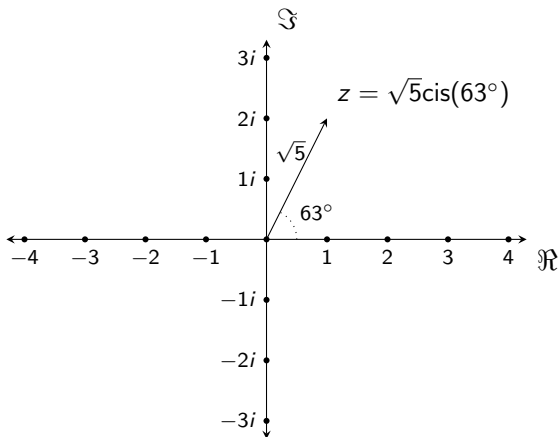
Numbers

Complex Numbers

Complex numbers:

- ▶ real length at an angle to the real axis
- ▶ $z = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$

Geometrical interpretation (Argand plane):



Numbers

Complex Numbers

Multiplication and Division revisited:

- ▶ $\text{cis}(\theta_1) \times \text{cis}(\theta_2) = \text{cis}(\theta_1 + \theta_2)$
- ▶ $r_1 \text{cis}(\theta_1) \times r_2 \text{cis}(\theta_2) = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$
- ▶ $r_1 \text{cis}(\theta_1) \div r_2 \text{cis}(\theta_2) = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$

Euler Identity:

$$e^{i\theta} = \text{cis}(\theta) = \cos \theta + i \sin \theta$$

- ▶ 'one of the most remarkable, almost astounding, formulas in all of mathematics' (R.P. Feynman)
- ▶ central to *complex analysis*; used everywhere in physics, electrical engineering, signal processing...
- ▶ connection to signals' amplitude and phase: Fourier analysis

Systems

Complex Numbers

- ▶ The Number Line;
- ▶ Pythagoras and the Rationals;
- ▶ The irrationality of $\sqrt{2}$;
- ▶ Solving quadratic equations;
- ▶ Cubic equations and Cardano;
- ▶ The Argand Plane;
- ▶ Arithmetic properties of Complex Numbers;
- ▶ Geometric interpretation of Complex Arithmetic;
- ▶ Phase and Logarithms;
- ▶ The Complex Exponential.