Abstract. The purpose of this paper is to show a contributor the required style for a paper for the 2014 AISB convention, AISB-50. The specifications for layout are described so that non-LATeX users can create their own style sheet to achieve the same layout. The source for the sample file is available for LATeX users. The PostScript and the PDF file is available for all.

1 PAGE LIMIT

The page limit for AISB-50 papers is no more than eight pages.

2 GENERAL SPECIFICATIONS

The following details should allow contributors to set up the general page description for their paper:

1. The paper is set in two columns each 20.5 picas (86 mm) wide with a column separator of 1.5 picas (6 mm).
2. The typeface is Times Modern Roman.
3. The body text size is 9 point (3.15 mm) on a body of 11 point (3.85 mm) (i.e. 61 lines of text).
4. The effective text height for each page is 56 picas (237 mm). The first page has less text height. It requires an additional footer space of 3.5 picas (14.8 mm) for the copyright inserted by the publisher and 1.5 picas (6 mm) of space before the title. The effective text height of the first page is 51 picas (216 mm).
5. There are no running feet for the final camera-ready version of the paper. The submission paper should have page numbers in the running feet.

3 TITLE, AUTHOR, AFFILIATION, COPYRIGHT AND RUNNING FEET

3.1 Title

The title is set in 20 point (7 mm) bold with leading of 22 point (7.7 mm), centered over the full text measure, with 1.5 picas (6 mm) of space before and after.

3.2 Author

The author’s name is set in 11 point (3.85 mm) bold with leading of 12 point (4.2 mm), centered over full text measure, with 1.5 picas (6 mm) of space below. A footnote indicator is set in 11 point (3.85 mm) medium and positioned as a superscript character.

3.3 Affiliation

The affiliation is set as a footnote to the first column. This is set in 8 point (2.8 mm) medium with leading of 8.6 point (3.1 mm), with a 1 point (0.35 mm) footnote rule to column width.

3.4 Copyright

The copyright details will be inserted by the publisher.

3.5 Running feet

The running feet are inserted by the publisher. For submission you may insert page numbers in the middle of the running feet. Do not, however, insert page numbers for the camera-ready version of the paper.

4 ABSTRACT

The abstract for the paper is set in 9 point (3.15 mm) medium, on a body of 10 point (3.5 mm). The word Abstract is set in bold, followed by a full point and a 0.5 pica space.

5 HEADINGS

Three heading levels have been specified:

1. A level headings
   - The first level of heading is set is 11 point (3.85 mm) bold, on a body of 12 point (4.2 mm), 1.5 lines of space above and 0.5 lines of space below.
   - The heading is numbered to one digit with a 1 pica space separating it from the text.
   - The text is keyed in capitals and is unjustified.
2. B level headings
   - The second level of heading is set is 11 point (3.85 mm) bold, on a body of 12 point (4.2 mm), 1.5 lines of space above and 0.5 lines of space below.
   - The heading is numbered to two digits separated with a full point, with a 1 pica space separating it from the text.
   - The text is keyed in upper and lower case with an initial capital for first word only, and is unjustified.
3. C level headings

1 University of Leipzig, Germany, email: someusername@informatik.uni-leipzig.de
The third level of heading is set is 10 point (3.5 mm) italic, on a body of 11 point (3.85 mm), 1.5 lines of space above and 0.5 lines of space below.

The heading is numbered to three digits separated with a full point, with a 1 pica space separating it from the text.

The text is keyed in upper and lower case with an initial capital for first word only, and is unjustified.

4. Acknowledgements
This heading is the same style as an A level heading but is not numbered.

6 TEXT
The first paragraph of text following any heading is set to the complete measure (i.e. do not indent the first line).
Subsequent paragraphs are set with the first line indented by 1 pica (3.85 mm).
There isn’t any inter-paragraph spacing.

7 LISTS
The list identifier may be an arabic number, a bullet, an em rule or a roman numeral.
The items in a list are set in text size and indented by 1 pica (4.2 mm) from the left margin. Half a line of space is set above and below the list to separate it from surrounding text.
See layout of Section 5 on headings to see the results of the list macros.

8 TABLES
Tables are set in 8 point (2.8 mm) on a body of 10 point (3.5 mm). The table caption is set centered at the start of the table, with the word Table and the number in bold. The caption is set in medium with a 1 pica (4.2 mm) space separating it from the table number.
A one line space separates the table from surrounding text.

Table 1. The table caption is centered on the table measure. If it extends to two lines each is centered.

<table>
<thead>
<tr>
<th>Window</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1273</td>
<td>110</td>
<td>21.79</td>
<td>89%</td>
</tr>
<tr>
<td>2</td>
<td>2145</td>
<td>116</td>
<td>10.99</td>
<td>50%</td>
</tr>
<tr>
<td>3</td>
<td>3014</td>
<td>117</td>
<td>41.77</td>
<td>89%</td>
</tr>
<tr>
<td>4</td>
<td>4753</td>
<td>151</td>
<td>71.55</td>
<td>77%</td>
</tr>
<tr>
<td>5</td>
<td>5576</td>
<td>148</td>
<td>61.60</td>
<td>80%</td>
</tr>
</tbody>
</table>

◊ execution time in ticks □ speed-up values △ efficiency values

9 FIGURES
A figure caption is set centered in 8 point (2.8 mm) medium on a leading of 10 point (3.5 mm). It is set under the figure, with the word Figure and the number in bold and with a 1 pica (4.2 mm) space separating the caption text from the figure number.
One line of space separates the figure from the caption. A one line space separates the figure from surrounding text.
12 THEOREMS

The text of a theorem is set in 9 point (3.15 mm) italic on a leading
of 11 point (3.85 mm). The word Theorem and its number are set in
9 point (3.15 mm) bold.

A one line space separates the theorem from surrounding text.

Theorem 1 Let us assume this is a valid theorem. In reality it is a
piece of text set in the theorem environment.

13 FOOTNOTES

Footnotes are set in 8 point (2.8 mm) medium with a leading of 11 point (3.15 mm), with a 1 point (0.35 mm) footnote rule to column
width.

Footnotes are set in 8 point (2.8 mm) medium with a leading of 11 point (3.15 mm). The word Theorem and its number are set in
9 point (3.15 mm) bold.

The text of a theorem is set in 9 point (3.15 mm) italic on a leading
of 10 point (3.5 mm), and appears in the sequence in
which it is cited in the paper.

14 REFERENCES

The reference identifier in the text is set as a sequential number in
square brackets. The reference entry itself is set in 8 point (2.8 mm)
with a leading of 10 point (3.5 mm), and appears in the sequence in
which it is cited in the paper.

15 SAMPLE CODING

The remainder of this paper contains examples of the specifications
detailed above and can be used for reference if required.

16 PROGRAMMING MODEL

Our algorithms were implemented using the single program, multi
ple data model (SPMD). SPMD involves writing a single code that
will run on all the processors co-operating on a task. The data are
partitioned among the processors which know what portions of the
data they will work on [7].

16.1 Structure of processes and processors

The grid has \( P = P_r \times P_c \) processors, where \( P_r \) is the number of
rows of processors and \( P_c \) is the number of columns of processors.

16.1.1 Routing information on the grid

A message may be either broadcast or specific. A broadcast message
originates on a processor and is relayed through the network until it
reaches all other processors. A specific message is one that is directed
to a particular target processor.

Broadcast messages originate from a processor called central
which is situated in the ‘middle’ of the grid. This processor has co
ordinates \(|P_r/2|, |P_c/2|\). Messages are broadcast using the row–
column broadcast algorithm (RCB), which uses the following strategy.
The number of steps required to complete the RCB algorithm
(i.e. until all processors have received the broadcast value) is given by
\(|P_r/2| + |P_c/2|\).

A specific message is routed through the processors using the find
row–find-column algorithm (FRFC) detailed in [5]. The message is
sent from the originator processor vertically until it reaches a pro
cessor sitting in the same row as the target processor. The message is
then moved horizontally across the processors in that row until it
reaches the target processor. An accumulation based on the recursive
doubling technique [9, pp. 56–61], would require the same number of
steps as the RCB requires. If either the row or column of the origin
ator and target processors are the same then the message will travel
only in a horizontal or vertical direction, respectively (see [12]).

17 DATA PARTITIONING

We use data partitioning by contiguity, defined in the following way.
To partition the data (i.e. vectors and matrices) among the processors,
we divide the set of variables \( V = \{ i \}_{i=1}^{N} \) into \( P \) subsets \( \{ W_p \}_{p=1}^{P} \)
of \( s = N/P \) elements each. We assume without loss of generality
that \( N \) is an integer multiple of \( P \). We define each subset as \( W_p = \{(p-1)s+j\}_{j=1}^{s} \) (see [11], [4] and [2] for details).

Each processor \( p \) is responsible for performing the computations
over the variables contained in \( W_p \). In the case of vector operations,
each processor will hold segments of \( s \) variables. The data partitioning
for operations involving matrices is discussed in Section 18.3.

18 LINEAR ALGEBRA OPERATIONS

18.1 Saxpy

The saxpy \( w = u + \alpha v \) operation, where \( u, v \) and \( w \) are vectors
and \( \alpha \) is a scalar value, has the characteristic that its computation is
disjoint elementwise with respect to \( u, v \) and \( w \). This means that we
can compute a saxpy without any communication between proces
sors; the resulting vector \( w \) does not need to be distributed among
the processors. Parallelism is exploited in the saxpy by the fact that
\( P \) processors will compute the same operation with a substantially
smaller amount of data. The saxpy is computed as

\[
    w_i = u_i + \alpha v_i, \quad \forall i \in \{ W_p \}_{p=1}^{P}
\]

18.2 Inner-product and vector 2-norm

The inner-product \( \alpha = u^T v = \sum_{i=1}^{N} u_i v_i \) is an operation that in
volves accumulation of data, implying a high level of communication
between all processors. The mesh topology and the processes architec
ture used allowed a more efficient use of the processors than, for in
stance, a ring topology, reducing the time that processors are idle
waiting for the computed inner-product value to arrive, but the prob
lem still remains. The use of the SPMD paradigm also implies the
global broadcast of the final computed value to all processors.

The inner-product is computed in three distinct phases. Phase 1 is
the computation of partial sums of the form

\[
    \alpha_p = \sum_{\forall i \in \{ W_p \}} u_i v_i, \quad p = 1, \ldots, P
\]

The accumulation phase of the inner-product using the RCA algo
rithm is completed in the same number of steps as the RCB algorithm
(Section 16.1.1). This is because of the need to relay partial values
between processors without any accumulation taking place, owing to
the connectivity of the grid topology.

The vector 2-norm \( \| u \| = \sqrt{u^T u} \) is computed using
the inner-product algorithm described above. Once the inner-product
value is received by a processor during the final broadcast phase, it
computes the square root of that value giving the required 2-norm
value.
18.3 Matrix–vector product

For the matrix–vector product \( v = Au \), we use a column partitioning of \( A \). Each processor holds a set \( W_p \) (see Section 17) of \( s \) columns each of \( N \) elements of \( A \) and \( s \) elements of \( u \). The \( s \) elements of \( u \) stored locally have a one-to-one correspondence to the \( s \) columns of \( A \) (e.g. a processor holding element \( u_j \) also holds the \( j \)-th column of \( A \)). Note that whereas we have \( A \) partitioned by columns among the processors, the matrix–vector product is to be computed by rows.

The algorithm for computing the matrix–vector product using column partitioning is a generalization of the inner-product algorithm described in Section 18.2 (without the need for a final broadcast phase). At a given time during the execution of the algorithm, each of \( P - 1 \) processors is computing a vector \( w \) of \( s \) elements containing partial sums required for the segment of the vector \( v \) in the remaining ‘target’ processor. After this computation is complete, each of the \( P \) processors stores a vector \( w \). The resulting segment of the matrix–vector product vector which is to be stored in the target processor is obtained by summing together the \( P \) vectors \( w \), as described below.

Each processor other than the target processor sends its \( w \) vector to one of its neighboring processors. A processor decides whether to send the vector in either the row or column direction to reach the target processor based on the FRFC algorithm (see Section 16.1.1). If a vector passes through further processors in its route to the target processor the \( w \) vectors are accumulated. Thus the target processor will receive at most four \( w \) vectors which, when summed to its own \( w \) vector, yield the desired set of \( s \) elements of \( v \).

18.4 Matrix–vector product—finite-difference approximation

We now consider a preconditioned version of the conjugate-gradients method [7]. Note that we do not need to form \( A \) explicitly. This implies a very low degree of information exchange between the processors which can be effectively exploited with transputers, since the required values of \( u \) can be exchanged independently through each link.

The preconditioning used in our implementations is the polynomial preconditioning (see [10], [6], [11] and [8]), which can be implemented very efficiently in a parallel architecture since it is expressed as a sequence of saxpys and matrix–vector products.

We have \( l \) rows and columns in the discretization grid, which we want to partition among a \( P_l \times P_c \) mesh of processors. Each processor will then carry out the computations associated with a block of \( \lfloor l/P_l \rfloor + \text{sign}(l \text{ mod } P_l) \) rows and \( \lfloor l/P_c \rfloor + \text{sign}(l \text{ mod } P_c) \) columns of the interior points of the grid.

The matrix–vector product using the column partitioning is highly parallel. Since there is no broadcast operation involved, as soon as a processor on the boundary of the grid (either rows or columns) has computed and sent a \( w_p \) vector destined to a processor ‘\( A \)’, it can compute and (possibly) send a \( w_p \) vector to processor ‘\( B \)’, at which time its neighboring processors may also have started computing and sending their own \( w \) vectors to processor ‘\( B \)’.

At a given point in the matrix–vector product computation, the processors are computing \( w \) vectors destined to processor \( A \). When these vectors have been accumulated in the row of that processor (step 1), the processors in the top and bottom rows compute and send the \( w \) vectors for processor \( B \), while the processors on the left and right columns of the row of processor \( A \) send the accumulated \( r \) vectors to processor \( A \) (step 2). Processor \( A \) now stores its set of the resulting \( v \) vector (which is the accumulation of the \( w \) vectors). In step 3, the processors in the bottom row compute and send the \( w \) vectors for processor \( C \) while the processor at the left-hand end of the row of processor \( B \) sends the accumulated \( w \) vectors of that column towards processor \( B \). The next steps are similar to the above.

In our implementation, we exploit the geometry associated with the regular grid of points used to approximate the PDE. A geometric partitioning is used to match the topology and connectivity present in the grid of transputers (Section 16.1).

The discretization of the PDE is obtained by specifying a grid size \( l \) defining an associated grid of \( N = l^2 \) interior points (note that this is the order of the linear system to be solved). With each interior point, we associate a set of values, namely the coefficients \( C, N, S, E \) and \( W \).

19 CONCLUSION

We have shown that an iterative method such as the preconditioned conjugate-gradients method may be successfully parallelized by using highly efficient parallel implementations of the linear algebra operations involved. We have used the same approach to parallelize other iterative methods with similar degrees of efficiency (see [4] and [3]).

ACKNOWLEDGEMENTS

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REFERENCES