Lognormality: An Open Window on Neuromotor Control

Réjean Plamondon[1]([✉]), Asma Bensalah[2], Karina Lebel[3], Romeo Salameh[4], Guillaume Séguin de Broin[1], Christian O'Reilly[5], Mickael Begon[11], Olivier Desbiens[1,6], Youssef Beloufa[1,6], Aymeric Guy[6], Daniel Berio[7], Frederic Fol Leymarie[7], Simon-Pierre Boyoguéno-Bidias[1], Andreas Fischer[8], Zigeng Zhang[1], Marie-France Morin[12], Denis Alamargot[13], Céline Rémi[9], Nadir Faci[1], Raphaëlle Fortin[14], Marie-Noëlle Simard[15], and Caroline Bazinet[10]

[1] Laboratoire Scribens, Département de Génie Électrique, Polytechnique Montréal, Montréal, Canada
{rejean.plamondon,guillaume.seguin-de-broin,olivier.desbiens,
youssef.beloufa,simon-pierre.boyogueno-bidias,zigeng.zhang,
nadir.faci}@polymtl.ca

[2] Computer Vision Center, Computer Science Department, Universitat Autònoma de Barcelona, Bellaterra, Spain
abensalah@cvc.uab.es

[3] Département de Génie Électrique et Informatique, Université de Sherbrooke, Sherbrooke, Canada
karina.lebel@usherbrooke.ca

[4] Faculté de Médecine, Université de Montréal, Montréal, Canada
romeo.salameh@umontreal.ca

[5] Department of Computer Science and Engineering, University of South Carolina, Columbia, USA
christian.oreilly@sc.edu

[6] LifeEngine Technologies Inc., Montréal, Canada
aymeric.guy@lifeengine.ca

[7] Computing Department, Goldsmiths, University of London, London, UK
ffl@gold.ac.uk

[8] University of Applied Sciences and Arts, Fribourg, Switzerland
andreas.fischer@hefr.ch

[9] LAMIA, Université Des Antilles, Guadeloupe, France
celine.remi@univ-antilles.fr

[10] AleoVR Inc., Montréal, Canada
caroline.bazinet@aleovr.com

[11] Faculté de Médecine, École de Kinésiologie, Université de Montréal, Montréal, Canada
mickael.begon@umontreal.ca

[12] Laboratoire GRISE - Groupe de Recherche et d'Intervention sur les Adaptations Sociales de l'Enfance, Université de Sherbrooke, Sherbrooke, Canada
marie-france.morin@usherbrooke.ca

[13] Laboratoire CeRCA-CNRS - Centre de Recherche sur la Cognition et l'Apprentissage , Université de Poitiers, Poitiers, France
Denis.Alamargot@univ-poitiers.fr

[14] Département de Psychologie, Université de Montréal, Montréal, Canada
`raphaelle.fortin.1@umontreal.ca`
[15] Faculté de Médecine, École de Réhabilitation, Université de Montréal, Montréal, Canada
`marie-noelle.simard@umontreal.ca`

Abstract. This invited special session of IGS 2023 presents the works carried out at *Laboratoire Scribens* and some of its collaborating laboratories. It summarises the 17 talks presented in the colloquium #611 entitled *« La lognormalité: une fenêtre ouverte sur le contrôle neuromoteur»* (Lognormality: a window opened on neuromotor control), at the 2023 conference of the *Association Francophone pour le Savoir* (ACFAS) on May 10, 2023. These talks covered a wide range of subjects related to the Kinematic Theory, including key elements of the theory, some gesture analysis algorithms that have emerged from it, and its application to various fields, particularly in biomedical engineering and human-machine interaction.

Keywords: Kinematic Theory · Lognormality Principle · Typical Applications

1   Introduction

The Kinematic Theory of rapid human movements describes, using a fundamental equation called the "lognormal function", the speed of an end effector. Various software packages have been developed to reverse-engineer movements by reconstructing them with lognormals. This reconstruction provides central parameters that represent the state of the brain, and peripheral parameters that describe the properties of the neuromuscular systems that produced the movement. Over the years, the theory has been tested and validated in numerous experiments, and successfully used to describe the essential properties of the velocity profiles of the fingers, wrist, trunk, head and eyes, etc. This led to postulate the Lognormality Principle, which states that the lognormal impulse response of a neuromuscular system emerges from a convergent process driven by the central limit theorem. This optimal global pattern reflects the behaviour of individuals who have perfect control over their movements. The production of complex movements is achieved by the temporal superposition and summation of lognormal velocity vectors, with the aim of minimising their number in a given task, to produce efficient and fluid gestures, optimising the energy required to generate them. As a corollary, motor control learning in children can be interpreted as a migration towards lognormality. Then, for most of their lives, normal adults take advantage of their lognormality to control their movements. Finally, as ageing and potential health problems increase, there is a progressive deviation from lognormality.

This manuscript presents the works carried out at the Scribens laboratory and some of its collaborating laboratories. It summarises the 17 selected talks presented in French in the colloquium #611 entitled « La lognormalité: une fenêtre ouverte sur le contrôle neuromoteur» (Lognormality: a window opened on neuromotor control), at the 2023 conference of the Association Francophone pour le Savoir (ACFAS) on May 10, 2023

https://www.acfas.ca/evenements/congres/programme/90/600/611/c. The ACFAS is a Canadian non-profit organization, based in Québec. Its community (4500 active members from 32 countries) promotes scientific activity, stimulates research and disseminates knowledge in French. Our workshop program focused on the key elements of the theory, some gesture analysis algorithms that have emerged from it, and provided an overview of various applications, particularly in the fields of biomedical engineering and human-machine interaction. Throughout this paper, we look back on these studies, as well as forward, and therefore cover past, current and future works. In addition to specialists in signal processing, neuropsychology, neuroscience, education, kinesiology, occupational therapy, pediatrics, students who have completed internships or studies at the Scribens laboratory and student entrepreneurs who plan to use lognormality as a metric in their products, have participated to this colloquium.

More specifically, this paper is an overview of the special session held and presented at the IGS 2023 conference by the first author.

2   The Lognormality Principle: Theory and Overview of Some Applications

2.1   Context

The asymmetric bell-shaped velocity profiles of rapid aimed movements and their invariant properties have been a subject of investigations for many decades in the last century. Among the various models that have been developed to explain these phe-nomena, the Kinematic Theory [118–120, 124] proposed an emergent ecological approach based on the central limit theorem to predict that these asymmetric bell-shaped velocity profiles can be optimally described with lognormal functions. Indeed, assuming that the invariant properties of these simple movements reflect the asymptotic behaviour of complex systems, composed of a large number of time coupled neuromuscular networks, such a neuromuscular system will have a lognormal impulse response that reflects its ideal behaviour, as long as such a neuromuscular system is made up of a large number of coupled subsystems and that the coupling is driven by a proportionality relationship between the subsystem cumulative time delays. This emergence towards lognormality is achieved from asymptotic convergence established over the years, from the exploratory oscillations of the baby's arm to the learning of precise gestures, as in handwriting exercises and sports.

2.2   The Lognormality in practice

Over the last 25 years, the Kinematic Theory has been very useful in terms of signal processing, as a reverse engineering methodology to reconstruct any movements and extract central and peripheral lognormal parameters:

t_0: Represents the time at which the motor command is emitted by the central nervous system. In psychomotor tests, this corresponds to the moment when the nervous system initiates a response after receiving a start signal, such as a sound or visual stimulus. The parameter t_0 makes the Kinematic Theory a causal theory, distinguishing it from all the other models in use nowadays [107].

D: Denotes the amplitude of a lognormal stroke. It corresponds to the total distance covered by the trajectory associated with the specific movement primitive.

μ: Reflects the logarithmic time delay. Exp(μ) defines the time required to reach the median of the motion distance. This parameter provides insight into the overall speed of the reaction.

σ: Represents the logarithmic response time, characterizing the duration of the motion.

θ_{start} and θ_{end}: Indicate the start and end angles of the motion, respectively, measured in radians.

SNR: The signal-to-noise ratio compares the quality of the reconstructed velocity profile to the recorded velocity. A higher SNR value signifies a more accurate reconstruction.

nbLog: This parameter represents the number of lognormal functions used to reconstruct a velocity profile. It serves as an index of motion smoothness, with lower values indicating smoother motion.

SNR/nbLog: This ratio reflects the fluidity of the movement and is calculated by dividing the SNR value by the nbLog.

Figure 1 (adapted from Faci et al. 2021) highlights the effect of these neuromotor parameters on a lognormal impulse response:

$$\Lambda\left(t;t_0,\mu,\sigma^2\right)=\begin{cases}\dfrac{1}{\sigma\sqrt{2\pi}\left(t-t_0\right)}\exp\left\{-\dfrac{1}{2\sigma^2}\left[\ln\left(t-t_0\right)-\mu\right]^2\right\} & \text{for } t>t_0 \\ 0 & \text{elsewhere}\end{cases}$$
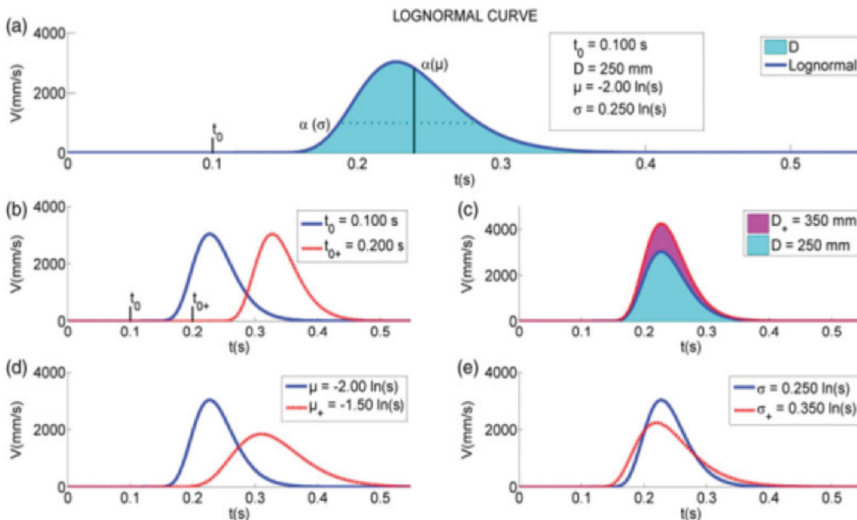

Fig. 1. Effect of the main parameter on a lognormal impulse response

Two major families of algorithms have been developed over the years, the Delta-Lognormal extractors used to reconstruct simple straight pointing movement with two

lognormals, one agonist and the other one antagonist [67, 126, 41, 108, 19] and the Sigma-Lognormal extractors used to reconstruct any 2D [54, 105], and 3D [56, 59] complex movements. As it has been shown time and again, reconstructing various gestures with lognormal patterns provides a powerful representation of the underlying neuromotor processes involved in different gestures. So far, the Kinematic Theory has been extensively tested by more than 20 research teams from 8 countries with around 300 000 samples, 11 000 participants, 18 tablet models, 10 other motion capture devices with sampling frequency ranging from 15 to 240 Hz.

In summary, the Kinematic Theory offers researchers a strong realistic theoretical paradigm, a general equation and a set of physiologically meaningful parameters and a set of robust parameter extraction algorithms. [115, 116, 121, 158].

2.3 Workshop Program

The following sections present typical applications of this methodological approach. This can be seen as the tip of an iceberg. There are more projects going on all over the world. Those that were selected for the colloquium were those that could be presented by a French speaker.

The whole workshop has been divided into four themes.

Section 3 presents three papers on AGING: a proof of concept regarding the use of lognormality to monitor brain stroke rehabilitation, the development of a kinematic signature for people with Parkinson's and psoriatic arthritis and a search for Parkinson's disease kinematic biomarkers.

Section 4 deals with PERFORMANCE. The first paper deals with the modelling of electrocardiogram using lognormals, a novel set signals where lognormality can be exploited. The second report two recent studies, one characterizing muscular fatigue and the second interpreting lognormality in terms of optimal control. The third aims at providing tolls for an objective analysis of surgical performance, a brand-new field of potential applications. The fourth summarizes previous studies dealing with the kinematic reconstruction of static calligraphic traces to infer a physiologically plausible motion from an input trace image.

Section 5 deals with TECHNIQUES. The first paper presents a globally optimal delta lognormal parameter extractor based on a branch and bound search method combined with the interval arithmetic. The second describes the first 3D Sigma-Lognormal extractor that has been recently developed and tested. The third compares symbolic and connectionist algorithms to correlate the age of healthy children with Sigma-Lognormal neuromotor parameters.

Section 6 deals with CHILDHOOD. The first two deals with handwriting learning, one with the assessment of graphomotor skills in kindergarten and first grade students in France and Québec and the second with the characterization and analysis of graphomotor behaviours involving young learners in a school context, both studies based on the Kinematic Theory and its lognormal models. The next three papers deal with neurodevelopmental problems and investigate the usefulness of the pencil strokes test: a pilot study dealing with strokes produced by children with mild traumatic brain injury, another one with strokes produced by children with ADHD. The third one, a work in progress, dealing with the characterization of children born prematurely to evaluate the

risk of developmental difficulties at preschool age. Finally, the last paper is a brand-new research proposal that aims at exploring the benefits of combining virtual reality and lognormality for prescreening ADHD in children.

3   AGING

3.1   Remote Monitoring of Stroke Patients via 3D Kinematics and Artificial Intelligence

Context. Being one of the top leading reasons for motor and cognitive impairment [30], stroke patient early detection and post-stroke monitoring has become major human concerns and research focus. Namely, post-stroke patient monitoring during the first weeks can optimise the rehabilitation process and lessen the human and financial burden on both the patients and caregivers. We propose a whole movement spotting and analysis pipeline, that have been validated in a clinical institution.

Experimentation Protocol. Our experimentation protocol has been influenced by the Fugl-Meyer clinical assessment, in an effort to make it as realistic as possible. It consists of four key target movements:M1: shoulder extension/flexion, M2: shoulder abduction/abduction, M3: external/internal shoulder rotation, M4: elbow flexion/extension.

We have designed two experimental scenarios:

- Scenario L1: the individual alternates between the four key movements, many times.
- Scenario L2: the individual performs a sequence of key target movements and non-target movement drawn from daily activities [12].

To record data individuals have had to wear an Apple Watch Series 4, in each wrist. A smartwatch application has been developed to extract the watch's signals and synchronize both watches.

Movement Spotting.   Before analysing the movements, we needed to spot and recognise them. Therefore, we have implemented an architecture inspired from the work of [82]. The architecture starts with a convolution size set to be half the sampling rate, followed by two other wise separable convolutions. As well as, using SVM as a baseline classification.

Since it is difficult to perform the action spotting in scenario L2, given that there are many movement classes, we have opted for clustering the movements that are similar. Concretely, we have clustered all movements into two classes (C0, C4): C4: being all movements similar to M4; C0: the rest of movements.

Kinematic Analysis.   In order to analyse movements and estimate the patients' progress, we have used a 3D algorithm [59] based on the Kinematic Theory of rapid human Movements [118–120, 124].

Results. *Spotting.* SVMs (accuracy = 84%) have outperformed CNNs(accuracy = 65%) for both healthy subjects, for scenario L1. The same pattern has been observed within patients. This is due to the lack of sufficient data for training, in the case of CNNs. For scenario L2, accuracy decreases to 61% for SVMs and 59% for CNNs for healthy

samples and lower than that for the patients. For the patients, the task was even harder because of their motricity lack.

Kinematic Analysis. The SNR/nbLog (Signal-to-noise-ratio per lognormal) for patients is significantly lower than for healthy individuals.

Additionally, the contrast between patients and healthy individuals, in terms of SNR/nbLog is remarkably higher for movement M4. One possible explanation for that, could be the difficulty of executing M4. Furthermore, no big difference was observed between the affected and non-affected arms for the patients, the reason behind that could be the fact patients were moving both arms at the same time, thus the affected arm impacts the non-affected arm performance.

Outcomes. For the first time, the 3D Kinematic Theory has been applied to analyse movements for stroke patients on smartwatches. The experiments have proved that it is an efficient non-invasive biomarker to assess stroke patients' progress. Further work can be done on the design of experimental scenarios by focusing on analytic movements.

3.2 Kinematic Signature in People with Parkinson's and Psoriatic Arthritis: Potential of the Sigma-Lognormal Approach

Context. Functional mobility, defined as one's ability to accomplish basic activities of daily living, is traditionally assessed using questionnaires or clinical performance tests [164]. These approaches are mainly based on subjective assessment, somehow limiting their ability to assess changes. In research labs, mobility can be assessed objectively using diverse high-end equipment [98, 164]. Yet, advances in technology, including but not limited to inertial measurement units (IMU), increase the potential for objective functional mobility appraisal outside traditional laboratories, including the clinic and the home [95]. However, these so-called wearable systems work on different basic principles, which may require to rethink some of the traditional variables used to describe mobility. For example, gait is often characterized using stride length, calculated by the displacement of the foot. With IMU, such metric requires a double integration of the aligned acceleration signal, resulting in significant integration errors. To overcome these limitations, modelling approaches can be used to characterize movement signatures [127]. Among these, the Sigma-Lognormal model, based on the Kinematics Theory, aims at characterizing the velocity profile during a pointing task. It has been extensively used to assess scripted 2D signature. Yet, mobility tasks also follow some sort of signature, though in a less controlled context. For example, turning while walking involves a specific cranio-caudal sequence where the head initiates the movement, rotating towards the new desired direction, followed by the trunk and the pelvis, until body is fully realigned [73]. In Parkinson's disease, this signature is modified due to an increased axial coupling [146]. In other words, turning while walking can be seen as a pointing task in the orientation domain, which signature varies according to the ability of a person to perform the turn safely. Similarly, gait can be seen as the foot following a specific movement signature to enable a shift in the center of mass, leading to body's displacement. This section presents the potential of the Sigma-Lognormal model to assess turn and gait.
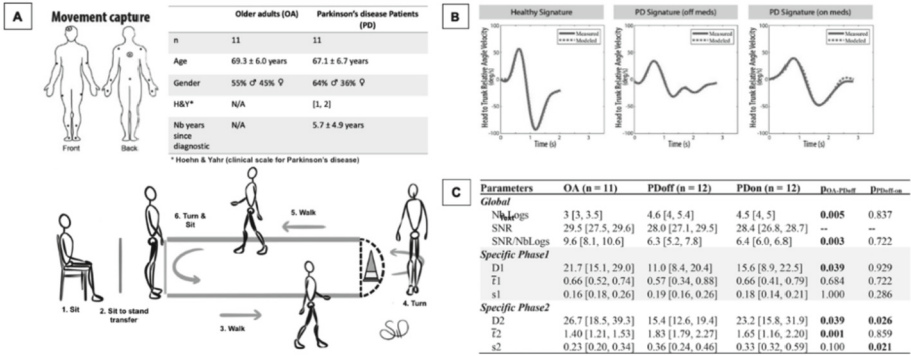
Fig. 2. Sigma-lognormal model to characterize the turn signature. (A) Experimental protocol. Participants were equipped with 17 inertial sensors. Head and trunk sensors were used to assess the turn. A total of 22 participants performed a timed-up and go where participant stands up, walks for 3m, turns around, and comes back to its initial seated position. The turn phase was manually segmented for analysis. (B) Representative turn signature for a healthy individual and a PD patient, on/off medication. (C) Sigma-lognormal parameters analysis. Phase 1 corresponds to the turn initiation by the head, while phase 2 relates to the command given to the trunk to realign with the head.

Turn signature with the Sigma-Lognormal Model. Fifteen healthy older adults (OA) and 14 Parkinson's disease participants (PD) performed a timed-up and go while equipped with IMUs (Fig. 2A). Relative orientation of the head to the trunk was calculated and derived to obtain relative orientation velocity [85, 86]. This signal was then modelled using the sigma-lognormal approach, and the resultant parameters, analyzed [83, 84].

Figure 2B illustrates the ability of the model to reproduce the signature for all participants and conditions. The overall mean signal over noise ratio (SNR) of 28.6 confirms the fit of the model with the turn signature. The various sigma-lognormal parameters (D, t and s) were then analyzed to assess (i) the ability of the model to discriminate between older adults and early Parkinson's disease, and (ii) its sensitivity to change through analysis of the PD on/off medication trials. Results have shown that the SNR/nbLogs ratio, defined as the quality of the model over the number of logs re-quired to fit the signal, have significantly changed between OA and PD (OA: 9.6 [8.1, 10.6]; PD: 6.3 [5.2, 7.8], p = 0.003). These results support the idea that motor control deteriorates with Parkinson's disease. Detailed analysis of the Sigma-Lognormal parameters also revealed a significant change between OA and PD in the D1 parameter, associated with the amplitude of the command given by the neuromuscular system to initiate the turning task (OA: 21.7 [15.1, 29.0]; PD: 11.0 [8.4, 20.4], p = 0.039). Impact of medication was also captured in the D parameters, with D1 showing a tendency to increase and D2 revealing a significant increase in command's amplitude engaging the trunk into the motion (Fig. 2C). These results confirm the usability of the Sigma-Lognormal model to assess turn signature. This study reveals the model's potential to be used in the orientation domain, on complexed tasks involving multiple segments.

Gait Signature Using the Sigma-Lognormal Model. Gait has been studied extensively, though most studies concentrate on controlled laboratory conditions [77]. Nowadays, there is an increased interest in evaluating gait in natural environments. To do so, IMUs are often used due to their portability and low cost [164]. Though these systems can detect temporal parameters accurately (e.g. cadence), they still struggle to estimate spatial information like stride length [155]. This study investigates the potential of using the Sigma-Lognormal model to (i) characterize gait, and (ii) estimate stride length. Twenty-four healthy individuals (mean age: 31 ± 10 years old) and 20 persons with psoriatic arthritis (mean age: 54 ± 9 years old) performed 2-min walking trials on a treadmill at slow, normal, and fast speeds. Participants were instrumented with 39 markers to enable full-body motion capture (OptiTrack by Natural Point, Corvallis, OR, USA). Each trial was segmented into strides, to be further analyzed. Velocity of the foot in the direction of motion was processed using the Sigma-Lognormal approach. Figure 2A illustrates the ability of the model to reconstruct a stride. The overall mean SNR of 78.5 confirms the representativity of the model. Linear regression was then performed on Sigma-Lognormal parameters (D, μ, σ) from the first two strokes to determine the ability of the model to estimate stride length. The obtained linear regression model resulted in an excellent fit ($R^2 = 0.9769$). Mean error of 0.0007cm also confirms the potential of the approach to estimate stride length. Using a Bland & Altman approach, the 95% limits of agreement were determined to be ± 9 cm. In other words, the regression model estimates stride length with an accuracy of ± 9cm in 95% of the cases. To improve these results, analysis was performed using the median stride for each individual, per speed. This approach reduced the limits of agreement to [-5.5, 4.2] cm. This study thus demonstrated the ability of the Sigma-Lognormal model to characterize gait and revealed its potential to estimate stride length.
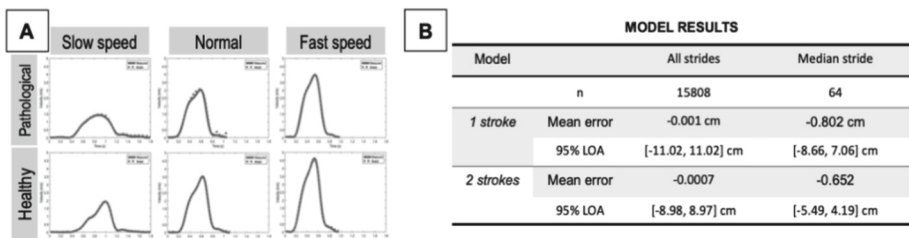

Model		All strides	Median stride
	n	15808	64
1 stroke	Mean error	-0.001 cm	-0.802 cm
	95% LOA	[-11.02, 11.02] cm	[-8.66, 7.06] cm
2 strokes	Mean error	-0.0007	-0.652
	95% LOA	[-8.98, 8.97] cm	[-5.49, 4.19] cm

Fig. 3. Sigma-Lognormal model to characterize the gait. (A) Representative gait signature for healthy and pathological individuals at slow, normal and fast speeds. (B) Models precision results

3.3 Contribution of Lognormality in the Identification of Kinematic Biomarkers in the Identification and Early Differential Diagnosis of Parkinson's Disease

Context. Parkinson's disease (PD) affects an estimated 6 000 000 people worldwide [140], making it the second most common neurodegenerative disease, only behind Alzheimer's disease.

The objective of designing an effective diagnostic method has not been reached yet, hindering research and development efforts for better treatments and proper management. Indeed, various disorders including atypical parkinsonian syndromes, hereditary parkinsonism, as well as secondary parkinsonism due to external causes such as drugs or infections, can often be mistaken for PD, especially during the first years of symptomatic disease progression. One clinicopathologic study found only 26% accuracy for a clinical diagnosis of PD proposed at the first consultation visit to a neurologist, in cases that received diagnostic confirmation by autopsy [1].

The search for biomarkers has been a main focus of PD research for several decades. An appropriate, easily measurable biomarker would allow early detection of the disease, at a time when clinical diagnosis can be uncertain, monitor progression as well as treatment response. Various genetic, biochemical, and multimodal imaging biomarkers have been explored with promising results [36], but cost, access, and data reproducibility often limit widespread applicability. In addition, deep phenotyping of motor and non-motor features of PD has been developed, using clinical scales, kinematic platforms, and body-worn sensors for data acquisition, combined with different data mining methods. Physiological eye, limb, or axial (posture and gait) movements or tasks have been recorded, in attempts to capture a neuromuscular signature that would reflect the pathological alterations that distinctly affect motor control in PD and related disorders.

These quantitative approaches offer the salient advantage of assessing the entire neuronal network recruited to prepare and execute a motor command, and multiple relevant motor features simultaneously. Several handwriting and geometric tasks have been evaluated, discriminating PD patients from healthy participants [34, 98, 147]. However, the applicability of these signals in early disease as tools to differentiate PD from other parkinsonian syndromes remains to be determined.

Study Parameters. One of the main purposes of this study is to assess whether the Lognormality Principle and the Sigma-Lognormal model can be used as a diagnosis tool for detection and differentiation of PD and atypical Parkinsonian syndromes.

The Script-Studio software [105] can be used to extract six neuromotor parameters from a pen stroke, and two global parameters.

At this point in time, and regarding the amount of data gathered, it has been found more pertinent to focus mainly on the global parameters: the Signal to Noise Ratio (SNR) and the number of lognormal impulses (nbLog) required to reconstruct the pen stroke. One last main study parameter is the SNR divided by the number of lognormal im-pulses required: The SNR/nbLog, which gives a general overview of the quality of reconstruction and fine motor control of the patient. A high SNR/nbLog ratio tends to indicate a good reconstruction and a patient in good control of its fine motricity.

Method. *Participants.* Building on prior experience [26], we collected data on four blocks of tasks involving distinct neuromuscular programs implicated in ocular pursuit and saccades, hand graphics, arm movements, and vocal sounds, according to the kinematic theory of rapid human movements performed in 2D and 3D. The objective of this study is to collect data for at least 30 patients in each of the three groups (PD, related parkinsonian syndromes and healthy patients.) Patients between age 50–75 with a clinical diagnosis of PD (N = 10) or related Parkinsonian syndromes (N = 1) were recruited within the first 6 years of motor symptoms, and compared to age-matched

healthy participants (N = 3). All provided informed consent. Patients were tested in the practically-defined OFF state, at least 12 h following the last intake of antiparkinsonian medication.

Tasks. Parkinsonian signs were assessed using a validated scale by a qualified neurologist. Eye movements were recorded with a standard eye tracker system. Following a visual or auditory cue, participants were instructed to make 30 linear strokes on a WACOM tablet using an electronic pen, repeatedly connect two or three (triangular) dots as quickly as possible, and to draw cursive connected "$\ell\ell\ell\ell\ell\ell$" and a spiral. They were asked to hold the tablet horizontally with arms stretched for 10 s, and to make triangle-shaped movements of the arms in the horizontal and vertical plane for 10 s while still holding the tablet with built-in accelerometer and gyroscope. They were asked to sustain a vowel or repeat 3 alternating vowels for 4 s, and this sequence was repeated 5 times. Velocity profiles were generated, and position signal data fed into the Sigma-Lognormal estimator. The lognormal parameters were calculated using low-pass filtered signals.

Results. Preliminary analysis for this pilot study reveals flagrant differences in SNR, number of lognormal impulses (and thus SNR/nbLog ratio) between healthy and PD patients on the "$\ell\ell\ell\ell\ell\ell$" tests. The comparison with the atypical group (typical vs atypical PD) also seems promising, though not reliable at this stage with the limited number of participants.

We cannot draw conclusions for this study until we reach a higher number of participants in all three study groups. Furthermore, a more complete analysis including all study parameters (fine and global) could prove to be an insightful discriminating tool. Lastly, a variety of tests aiming at different motor control skills could better differentiate symptoms between PD, related Parkinsonian syndromes, and healthy patients.

4 Performances

4.1 Deep Reinforcement Learning for ECG Modelling Using Lognormals

Context. In a recent study [109], we discussed the development of a model-driven approach for the analysis of the electrocardiogram (ECG) signals. This approach is motivated by the need to improve our capacity to understand the dynamics of complex systems represented in high dimensional space using comparatively sparse experimental data. This combination results in ill-posed problems that we can attempt to regularize by informing (constraining) our analysis using prior knowledge. We can operationalize this idea by embedding pre-existing knowledge in models used for inference. Further, by using biophysically-relevant models with parameters representing latent variables of interest, the inverse modelling process allows investigating processes that may not be experimentally accessible.

Previous models proposed for the ECG have mostly been limited to forward modelling and relied on systems of differential equations [24, 133]. Although very interesting, these oscillatory models operate near chaotic regimes, which makes them notoriously difficult to fit during inverse modelling. Alternatively, the PQRST complex of the ECG has been modelled by fitting a pair of Gaussian equations for each component of this

complex [9]. This approach is valuable for applications relying on high-quality fitting (e.g., signal compression), but the absence of biophysical motivation for this model is limiting.

Method. Here, we propose to model the PQRST complex as a set of lognormal equations. The motivation for adopting the lognormal is well-established in the context of the Kinematic Theory [118–120, 124]. The P, Q, R, S and T components of the ECG are associated with subsequent waves of depolarization and repolarization generated by the propagation of action potentials through gap junctions across the network of myocardial cells constituting the different structure of the heart (i.e., the sinoatrial node, the walls of the atria, the atrioventricular node, the His-Purkinje system, and the walls of the ventricles). We modelled each wave of the PQRST complex with one lognormal, except for the T wave that we decomposed in two lognormals (T + and T-) because its shape was not sufficiently well captured by a single lognormal.

For inverse modelling, we used a prototype-based approach (O'Reilly & Plamondon, 2010), where a prototype was used (Fig. 4) as an initialization condition for a deep reinforcement learning approach using as a reward the difference in signal-to-noise ratio (SNR) between two consecutive steps of the iterative learning algorithm [109]. We constrained this optimization process in a box. The envelope of all solutions compatible with these constraints can be calculated [108], allowing us to validate that this envelope encompasses the PQRST complexes observed in our dataset. We also enforced model-plausibility constraints to ensure that the model obtained from the fitting operation is plausible according to our knowledge of the targeted system. In our case, the order of the waves in the PQRST complex must be conserved. Thus, we enforced that the peaks of the lognormal equations modelling each of these components are not allowed to move temporally in a way that would inverse their order. Such an alteration of the temporal ordering of components is common in lognormal modelling, with significantly higher SNR being sometimes achievable by moving components in positions that are not plausible in a physiological sense but that model sources of noise accurately.

We validated our approach with a dataset of 150 ECG recordings collected from 40 infants between 1 week and 24 months of age. We divided these recordings into 9212 60-s segments of uninterrupted ECG recordings. Heartbeats were automatically detected using the Python library HeartPy. We rejected 803 segments (8.72%) because heartbeats could not be detected (i.e., a BadSignalWarning error was raised by HeartPy or less than 20 beats were detected). We made beats comparable by epoching and normalizing the beat duration as follows. Considering three subsequent R peaks occurring at time t_1, t_2, and t_3, the epoched and normalized version of the peak corresponding to t_2 is obtained by linearly interpolating the ECG between t_2-α and $t_2 + \alpha$ over 500 regularly spaced samples, with $\alpha = (t_3-t_1)/2$. This approach interpolates the EEG signal roughly (exactly when t_3-$t_2 = t_2$-t_1) from t_1 to t_3 on 500 points, with t_2 in the middle of that window. Note that this approach is slightly different than what we used in [105]. This deviation is adopted to correct the fact that the method in [105] concatenating two windows interpolated on $[t_1, t_2]$ and $[t_2, t_3]$ could introduce a slight distortion in the shape of the R peak when the cardiac rhythm is accelerating or decelerating. We mapped these 500-point epochs to a $[-1, 1]$ interval and refer to the variable along that dimension as the normalized time. For each segment, we computed a mean beat by averaging across

these epochs. We characterized the stability of the PQRST profile within a segment by computing the following signal-to-noise ratio between every beat and the mean beat. We rejected every segment that has a mean SNR across all its PQRST lower than 5 dB (N = 578; 6.3%). Such low SNR indicates PQRST complexes that are not similar across the recordings due to issues like R peak detection and various sources of artifacts.

We used the Stable_baseline3 and OpenAI Gym Python packages to train the reinforcement learning model and to apply it for parameter inference. The details of this procedure can be found in [109]. Parameters learned on time-normalized ECG signals can be mapped to corresponding values on the original time scale using the following relationship: $\{\mu^*, \sigma^*, t_0^*, D^*\} = \{\mu + log(\alpha), \sigma, \alpha t0, \alpha D\}$. The code used for the analyses is available at https://github.com/lina-usc/ecg_paper (accessed on 19 June 2023).

Results. We extracted the PQRST complexes for all segments (N = 1,008,784 PQRST complexes). We excluded from further analyses beats fitted with an SNR < 5 dB (8.8%). The fitting SNRs are generally lower than for fitting movement kinematics, with an average of 10.11 dB. For example, an average SNR of 20.75 dB was reported for a prototype-based lognormal modelling of the speed of triangular motion [106]. We believe this lower fitting accuracy for ECG signals is partly due to systematic offsets in the resting potentials. Such systematic offsets significantly contribute to the modelling error and can be observed at a steady state for electric potential but not for the speed of human movements.

As a proof of concept, we validated that modelling parameters are sensitive to a factor expected to have a significant effect on ECG: age. We evaluated the significance of the relationship between age and modelling parameters using the Kendall rank correlation coefficient. We used this non-parametric test to account for the non-normality of the data. Out of 24 parameters, 14 showed a statistically significant relationship with age at p_{adj} < 0.05 with a conservative Bonferroni adjustment for 24 independent tests (Table 1).

Table 1. Kendall correlation coefficients and associated p-values for the relationships between model parameters and age. Bold red values indicate statistical significance as p_{adj} < 0.05.

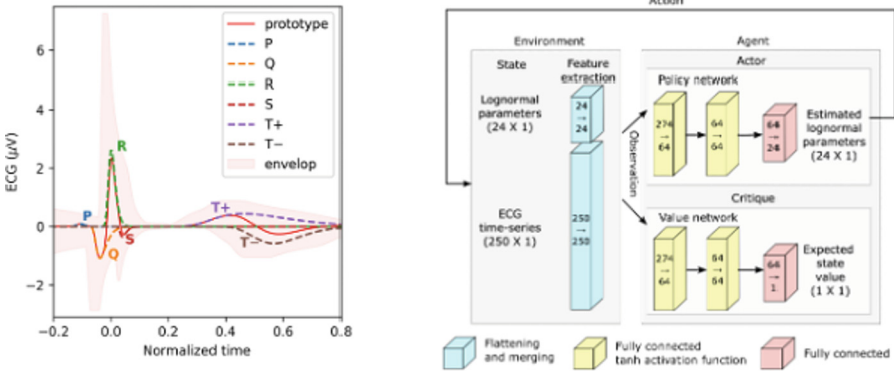|	D		μ		σ		t₀	
	τ	p_{adj}	τ	p_{adj}	τ	p_{adj}	τ	p_{adj}
P	0.135	0.764	.245	2.28e-03	-0.0842	4.31	-0.374	6.04e-08
Q	-0.00487	10.5	0.178	0.110	5.10e-04	23.8	-0.322	7.07e-06
R	0.0669	6.88	0.377	4.59e-08	-0.113	1.71	-0.452	1.36e-11
S	-0.158	0.288	0.188	0.0673	0.0332	14.3	0.397	5.87e-09
T+	0.292	7.61e-05	0.382	2.92e-08	0.249	1.80e-03	0.377	4.37e-08
T-	-0.302	3.48e-05	0.422	4.22e-10	0.248	1.86e-03	0.232	5.26e-03

Fig. 4. Left: Prototype for the PQRST complex. The shaded region shows the envelope defined by the bounding box constraints on the value of the parameters. Right: Schematic of the deep reinforcement learning model implemented for parameter estimations. Reproduced from [109].

Discussion. We expect the fitting accuracy from an approach such as [9] to be higher than what was obtained with our model, although we did not explicitly compare accuracies. Published values may not be comparable because they were obtained on a different dataset, with different preprocessing, targeting different populations. Furthermore, our approach uses only 24 parameters, whereas the approach using pairs of normal equations in [9] uses 35. This approximative 50% increase in modelling parameters is expected to provide more flexibility to improve fitting accuracy. More importantly, we aimed to develop a biologically relevant model rather than obtain maximal fitting accuracy. High fitting accuracy is desirable for some applications, such as the signal compression application mentioned in [9]. However, for physiological interpretability, the biological relevance of the model and the preservation of component order are more important and should be prioritized even when it results in some loss in fitting accuracy. These arguments should be familiar to anyone who pondered on the issue of model overfitting.

Outcomes. As demonstrated by these initial results, the proposed model is sensitive to factors influencing the ECG signal. Given the interpretability of this model in terms of the convolution of a large number of coupled subsystems, this model-driven approach to the analysis of ECG is poised to offer a more principled way to analyze these biosignals.

4.2 Kinematic Theory, Muscle Fatigue and Optimality: Contribution to the Biomechanics of the Upper limb

Context. The laboratory of Simulation and Movement Modelling (Montréal, Canada) is recognized for its research on upper-limb biomechanics. Particularly, it focused on 1) shoulder fatigue [65, 66], a component of the injury production mechanism [32], and, more recently, 2) predictive simulation using the optimal control theory [100]. Both applications were recently studied in line with the Kinematic Theory (KT) of rapid human movements. Existing tools like visual analog scales, questionnaires, and electromyography (EMG) have provided valuable insights into shoulder fatigue prevention but remain limited or complicated to use in clinics, sports, or occupational environments.

Differentiating between central and peripheral shoulder fatigue is also critical for tailoring appropriate recovery interventions. KT, which models the neuromotor impulse response through lognormal functions, offers a robust framework for detecting pathologies. This theory provides an idealized model of motor control, where changes in the neuromuscular system are manifested through modifications in parameters defined by this theory. A relevant tool that relies on KT must be sensitive to shoulder fatigue, and its parameters should be reliable. The objectives of two recent papers [80, 81] were to assess if shoulder fatigue might change KT central and peripheral parameters and their test-retest reliability.

Invariants commonly observed in human movements provide valuable insights into movement generation and control mechanisms. According to KT, the velocity profile's invariance derives from the human system's complexity and the interconnection of its numerous subsystems. It results in an asymmetrical bell-shaped velocity profile of the end-effector, as observed in rapid human movements. Concurrently, optimal control theory suggests that a system operates in the most efficient manner possible, considering both cost function and constraints. Interestingly, no identified cost function has been able to reproduce the speed profile suggested by KT. The objectives were twofold: 1) to assess various cost functions by expressing them in terms of parameters derived from KT, and 2) to propose a novel cost function that aligns coherently with KT velocity profiles.

Shoulder Fatigue Assessment. Twenty healthy participants performed two sessions of handwriting tasks on a tablet put vertically at shoulder height, both pre- and post-fatigue of the shoulder (50% of maximum voluntary contraction in concentric at 90°/s till 9/10 on Borg CR10 scale). In one session, the fatigue was induced through internal rotation, and in the other, through external rotation. The writing tasks involved basic strokes, triangles, and horizontal and vertical oscillations. Parameters from these strokes were determined following the Sigma-Lognormal model. Both intra-subject and inter-subject changes in parameters due to fatigue were evaluated using U-Mann-Whitney tests. An additional 20 participants perform two sessions of pre-fatigue strokes only. Intraclass correlation coefficients (ICC) were calculated from the 40 participants to quantify the parameter reliability. We also reported the standard errors of measurement and minimal detectable changes.

Central and peripheral parameters were significantly modified after fatigue, but responses were subject-specific. Still, when considering our sample, parameters that describe the motor program execution increased significantly after fatigue. Reliabilities of the KT main parameters were moderate to excellent for all tests. Particularly, the parameters that best explained shoulder fatigue exhibited good to excellent reliability, accompanied by low standard errors of measurement. Overall, the setup and handwriting tests were appropriate for shoulder fatigue detection. Further research is required to detect lower levels of shoulder fatigue and determine its feasibility in clinical, sports, and occupational environments.

Optimal Control and Kinematic Theory. Common cost functions (least squared velocity, acceleration, and jerk, as well as minimal time: $\int t2\ dt$) were expressed as functions of the lognormal parameters: μ and σ that are the log-time delay and response time, respectively. We found that minimizing the least squared velocity, acceleration, and jerk amounts to maximize μ and σ, which is not "physiological". In-deed, previous

studies proposed boundaries of μ and σ for handwriting [111]. In contrast, minimizing time corresponds to minimizing μ and σ. Consequently, we proposed a cost function composed of minimal jerk, kinetic energy (i.e., weighted squared velocity), and time. Such a cost function admits a minimum within the μ and σ boundaries. We simulated arm movement in the horizontal plane by minimizing this cost function. We could predict an asymmetrical bell-shaped velocity profile of the end-effector like the one expected by KT. The asymmetry comes from the minimum time, while the concavity of the deceleration is mainly explained by the kinetic energy. The proposed cost function needs further validation; weights could be identified using inverse optimal control.

KT has paved the way for fresh perspectives, promising to deepen our comprehension of the mechanisms underlying human motion generation and its adaptation during fatigue-inducing tasks.

4.3 Objective Analysis of Surgical Performance thanks to a Simulator Augmented by Artificial Vision

Context. Surgical skill assessment is essential for the continuous improvement of surgeons. However, current methods such as evaluation using scoring systems like the OSATS [8] require at least one expert evaluator. This limits the frequency of assessments and makes them prone to bias and variability.

Many methods have been proposed in the past years for the automatic and objective evaluation of surgeons. Those methods use various data acquisition devices to capture surgical movements, such as cameras [63, 64, 72], surgical robots [52, 110], accelerometers [165], EMG sensors [148], among others. The data acquired is usually paired with metrics evaluation algorithms or machine learning based techniques to assess surgical skills [166].

The Leap Motion Controller (LMC) (Ultraleap Ltd, Bristol, UK) provides a low-cost solution to capture relevant hand movement data in three dimensions (3D) through its integrated hand-tracking software and presents a potential method to acquire kinematic data for surgical skill evaluation.

To analyze complex patterns using kinematic data, we have exploited the Sigma-Lognormal model which has shown validity in many fields of application [126] using the Lognometer, a system that integrates this model to allow the acquisition and analysis of precise 2D handwriting movements of varying complexity [49].

The aim of this study was to validate the use of the LMC to accurately capture dominant hand movements and assess its potential to be used as a data acquisition tool for surgical performance evaluation.

Methods. Three subjects participated in the data acquisition: one left-handed male, one right-handed male and one right-handed female. Two different tasks were performed for 30 repetitions each, on the Lognometer. The Lognometer comprises a digital pen and tablet (Cintiq 13HD, Wacom Co., Kazo, Japan), and captures the position of the tip of the pen at a 300 Hz frequency.

The task execution was simultaneously recorded with the LMC, which saves infrared video files and 3D positions of various hand markers with a variable acquisition frequency (60–90 Hz). The central palm marker coordinates were used to evaluate velocity profiles

in this study, a good compromise between tracking stability, precision, and proximity to the end-effector.

The first task was the drawing of a single stroke with the pen on the tablet after a visual stimulus and aimed to verify the reliability of the LMC to capture fast movements. The second task was to draw a continuous line connecting three targets to form a triangle and aimed to verify the capability of the LMC to accurately reproduce velocity profiles from the recorded 3D coordinates for more complex movements. Even though 2D pen strokes on the Lognometer were compared with 3D recordings on the LMC, most of the movement was along the 2D plane of the Lognometer tablet. The position data from each device were used to obtain velocity profiles that were aligned and compared one-by-one. Normalized Cross-Correlation was used to obtain a Pearson's correlation coefficient, quantifying the similarity between the two signals between -1 and 1.
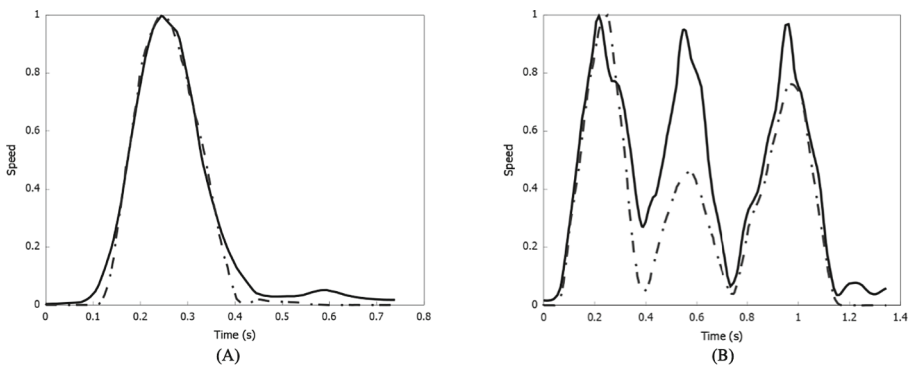
Fig. 5. Velocity profiles from the Lognometer (dotted line) and LMC (full line) for a repetition of the single pen stroke task (A) and the three-target triangle task (B).

Results. In total, 180 movements were recorded and compared. Figure 5 shows the superimposed velocity profiles from both devices for an example recording of each task. For the single pen stroke task, average Pearson correlation coefficients of 0.90 ± 0.13, 0.91 ± 0.17 and 0.98 ± 0.03 were obtained for subjects 1, 2 and 3 respectively. Four repetitions out of 30 were partially cut for subjects 1 and 2, due to the movement being too rapid for proper hand detection by the LMC. Excluding these outliers, the average Pearson correlation coefficients were 0.94 ± 0.04 and 0.98 ± 0.01 for subjects 1 and 2 respectively. For the three-target triangle task, the average Pearson correlation coefficients were 0.87 ± 0.03, 0.84 ± 0.03 and 0.91 ± 0.01 for subjects 1, 2 and 3 respectively. There were no detection interruptions for all repetitions of this task across subjects.

Discussion. This study analyzed the LMC's potential to be used in complex hand tracking movement analysis in a surgical evaluation context. The data acquisition protocol was robust, and the resulting recordings were of high quality when compared to a reference Lognometer.

With outliers removed, the Pearson correlation coefficients obtained from the single pen stroke task were very strong, with a total average across the three subjects of 0.97 $\pm$ 0.03, signifying a close adequation as observed on Fig. 5A.

For the more complex three-target triangle task, the average Pearson correlation coefficient of 0.87 ± 0.04 across the three subjects was lower than the coefficient of the simpler pen stroke task, but still represents a significant similarity between the two velocity profiles. As seen on Fig. 5B, the velocity profiles for the triangle task show similarities in peak timing. However, differences in amplitude and shape were observed: the second peak measured by the LMC was much higher, and less smooth. This may be due to the fact that, unlike the Lognometer that captures movements of the tip of the pen on a single 2D plane, the LMC captures 3D hand movement data, including pronation, supination and varying hand placement along the movement, thereby changing the shape and amplitude of some parts of the velocity profile due to additional movements being detected.

Momentary loss of detection was also observed in certain recordings for the single pen stroke task, as the LMC software cannot detect hands moving at a very high speed. No loss of detection was observed for the triangle task, since this task requires higher accuracy which is translated in a slower drawing speed: the slower movement allows for reliable detection of the hand.

To fully assess the possibility to use the LMC to track 3D hand movements, its data acquisition should be compared with devices also capable of 3D tracking. However, even when compared with reference 2D data, similar speed profiles were achieved which confirms the potential of the Leap Motion Controller for the purpose of surgical skill evaluation. This data could be evaluated through the Sigma-Lognormal model, to further compare the movements captured from both devices [116]. The lognormal parameters extracted from the model could also be used as metrics to classify different levels of expertise based on the quality of their movements.

Outcomes. The Ultraleap LMC is a promising tool to capture 3D kinematic data, which could potentially be used to assess surgical performance and analyze complex movements through the Sigma-Lognormal model.

4.4 Kinematic Reconstruction of Static Calligraphic Traces from Curvilinear Features

Context. Most of the existing works are aimed either at a precise analysis of the kinematics of a digitized input or at the segmentation of a handwriting trace into components for biometric or pattern recognition purposes. On the other hand, our specific aim is perceptually and artistically driven, and we seek to infer a physiologically plausible motion from an input trace, the kinematics of which may be unavailable, such as when using vector graphics inputs, or may be degraded or unreliable due to the poor quality of a digitization device, such as when using low-cost tablets or trackpads. The motivation for this approach is grounded on the hypothesis that the visual perception of marks made by a drawing hand triggers activity in the motor areas of the brain [61, 93], and further induces an approximate mental recovery of the likely movements and gestures underlying the artistic production [62, 114]. We argue that this is particularly true for

certain art forms such as expressed in calligraphy [60] and graffiti art [14, 99], in which the mastery of a skillful movement in large part determines the aesthetic quality of the resulting artefact.

In our proposed method, we first represent an input trace as a series of closely fitted circular arcs. We then exploit this spatial and structural geometric representation to infer the kinematics of a likely generative movement—as would be performed by a skilled human expert or artist, as predicted by the Lognormality Principle. To do so, we rely on the Kinematic Theory of rapid human movements [127], a family of models of reaching and handwriting motions, in which a movement is described as the result of the parallel and hierarchical interaction of a large number of coupled neuromuscular components. The resulting method allows the reconstruction of physiologically plausible velocity profiles for the geometric trace of an input movement given as an ordered sequence of points.

Method. Our first step is to take advantage of the duality between curvature and symmetry axes [92] in order to extract more robustly curvilinear shape features (CSFs), such as those based upon extrema (of some curvature measure or approximation) along a handwriting or drawing trace. The method is also directly adaptable to open contours, to contours with breaks in curvature, and can further be used to identify loops where a trace overlaps itself. Each CSF is also explicitly paired with corresponding contact circles and a pair of curvilinear support regions: contour traces on each side of an identified contact circle or extremum, where curvature is approximately monotonic. We have introduced, defined and described how to retrieve CSFs in recent works [13, 16, 17].

In between each contact circle segment, as a second step, we fit Euler spirals to the trace of the support regions. Euler spirals or clothoids are a useful type of curves in which curvature varies linearly with arc length, permitting the description of variably curved segments which may contain an inflection. To select initial parameter values of each Euler spiral segment, we use a secant method described by Levien [91]. We proceed to refine this initial fit with a least squares optimisation based on the classic Gauss-Newton method. Once spirals are optimally fitted, we can identify inflection points and obtain a final segmentation of the entire trace as a set of circular arcs. More details can be found in [17]. This representation of the input trace, as a series of circular arcs, is now ready to be exploited together with the Signal Lognormal ($\Sigma\Lambda$) model [105, 117].

An important practical assumption is typically made when initiating the $\Sigma\Lambda$ model: handwriting movements are mostly made with rotations of the elbow or wrist. The corollary is then that the curvilinear evolution of a drawing stroke can be approximated by a circular arc. This has for consequence to simplify the computation of the angular evolution of a stroke as represented by the $\Sigma\Lambda$ model.

Each stroke is to be represented via aiming target locations. The initial set of aiming targets (aka "virtual targets") consists of three types of feature points or features for short: from CSF analysis (i) recovered curvature maxima loci, and from Euler spiral analysis (ii) inflections, and (iii) splits (of wide angled circular arcs). We can either directly used these loci or find their nearest neighbors, on the original input trace, which leads to slightly more accurate reconstructions. An initial estimation of the trajectory parameters is performed using these virtual targets.

To improve the reconstruction, we adopt an iterative refinement scheme in which we adjust the curvature and time overlap parameters together with the target positions

in order to minimize the difference between the reconstructed and original trajectories. We optimize the quality of the reconstruction by maximizing an error criterion based on a signal-to-noise ratio or SNR [17]. Because we do not take into consideration the kinematics of the input, we evaluate the quality of the reconstruction using the SNR computed between the reconstructed and input trajectory. Our proposed method consistently produces accurate (>15dB SNR) reconstructions of the input, while providing flexibility for the use of additional constraints that can be exploited in order to generate interactive stylizations and variations.

Discussion. The $\Sigma\Lambda$ model directly reflects the characteristics of a smooth human movement at the planning and neuromotor level. We therefore expect and observe that parameter perturbations result in variations of a trace that are similar to the one that would be seen in multiple instances of handwriting or drawing made by one or more subjects. We have found that applying the perturbation with a variance inversely proportional to the temporal overlap parameters improves the legibility of the variations. This is equivalent to imposing a higher precision requirement at trajectory locations with higher curvature, which are known to be the most informative [53]. This is also related to the "minimum intervention principle" [151], Suggesting that human movement variability is higher where it does not interfere with the performance required for a task.

The smooth kinematics produced by the $\Sigma\Lambda$ model can be exploited to generate expressive brush renderings of the trajectory. We have designed and applied a brush model that builds upon the assumption that the amount of paint deposited is inversely proportional to the speed of the drawing tool. We can also sweep a texture along the generated trajectory with width also inversely proportional to speed [16], which generates patterns that are highly evocative of some instances of calligraphy as well as graffiti made with markers or spray paint. The trajectory generated by the reconstruction, as well as the brush rendering parameters can be edited in real time with an intuitive user interface [16]. Also, the resulting kinematics reproduce natural human-like movements that can be exploited to create stroke animations of the input as well as to generate smooth motion paths for virtual characters or even humanoid robots [15]. Another related application of the $\Sigma\Lambda$ parametrization is to perform kinematic smoothing of a given trajectory [17].

5 Techniques

5.1 Separation Algorithm and Evaluation Applied to the Delta-Lognormal Model

Context. The present paper proposed a novel algorithm to extract lognormal parameters from handwriting gesture. The proposed algorithm is based on the branch and bound method combined with the interval arithmetic. The general idea is to exploit intervals arithmetic to bound the Delta-Lognormal function and its gradients and use the bounded functions in several ways in a branch and bound global optimization. The goal is to output the global and specific timing properties of a handwriting gesture in a unique bounding box. New tools could then exploit the confident interval of the bounding box to address the wide range of applications where the model can serve. The temporal properties extracted from the pointing gesture, allows to reconstruct the velocity profile

of the gesture and represent the planning and timing used to accomplish the pointing gesture. The new algorithm produces a unique high-quality solution with a processing time sufficiently short for practical applications. The accuracy of the extracted parameters that constitute the bounding box is quantified automatically.

Methodology. Before starting to detail the proposed algorithm, some definitions and notations need to be presented. An interval is denoted by a variable in upper case as presented in (1).

$$X = \left[\underline{x}, \bar{x}\right] \tag{1}$$

An interval vector or box is denoted by a variable in uppercase in bolt (2).

$$X^I = (X_1, X_2, \dots X_n) = \left(\left[\underline{x}_1, \bar{x}_1\right], \left[\underline{x}_2, \bar{x}_2\right], \dots \left[\underline{x}_n, \bar{x}_n\right]\right) \tag{2}$$

The Basic interval arithmetic operations and the one-variable transcendental functions operations are described in [19, 101].

Definition 1: The natural interval extension of a given function $f(x_1, x_2, \dots x_n)$ of n variables is given by the interval function $F(X_1, X_2, \dots X_n)$, which is obtained by replacing the real variable x with the corresponding interval variable X.

Fundamental Theorem. Let $F(X_1, X_2, \dots X_n)$ be the natural interval extension of $f(x_1, x_2, \dots x_n)$ then $f(X_1, X_2, \dots X_n) \subseteq F(X_1, X_2, \dots X_n)$, and for all intervals, $Y\_k \subset X\_k, for k 1 \dots n, f(Y\_1, Y\_2, \dots Y\_n) \subseteq F(Y\_1, Y\_2, \dots Y\_n)$,
where $f(Y_1, Y_2, \dots Y_n)\{f(x_1, x_2, \dots x_n) : x_k \in Y_k for k = 1, ..n\}$.

This theorem due to Moore [109] was extended and proved by Hansen [70] We use this theorem to bound de Delta-Lognormal function as a specific sequence of interval arithmetic operations.[19].

The global optimization problem that is considered is the following:

Minimize $f(p) subject to p \in P^I$, where f is a 7 dimensional continuously differentiable function subject to p $\mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ and $P^I \subseteq \mathbb{R}^{\mathbb{N}}$ is a 7 dimensional interval vector. Thus,

$$P^I = \left\{\left[\underline{D}_1, \bar{D}_1\right], \left[\underline{\mu}_1, \bar{\mu}_1\right], \left[\underline{\sigma}_1, \bar{\sigma}_1\right], \left[\underline{D}_2, \bar{D}_2\right], \left[\underline{\mu}_2, \bar{\mu}_2\right], \left[\underline{\sigma}_2, \bar{\sigma}_2\right], \left[\underline{t}_0, \bar{t}_0\right]\right\}$$ is

the bounding space.

The objective function and it's gradient are respectively:

$$f\left(P^I\right) \in F\left(P^I\right) = \left[\underline{F}\left(P^I\right), \bar{F}\left(P^I\right)\right] = \int \left(v_t(t) - \Delta\Lambda\left(t; P^I\right)\right)^2 dt \tag{3}$$

$$f'\left(P^I\right) \in \nabla F\left(P^I\right) = F'\left(P^I\right) = \frac{\partial f\left(P^I\right)}{\partial P_i^I} i = 1..7, \tag{4}$$

Λ is the lognormal impulse response function. Now that we have these definitions and notations, the proposed algorithm is called IAB&BPE which stands for Interval Arithmetic Branch and Bound $\Delta\Lambda$ Parameter Extractor. IAB&BPE is formulated as follows:

IAB&BPE:

1. Set the initial parameter P^I **initial rule.**

2. Set the working list $L_1 := \{ P^I \}$, the final list $L_2 := \{\}$
    and the upper bound $\overline{\tilde{f}} = f(m(P^I))$

While $(L_1 \neq \{\})$

3. Select optimal box P from L_1 **Selection rule**

4. Bisect P^I into P_i^I sub boxes $i = 1..2$ **Splitting rule**

For $i = 1$ to 2

5. Compute the bound $f(m(P_i^I)), \underline{F}(P_i^I)$ **Bounding rule**

6. If $\left(\left(\underline{F}(P_i^I) < \overline{\tilde{f}} \right) \text{and} \left(0 \in \nabla F(P_i^I) \right) \right)$ **Discarding rule**

7. $\overline{\tilde{f}} = min \left\{ \overline{\tilde{f}}, f(m(P_i^I)) \right\};$ **Update upper bound for f^***

8.   If P_i^I satisfy the ending criterion   **Termination rule**
          $L_2 += \{\underline{F}(P_i^I), P_i^I, \nabla F(P_i^I)\};$

9.   Else $L_1 += \{\underline{F}(P_i^I), P_i^I, \nabla F(P_i^I)\};$ **Storing rule**

10. Return L_2, f^*

The details concerning the different rule can be found in [22].

Tests and Results. The algorithm has been tested using real and synthetic human ges-
tures. We developed a database comprising 9000 and 500 synthetics and real human
gestures respectively. The real gestures were acquired with a Wacom Intuos2 digitizer,
sampled at 200 Hz. The first experiment consists in testing the algorithm with synthetic
gestures. In this experiment, the algorithm was tested in its ability to retrieve the global
Delta-Lognormal parameters representing each synthetic gesture. For the 9000 synthet-
ics gestures, the algorithm always finds the solution, not only the base line target within
an accuracy of $\varepsilon = 10^{-6}$, but also a confidence interval including the target value. The
second experiment has been conducted using data collected from human gestures. In
this experiment, parameters that are considered as solutions for a gesture must have an
accuracy of at least 25 dB SNR. For this criterion, the proposed algorithm converges
for all cases studied. Figure 6 shows an example of a human pen tip movement and
its corresponding original and reconstructed velocity profiles. Both the original and its
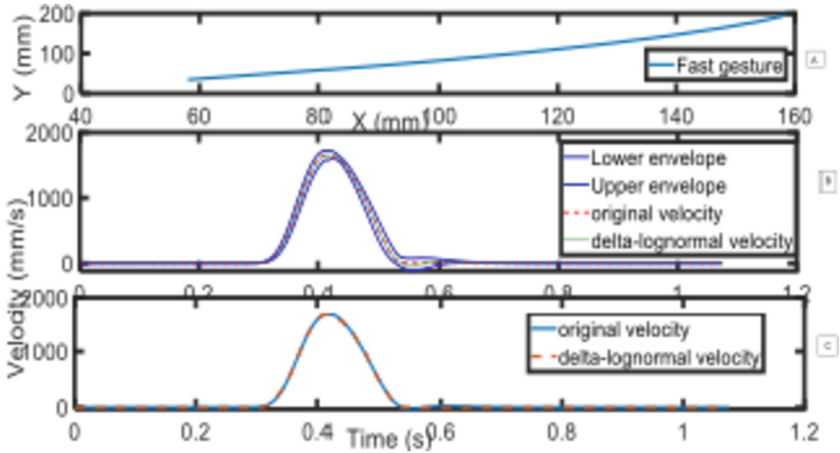chosen reconstructed are found in the bounding box returned by the algorithm.

Fig. 6. Example of human Handwriting strokes extracted by IAB&BPE: A. the (x, y) position of the pen tip movement of a writer, B. the real and reconstructed velocity profile with a 31dB SNR enclosing their envelopes, C. the reconstructed velocity profile.

Outcomes. In this paper we have shown that an interval arithmetic branch and bound algorithms can extract the Delta-Lognormal parameter with less computational costs. The effectiveness of the proposed algorithm is quite remarkable. This algorithm exploits the natural interval extension and the fundamental theorem of interval arithmetic to compute the bounding operations of the Delta-Lognormal function.

5.2 Analysis of Three-Dimensional Movements with the Sigma-Lognormal Model

Context. The Kinematic Theory of rapid human movements [118–120, 123, 124] describes movements as a sequence of elementary strokes, which are planned in the brain with specific execution times and distances to cover, and are then executed by the neuromuscular system with lognormal speed. For one-dimensional movements, the Delta-Lognormal model [122] considers two strokes in opposed direction, an agonist and an antagonist movement. For two-dimensional movements, the Sigma-Lognormal model [117] considers a vectorial sum of strokes, which overlap in time. To estimate the parameters of the strokes, the Robust XZERO algorithm [41] is generally used to extract the lognormal parameters from the velocity profile, complemented with an estimation of the start and end angle of each stroke [105]. In the following, we review a recent generalization of the Sigma-Lognormal model to three dimensions [59, 144], which naturally extends the model with two additional angles.

Model. In the 3D Sigma-Lognormal model [59], each stroke has 8 parameters,

$$s_{3D} = (t_0, D, \mu, \sigma, \theta_s, \theta_e, \phi_s, \phi_e) \tag{5}$$

where t_0 is the starting time, D is the distance to cover, μ and σ are the parameters of the lognormal speed, θ_s and ϕ_s are the starting angles, and θ_e and ϕ_e are the ending angles.

When compared with the 1D and 2D models, the same lognormal speed

$$|\vec{v}(t)| = \frac{D}{\sqrt{2\pi} \cdot \sigma(t - t_0)} \exp\left(-\frac{[\ln(t - t_0) - \mu]^2}{2\sigma^2}\right) \tag{6}$$

is considered for each stroke. When compared with the 2D model, the angles θ_s, θ_e are complemented with an additional pair of angles ϕ_s, ϕ_e to extend into three dimensions. The distance travelled at time t is

$$d(t) = \int_0^t |\vec{v}(\tau)| d\tau = \frac{D}{2}\left[1 + \text{erf}\left(\frac{\ln(t - t_0) - \mu}{\sigma\sqrt{2}}\right)\right] \tag{7}$$

and the angles at time t are

$$\theta(t) = \theta_s + (\theta_e - \theta_s)\frac{d(t)}{D} \tag{8}$$

$$\phi(t) = \phi_s + (\phi_e - \phi_s)\frac{d(t)}{D} \tag{9}$$

considering a pivoting movement. The three velocity components are calculated as

$$v_x(t) = \sum_{i=1}^n |\vec{v_i}(t)| \sin(\phi_i(t))\cos(\theta_i(t)), \tag{10}$$

$$v_y(t) = \sum_{i=1}^n |\vec{v_i}(t)| \sin(\phi_i(t))\sin(\theta_i(t)), \tag{11}$$

$$v_z(t) = \sum_{i=1}^n |\vec{v_i}(t)| \cos(\phi_i(t)) \tag{12}$$

and the final movement is a vectorial sum over a sequence of n individual strokes $\vec{v}(t) = \sum_{i=1}^n \vec{v_i}(t)$.

Parameter Estimation. The 8 parameters of the 3D Sigma-Lognormal model are estimated from an observed trajectory as follows. First, the trajectory is preprocessed by stopping the movement at the beginning and the end during 200ms (which leads to a more stable estimation of the first and the last stroke), interpolating the velocity profile with cubic splines and resampling at 200 Hz (which leads to a normalization of the sampling rate and supports parameter estimation for acquisition devices with a low sampling rate), and removing noise introduced by the acquisition device with a low-pass filter.

Afterwards, strokes are estimated iteratively, one stroke at the time. They are detected in the speed profile with respect to a minimum area under curve and the Robust XZERO algorithm [42] is used to estimate the parameters of the lognormal speed. Afterwards, the estimation of the angular parameters is based on characteristic times of the lognormal function, including the time of maximum speed and the inflection points. They are used to estimate the velocity components in the three dimensions and calculate the angles with trigonometric functions. For more details, we refer to [59].

The model quality is measured by means of the signal-to-noise ratio (SNR)

$$SNR = 10 \cdot \log \left(\frac{\int_{t_s}^{t_e} \left| \overrightarrow{v_o}(\tau) \right|^2 d\tau}{\int_{t_s}^{t_e} \left| \overrightarrow{v_o}(\tau) - \overrightarrow{v_r}(\tau) \right|^2 d\tau} \right) \tag{13}$$

comparing the observed velocity $\overrightarrow{v_o}$ with the reconstructed velocity $\overrightarrow{v_r}$ of the analytical 3D model.

Experimental Results. The 3D extension of the Sigma-Lognormal model has been tested on two action recognition datasets, HDM05 [104] and UTKi-nect [160], as well as an Air-Writing dataset [29]. For the HDM05 dataset, we consider a common subset of 249 motion samples from 11 actions performed by 5 subjects, recorded with a Vicon motion caption suit at 120 Hz. The UTKinect dataset contains 199 samples of 10 actions performed by 10 subjects, recorded with a Kinect camera at 30 Hz. For the Air-Writing dataset, we consider a common subset of 100 words written by 5 subjects in the air, recorded by a Leap camera at 60 Hz.

Table 2 shows the SNR results for the three datasets. For the two action recognition datasets, a high-quality SNR is achieved that is clearly above 15dB, which is generally considered as a quality threshold for kinematic analysis. Although the Air-Writing results are below this threshold, the reconstructed trajectories could be used in a word recognition experiment without significantly impacting the classification accuracy [59].

Table 2. SNR results of the 3D Sigma-Lognormal model in dB.

Database	HDM05	UTKinect	Air-Writing
SNR	18.52 ± 4.09	20.21 ± 4.40	12.52 ± 2.02

Outcomes. With a natural extension of the Sigma-Lognormal model to three dimensions we were able to reconstruct a variety of 3D movements, recorded with different acquisition devices, with a good model quality. The results are encouraging and open up promising possibilities to use the Kinematic Theory in three dimensions, for example in biomedical contexts or in robotics.

5.3 Comparison of Symbolic and Connectionist Algorithms to Correlate the Age of Healthy Children with Sigma-Lognormal Neuromotor Parameters

Context. Motor control, a crucial skill that is progressively acquired during childhood, profoundly influences a children's ability to learn and live well. Traditional methods of measuring motor control maturity, such as administered motor ability tests or behavior-based questionnaires, often require significant human or material resources [23, 57] and can be influenced by cultural differences. This study proposes a convenient and culturally neutral approach using handwriting, a typical fine motor control task. Employing the Kinematic Theory of rapid human movements [118–120, 123, 124] and its Sigma-Lognormal model [106], we extract specific parameters from children's handwriting

strokes on a tablet. Both this Theory and model have been used in various biomedical applications, including analyzing graphomotor performances in kindergarten children [45], assessing stroke risk [127], and identifying Attention-Deficit/Hyperactivity Disorder (ADHD) in children [79]. In this study, we extend this research and propose the use of a tablet-based system [45] to estimate motor control maturity in children, leveraging the Kinematic Theory. The Sigma-Lognormal model modelized the velocity profile of movements into lognormal functions, with each function capturing distinct kinematics related to neuromuscular commands. From each lognormal function, six parameters are derived: $\{t_0, D, \mu, \sigma, \theta_{start}, \theta_{end}\}$. Additionally, three parameters (SNR, nbLog, SNR/nbLog) were employed to evaluate the reconstruction.

Method and Experiments

Participants. We aimed to develop a model correlating Sigma-Lognormal parame-ters with motor control maturity in neurotypical children. A total of 513 children, aged 6 to 13 years, from three schools in the south-shore of Montréal participated in the tests. Children with reported neurological, psychological, or motor disorders were excluded.

Sigma-Lognormal Tests. Participants performed two tests: the simple stroke test and the triangular drawing test. For the simple stroke test, participants drew a straight line, and for the triangular drawing test, they drew a triangle crossing three round targets. The movements were recorded using a tablet [49].

Data Transformation. To facilitate model training, the one-hot encoding was used to represent the orientation of stroke drawings. Clockwise angles were converted to match counterclockwise angles.

Experiments. Different approaches were explored: training models on individual movements, calculating mean movement parameters per participant, and using all movements together. Models such as Recurrent Neural Network (RNN), Multilayer Perceptron (MLP), Ordinary Least Squares (OLS), Ridge Regression (RR), Huber Regression (HR), Support Vector Regression (SVR), XGBoost (XGB), Random Forest (RF), and K-Nearest Neighbors Regression (KNN) were tested and compared using nested cross-validation.

Results. In addition to assessing the regression model's performance using the coefficient of determination (R2), mean absolute error (MAE), and root mean squared error (RMSE) were computed to compare mean errors. The mean absolute percentage error (MAPE) was also used to evaluate errors relative to the participants' age.

The results, shown in Tables 3 and 4, point out significant differences in performance between the two tests. The models for the triangular test outperformed the models for the simple stroke test. The lower performances in the simple stroke test may be at-tributed to the test's simplicity, as even the youngest children were able to perform it well. On the other hand, the triangular test better differentiated age-related gains in performance. Nested cross-validations were performed, and one-way ANOVA analysis showed that the neural networks performed significantly better than other models, particularly with full trials.

Discussion and Future Work. The Sigma-Lognormal model proved effective in estimating the evolution of motor control maturity with efficiency and accuracy. Even simple linear regression yielded decent results when the movements were modeled using

Table 3. Regression model's performance for the triangular tests.

Data	Model	RMSE	MAE	MAPE	R2
	OLS	1.290	1.032	0.113	0.476
	HR	1.280	1.027	0.112	0.484
	RR	1.239	0.986	0.108	0.517
	KNN	1.365	1.136	0.126	0.416
Mean trial	RF	1.302	1.060	0.116	0.468
	XGB	1.268	1.029	0.113	0.493
	SVR	**1.210**	**0.971**	0.107	**0.539**
	MLP	1.278	1.010	0.110	0.486
	GRU	1.252	0.978	**0.106**	0.505
	OLS	1.385	1.166	0.128	0.400
	HR	1.382	1.162	0.128	0.403
	RR	1.386	1.167	0.128	0.399
	KNN	1.453	1.237	0.140	0.341
Full trials	RF	1.393	1.178	0.130	0.393
	XGB	1.364	1.149	0.127	0.418
	SVR	1.319	1.094	0.120	0.456
	MLP	1.244	0.991	0.108	0.515
	GRU	**1.196**	**0.937**	**0.102**	**0.548**

Table 4. Regression model's performance for the simple stroke tests.

Data	Model	RMSE	MAE	MAPE	R2
	OLS	1.516	1.220	0.133	0.283
	HR	1.514	1.218	0.133	0.285
	RR	1.484	1.206	0.132	0.313
	KNN	1.521	1.285	0.139	0.279
Mean trial	RF	1.460	1.215	0.133	0.335
	XGB	1.478	1.230	0.135	0.318
	SVR	**1.450**	**1.191**	**0.130**	**0.343**
	MLP	1.487	1.214	0.132	0.310
	GRU	1.501	1.222	0.133	0.297
	OLS	1.553	1.334	0.147	0.249
	HR	1.583	1.367	0.150	0.220
	RR	1.585	1.369	0.151	0.218
	KNN	1.616	1.392	0.156	0.186
Full trials	RF	1.541	1.324	0.146	0.261
	XGB	1.518	1.299	0.143	0.283
	SVR	1.497	1.270	0.140	0.302
	MLP	1.485	1.199	0.133	0.309
	GRU	**1.425**	**1.125**	**0.123**	**0.365**

the Sigma-Lognormal model. Handwriting, as a daily activity, can be easily acquired and analyzed for health monitoring purposes.

Among the algorithms compared, GRU and SVR performed best, highlighting the advantages of neural networks in customizing data structures to fit specific needs. However, symbolic algorithms, such as SVR, performed well and offered explanations within the context of the Kinematic Theory. Feature selection was not investigated in this study, but it may impact the performance of different models [71].

Future work includes studying the kinematics of additional tests and analyzing the original time series of movement kinematics. Symbolic algorithms may not be suitable for the larger dimensions of the original data, and larger neural networks may be

preferred. Exploring self-supervised learning [163] and pre-training techniques could optimize model performance with long sequential data. Additionally, investigating the combination of symbolic models (Sigma-Lognormal) and connectionist models (such as VAEs) could provide interesting insights and improve performance. This approach could enhance the understanding of human body motions involved in handwriting.

Outcomes.
Our study presents a novel approach using the Sigma-Lognormal model and neuro-muscular tests to predict motor control maturity in children. The complex triangular test, analyzed with the Sigma-Lognormal model, offers parameters for simple linear regression that accurately predict motor control maturity. Neural networks excel in this task, but symbolic models show promises. Future research should compare alternative tests and assess test-retest reliability. This approach has potential for detecting neurodevelopmental issues for children based on their motor control development.

6 Childhood

6.1 Interest of Kinematic Theory and its Lognormal Models in Assessing Graphomotor Skills in Kindergarten and First Grade Students in France and in Québec

Context. From a cognitive point of view, tracing letters with hand implies at least three steps of processing: retrieving from memory the allograph (shape) of each letter, programming the gesture allowing to trace each allograph and controlling the execution of the corresponding motor sequence [3, 103]. Successfully implementing and operating these processing steps requires acquiring and mobilizing a set of underlying skills such as visuo-motor coordination (allowing the pencil guidance according to the visual context) and graphomotor control (allowing programming and adjusting the motor realization of a graphic gesture). Moreover, strongly dependent on lessons in school, learning to write letters by hand is also highly constrained by the development of gross and fine motor maturation allowing a dual function of (i) gripping the pen and (ii) using the hand, forearm and arm working in synergy to move it and trace the letter [44, 69, 149].

The evaluation of handwriting and their underlying skills in young students is generally carried out through a set of measures mostly standardized like motoric tests (fine and gross evaluation scales, [131, 153]), visuo-motor tests [113, 142] and even handwriting variables allowing to assess the legibility of a produced letter as well as the kinematics of its production [2]. Nevertheless, regarding the graphomotor control (i.e. motor programming and execution of a gesture implied in tracing or writing by hand [35]), it is clear that few tests make it possible to evaluate this skill independently of letter production, as other underlying skills can be approached and described independently of the written tasks which mobilize them.

Accordingly, the objective of this article is to show how the Kinematic Theory, based on lognormal models [118–121 and above], can constitute a relevant objective developmental measure of graphomotor control of pen movements in French and Quebeckers children, according to (i) grade level of students, from kindergarten to grade 1 and/or (ii) a longer kindergarten prestation at school in France (3 years), compared to Quebec (1 year).

Method. *Participants*: Ninety-four students, including 47 French students and 47 Quebec students, of French mother tongue, participated in this study. French students were enrolled in five primary schools in the cities of Créteil, Orléans and Chateauroux. This French sample consisted of 27 students (including 15 girls) in kindergarten (average age = 5.32 years, ET = 0.23) and 20 students (including 11 girls) in the first year (average age = 6.48 years, ET = 0.18). Quebec students were enrolled in three primary schools in the cities of Chicoutimi and Sherbrooke. Developed according to the same criteria as the French sample, the Quebec sample consisted of 27 kindergarten students (including 15 girls) (average age = 5.35 years, ET = 0.23) and 20 first-year students (including 11 girls) (average age = 6.43 years, ET = 0.18). Each French student was matched with a Quebecker student of the same age (to the nearest month), the same sex, the same cognitive abilities (work memory and non-verbal intelligence, as assessed by background measures not detailed here).

Measures: A series of 4 main measures, leading to 12 variables, have been elaborated in order to assess handwriting abilities and their underlying motoric, visuo-motor and graphomotor skills. (i) Motoric skills were evaluated through two tests selected from the NP-MOT scale [153] to probe the different facets of fine and gross motor skills. Fine motor skills were assessed by a fingertip tapping task designed to evaluate finger dexterity (i.e. motor speed and rapid motor programming) for the left and right hand. This test was supplemented with another evaluation dedicated to gross motor skills and consisting in walking in a straight line, jumping from a height of 20 cm and standing on one foot with your eyes open. (ii) Visuo-motor skills were evaluated by two complementary tests, one measuring the ability to guide the pencil as quickly as possible between two lines of a course (Visuo-Motor Precision, subtest of the NEPSY: [78]), the other consisting in copying a series of figures more and more complex (Visuo-Motor Integration (VMI) test: [11]). (iii) Graphomotor control skills were assessed by asking the four groups of students to produce 30 pen strokes by hand, according to the protocol used in [116]. This allowed us to extract lognormal models as the main components of the Kinematic Theory. This theory, developed and tested by Plamondon [118, 119, 121, 122] and Plamondon et al., [117, 124, 126–128] is based on the assumption that all controlled movements, be they simple or complex, are made up of basic primitives (Lognormal function) that reflect the impulse responses of the neuromuscular systems involved in their production.

Figure 7 show the reconstruction of a specific stroke trace written with a pen by a kindergarten pupil, by using Script Studio software. The extraction shows here the existence of six lognormal functions, formalized by three general parameters: (i) nbLog: number of lognormal functions required to reconstruct the signal. This parameter represents the writer's fluidity of movement. The higher the nbLog, the less fluid the movement; (ii) SNR: signal-to-noise ratio between the original speed profile and the reconstructed speed profile, computed in decibels (dB). This is a measure of the quality of the sigma-lognormal reconstruction. The higher the SNR, the better the reconstruction; (iii) SNR/nbLog: performance criterion. The ability to reconstruct a movement's speed profile with lognormals can be interpreted as an indicator of motor control quality, as the lognormal speed profile corresponds to complete motor control [126]. The higher the SNR/nbLog, the closer the movement to ideal lognormal behavior. These three parameters, by evaluating the quality of the curve-fitting, reflect the general state
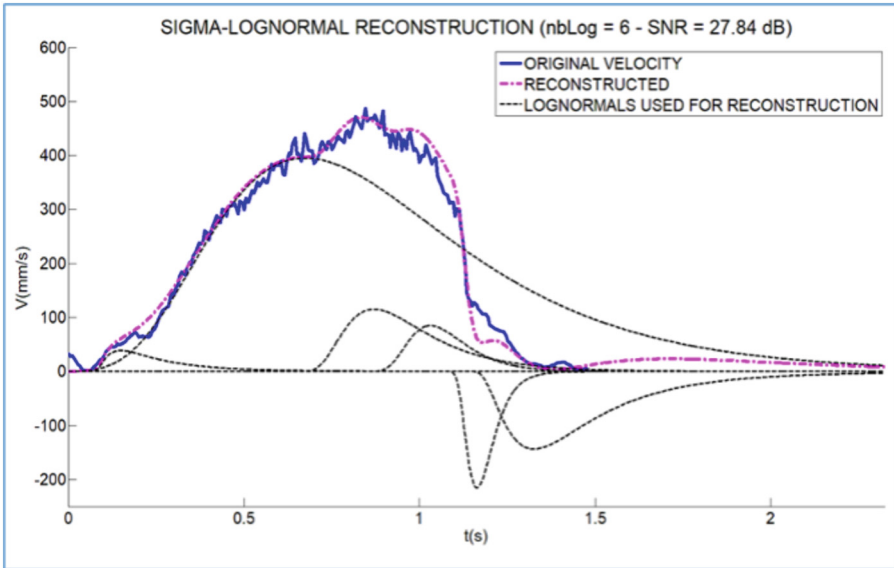
Fig. 7. Reconstruction of a specific kindergarten stroke trace

of the neuromotor system. As the Fig. 7 shows, the lognormal modeling means that the movement as produced by the pupil was based on a sequence of six successive commands, with: nbLog = 6; SNR dB) = 27.83; SNR/nbLog = 4.64. By opposite, normal adults in perfect control of their movements would have performed the stroke by using two lognormals. Indeed, if children use more lognormals than adults to execute a given stoke, this number decreases as they gradually master handwriting [45]. Accordingly, in this study, we focus on the 3 general parameters; nbLog, SNR and SNR/nbLog by seeking to understand to what extent these parameters vary significantly with the Grade and Country of students, under the effect of the development of maturation and different school learning. (iv) Finally, handwriting skills were elicited by the production of familiar letter allographs. Students were asked to write their firstname several times within 30 s, using their usual handwriting. This task is frequently used to assess handwriting, as it features the best known and doubtless most automatized letter sequence, allowing researchers to focus more purely and specifically on motor aspects [2, 5, 6, 130]. The accuracy and fluency of letters production was assessed by the 4 following variables: % of legible (recognizable) letters, letter accuracy fluency (legible letters per min), pen movement speed, number of pen pauses per letters and mean pause duration.

Apparatus: The graphomotor task (tracing 30 strokes) and handwriting task (firstname written recall) were both performed on a pen-display tablet (Wacom Cintiq Pro 13) connected to a laptop piloted by Eye and Pen© software [4]. The children wrote directly on the surface of the tablet using a stylus (Wacom Intuos 3 Grip Pen). This tablet records data at a sampling rate of 200 Hz, with a spatial resolution of 200 lines per millimeter, and the software records the timing, position, and status of the pen tip on the tablet screen in real time.

Results. Performances of the four groups of students were analyzed by running a two-ways MANOVA in order to determine e the main effects of the Grade (Kindergarten versus Grade 1) and Country (France versus Quebec) factors, on the 12 variables evaluating fine and gross motricity, visuo-motor integration, graphomotor control and handwriting skills. The multivariate effect of Grade was statistically significant. More precisely, the performance of the group of Grade 1 students, except for the number of pauses per letter, were significantly higher than those of kindergarten for all the other measurements, including the 3 lognormal general parameters: nbLog (Kindergarten: 6.36, Grade 1: 5.10; $p < .01$); SNR (Kindergarten: 26.68, Grade 1: 27.03; $p < .01$), SNR /nbLog (Kindergarten: 5.33, Grade 1: 6.35; $p < .002$). The multivariate effect of Country was also statistically significant. However, the effects for each of the variables are more contrasted. Thus, the group of French students (all grades combined) obtains a significantly higher score than that of Quebecker students for the task of gross motor maturation, the test of Visuo-Motor Integration as well as the % of legible letters in the first name written recall task. On the other hand, in the case of the pen movement speed and the fluency per legible letters score, it is the Quebecker students who attest higher performances than those of the French students. No other significant difference appears between the students of the two countries for the other measures, including those carried out by the analysis of general lognormal parameters.

Discussion. If we focus here on the analysis of the general lognormal parameters, results revealed significant differences on nbLog, SNR and SNR/nbLog between children in kindergarten and first graders. The mean value of nbLog was statistically lower for children in first grade than for children in kindergarten, and consistent with this, the mean values of SNR and SNR/nbLog were higher. Indeed, when first graders want to draw a line, they have more fluidity than kindergarten pupils, as reflected in a lower nbLog. Moreover, the quality of stroke reconstruction is better in Grade 1 than in kindergarten, as SNR and SNR/nbLog were significantly higher for the first graders, owing to improved neuromotor control and lognormality with age. It can be then argued that children's motor control improves as they grow older, as predicted by the Principle of Lognormality and the Kinematic Theory.

Moreover, the results of this study bring out two important facts. First, the extraction of Lognormal parameters from a relatively simple task (e.g., drawing 30 strokes) makes it possible to highlight coherent developmental differences between kindergarten and first grade, and this, in consistency with the effects observed for the other skills, actually motor and visuo-motor, involved in the development of handwriting. In this sense, the "stokes tracing task", independent of the tracing of letters, combined with the extraction of lognormal models, could be an interesting avenue to explore, in order to constitute a standardized and predictive test, in the long term.

Second, interestingly, lognormal parameters are here sensitive to grade level but not to country of schooling, unlike other abilities like gross motor maturation or visuo-motor integration. This result suggests that Lognormal modeling probably makes it possible to approach rather the neuromotor component of graphomotor control, which should be, as well as finger tapping performance, more strongly dependent on proximo-distal maturation than on school training.

Outcomes. Finally, if Kinematic Theory and its Lognormal models seems to represent an interest for assessing graphomotor control in young pupils, it remains to assess to what extent the fluidity of the gesture, when drawing a series of stokes, could be related to the dynamics of drawing letters and words, and more particularly to the frequency and duration of pen stops (pauses), supposed to indicate difficulties in controlling the execution of the strokes making up a letter [37, 112]. The presence of such a relation between strokes and letters could be investigated by applying the lognormal modeling to the production of letters of the alphabet and of the firstname, in addition to the production of a series of strokes.

6.2 The use of the Lognormality Principle for the Characterization and Analysis of Graphomotor Behaviours Involving Young Learners in a School Context

Context. Children learnings suppose successful mobilizations of specifical graphmotor gestures (GG) as pointing, drag and drop and handwriting since their beginning at kindergarten around 2–3 years old. Earliest mastery of each of these GG in various contexts is fundamental because they are involved by most of the scholar tasks that must be executed into tangible or digital ecosystems. The lognormality of adult's expert graphomotor behaviors and a tendency to a gradual migration to this optimal lognormal behavior through development and training have been established and validated thanks to the sigma-lognormal modeling of GG of kindergarten apprentice scripters and adults [136]. However, the GGs considered had been acquired in a strict clinical framework by considering psychomotor tasks quite different from real school tasks. This raises the question of the possibility of extending these conclusions to the cases of GG specific to school constraints, carried out and acquired in the less strict context of tasks of a school nature. To answer this question, we have conducted experiments in a school context for nearly a decade with the aim of answering the following questions: Is the reconstructive power of sigma-lognormal modeling, that was observed in strict clinical cases of rapid plotting of simple trajectories, robust enough to withstand the school environment noise and its constraints? Does considering the sigma-lognormal modeling of realistic and more complex traces than those considered in the clinical case makes it possible to distinguish levels of expertise in terms of levels of motor control acquired thanks to school training?

Methodology. To answer these questions, we exploited types of graphomotor gestures carried out in school activities collected during several experiments conducted on school time in fifteen schools from primary to secondary between 1997 and 2019.

These GGs were carried out by more than a thousand all-comers, aged 3 to 14 years and enrolled from the first year of kindergarten to the third class of middle school. Some of them were made in a tangible environment in paper-and-pencil on paper mode. They were acquired online as described in [46] thanks to several models of Calcomp and Wacom digitizers, driven by the Dekat'tras application, placed as a plotting support. Others were made and acquired online directly in a digital learning environment based on the platform Copilotr@ce [140]. Some of these school GGs were produced using an ink pen or non-ink writing tool, while others were produced by finger.

Various activities including spontaneous or constrained scribbling [136–138] for 20 s, tracing on predefined trajectories, copying isolated patterns from alphabets [45] or cursive words, writing common isolated words such as first name, days of the week under various conditions [139], were offered to students.

For each of the types of graphomotor gestures acquired, the batch processing procedure of the ScriptStudio tool exploiting the Robust X-Zero approach [105] was used to perform sigma-lognormal modeling of curvilinear velocity profiles and approximation in 2D space of the trajectory executed by the pupils. Two global kinematic parameters were then extracted. The first of these parameters, called nbLog, is an integer value. It specifies the number of lognormals needed to reconstruct the speed and trajectory with a signal-to-noise ratio defining the value of the second parameter extract-ed. This one is called SNR. From these two parameters a third: the SNR/nbLog ratio, was estimated for each type of GG considered. Then, acceptable rates of good reconstruction, i.e., with an SNR greater than or equal to 15dB, were established. Next, the distribution of SNR was determined for each grade level represented in the cohort of students who participated in the collection of the type of GG considered. Finally, the behavior of each of these three parameters according to grade level and, for some, according to the constraints imposed by the task to be carried out, was tested by means of statistical tests.

Main Results. A great majority of GGs acquired under real conditions at school, whether on graphic or touch tablets and all models of equipment combined, has been rebuilt with an SNR greater than or equal to the minimum threshold of 15 dB. This, in spite of their complexity, duration, continuous or not and the grade level of the pupils whose produced them.

This first observation makes it possible to validate the robustness of Sigma-Lognormal modeling (SLM) for use for the analysis of real childish graphomotor behaviors in the school context. This robustness is verified although the school context is more prone to disruptions in the operating conditions of the SLM than clinical environments are. The use of SLM is also possible from the first years of schooling and throughout primary and secondary schooling. This, by directly considering GG produced along usual pedagogical activities.

The second observation relates to the distribution of SNR values according to the degree of experience in the implementation of school GG translated by the pupils' grade levels. Regardless of the type of GG considered, it turns out that the higher the educational level, the higher the rates of high SNR values and the lower the rates of low values of SNR. Conversely, in the case of a low educational level the rates of low SNR values are higher.

Discussion. These results therefore argue in favor of the validation of the possibility of observing the principle of migration to lognormality according to effects of school trainings from kindergarten up to at least end of middle school and this, for most GGs taught and mobilized by the school.

Based on such results, it becomes possible to set up individual monitoring of the progression of the pupils' level of motor control during their school cursus thanks to the observation of the evolution of the SNR for each type of school GG.

By virtue of the Principle of Lognormality it is possible to postulate that at equivalent SNR level for an analogous type of graphomotor gesture, the higher is the number

of lognormals the more this ratio tends towards 0 which reflects a lower quality of motor control of the graphomotor gesture mobilized during the proposed task. Conversely, the lower is the number of lognormals, the higher this ratio will be, which will reflect a better motor control capability of the GG during the task.

Outcomes. The non-conservation of high SNR as so as inconsistent SNR/nbLog ratios between various constrained situations of scribbling and writing words while SNR remain high tend to show that the global parameters SNR and SNR/nbLog can play role of gauges of shortcomings in automating the planning and execution procedure of the types of GG concerned.

Therefore, a non-invasive and transparent monitoring of motor control growing seems feasible by comparing the values of these three global parameters directly through various real pedagogical situations at school. Such monitoring should also help teachers to decide on objective and quantifiable bases whether to continue, maintain or strengthen the use of some pedagogical approaches to learn school GGs.

However, to achieve such tools, solutions to quickly compute those parameters are needed.

6.3 Lognormality in Children with Mild Traumatic Brain Injury: a Pilot Study

Context. Pediatric traumatic brain injury (TBI) is a public health burden and the leading cause of disability worldwide [159]. Each year, millions of children sustain TBI, with mild traumatic brain injuries (mTBI) and concussions accounting for more than 90% of all TBI cases. Previous studies have shown that 15–30% of children with mTBI continue to experience PCS for several months following injury, which in turn can result in functional deficits and declines in quality of life [10, 102, 162]. However, there is currently a lack of accurate objective and developmentally appropriate tools to sensitively assess fine motor skills after mTBI. This pilot study investigates whether the Sigma-Lognormal model proposed by the Kinematic Theory can be used to detect a difference between simple handwriting gestures performed by children at different times after experiencing mild traumatic brain injury (MTBI) [48, 50].

Method. Participants included children and adolescent who presented to the two tertiary care pediatric hospital (i.e., Montreal Children's Hospital and CHU Sainte-Justine). 90 children and adolescent were initially recruited to the sub-study, but complete data was only available in 32 participants, aged 6 to 18 years old, with mild brain injury.

Each participant had to draw fast single strokes, one at a time, following a visual reaction time protocol. After the test, every participant should have produced 30 valid strokes. The trials were recorded at 100 Hz using a tablet digitizer (Wacom Intuos2). Every stroke had to begin from a starting point located at the middle a guide sheet, and to end at one of the sides of the sheet, as depicted in the Fig. 8.
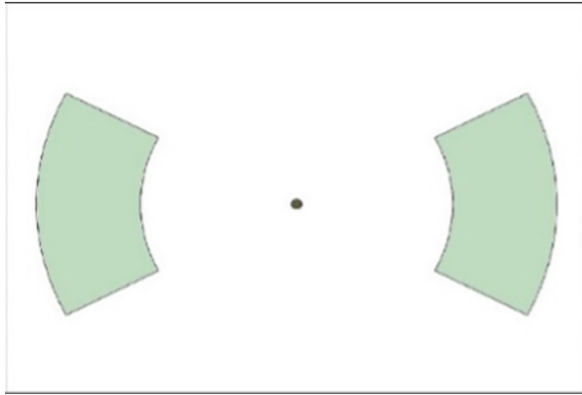
Fig. 8. The guide sheet

The direction side was set depending of the laterality of the participant. They were asked to produce handwriting strokes on a digital tablet at 1 month and 3 months after sustaining the injury. The Sigma-Lognormal model was used to analyze the executed movements.

The classification was done using the evolution of each parameter over time for each subject. To determine if the change in parameters is evolving positively, there has to be a significant statistical difference between the 1- and 3- months mean post-injury. Participants were considered as having an improved neuromotor system state when there was a decrease in the mean value of the following parameters: all rescaled t_0, first t_0, σ, number of rejected strokes, D and nbLog. Similarly, participants had a better neuromotor system state after 3 months post-injury if there was an increase of the mean value of the following parameters: μ, SNR and SNR/nbLog.

To measure the somatic, cognitive, emotional, and fatigue/sleep-related symptoms present in each participant, parents were asked to complete the Post-Concussion Symptom Inventory (PCSI) [167] to document their child's symptoms. The parent-report version of the PCSI consists of 26 items, where responses are rated for severity on a 7-point Likert scale (i.e., from 0 to 6; 0 = Not a problem, 6 = Severe problem). We examined how the number of reported symptoms, as endorsed on the parent PCSI, progressed over time. Specifically, the total score obtained on the parent PCSI (i.e., total number of symptoms) for each participant was compared at 1-month and 3-months post-injury. To this end, the children were placed in 4 different categories according to the evolution of their condition: Improvement, Deterioration, Stabilization and No judgement.

Results. This model showed significant differences between the set of traits produced by participants when comparing their results at 1 and 3 months post-injury. Of the 32 participants, 28 of them have significant differences for at least 1 lognormal parameter. We notice an improvement in the quality of the traits achieved over time. For example, there were 17 participants who had a significant difference, with the Bonferroni correction, for the SNR/nbLog. Only four of the participants showed no significant change during this period.

By examining PCSI quality-of-life questionnaires including the child's responses as well as those of the parents 1 month, 3 months after sustaining a concussion, the child's state of health were analysed to see how quality of life had changed during this period. There was a match between the parents' and the children's responses for 19 of the 32 children. In the case of 4 children, no judgement can be made for lack of data. In the case of 3 children, there was a contradiction between the child's answers and those of the parents, one stipulating an improvement and the other a deterioration. In the case of 6 children, there was a contradiction between the child's answers and those of the parents, one stipulating an improvement or deterioration, while the other stipulates a stable state.

Comparing the results of the PCSI questionnaire with those of the pencil line test, the results match for 9 out of 32 children. For the rest of children, however, the results did not match. For the 9 children where there was a match, 6 were improving, 2 were deteriorating and 1 was stable. In the other cases, there was a contradiction be-tween the child's and parent's answers, where one of the two answered that the child's condition had remained stable over this period of time. For the children for whom there was no correspondence between the results of the pencil line test and the PCSI questionnaire, for 7 children, the pencil line indicated an improvement in the child's condition, whereas the PCSI questionnaire indicated the opposite. For 9 children, the pencil test pointed out a stabilisation whereas the PCSI questionnaire indicated stability, the inverse for 2 and one case was non conclusive.

Outcomes. By comparing the results of the PCSI answers and the Sigma-Lognormal analysis, a concordance was observed between the two tests for only nine (32%) of the participants. This is not surprising as the PCSI questionnaire and Sigma-Lognormal parameters assess different aspects of functioning, and thus, are likely to yield different patterns of results. First, post-concussive symptoms were documented using subjective parents' reports on questionnaire, and it has been previously suggested that parents of children with mTBI may tend to over-report symptoms of their children on questionnaires [18]. Second, the Sigma-Lognormal analysis is an objective methodology based on the Lognormality Principle. It could be used to ponder subjective reports and provide unbiased data to confirm or infirm the evolution of the mTBI. In this perspective, these preliminary results will serve as a basis for further research into the benefits of using the Sigma-Lognormal model for the assessment of the integrity of neuromotor systems after traumatic brain injury in children.

6.4 Kinematic Analyses of Rapid Pencil Strokes Produced by Children with ADHD

Context. Most children with ADHD (Attention Deficit Disorder with or without Hyper-activity) have problems with gross and/or fine motor skills [75]. Children with ADHD often have greater difficulty planning and programming their movements effectively than their non-ADHD peers [47, 142, 161]. A proper assessment of the motor and grapho-motor skills of children, whether they have ADHD or not, seems relevant to guide intervention in the face of these problems and to better understand the nature of motor difficulties in ADHD. The kinematic analysis of writing movements can be used to study the factors involved in motor control and fine motor skills [41, 42, 106, 120, 125,

157]. When a person writes on a digitizer, the coordinates of their pencil movement are transformed into a velocity profile from which an analysis can reconstruct the movement with its corresponding sequence of lognormals. This theory uses the stroke to understand how motor control processes movement execution. According to the Kinematic Theory, when someone controls its movement, their velocity profiles will tend to approximate lognormality [126].

In the present study, the Sigma-Lognormal model was used to obtain a detailed description of children's pencil stroke velocity profiles. The first objective was to assess whether the parameters obtained from Sigma-Lognormal modeling of the fast pen stroke velocity profiles could effectively differentiate between children with and without ADHD. The hypothesis states that the quality of handwriting movement would be inferior in children with ADHD, particularly those with more severe symptoms. The second aim was to investigate the correlation between the lognormal motor behavior of children with ADHD and their performance on other assessments of graphomotor and fine motor skills. The anticipated hypothesis was that lognormal parameters would exhibit a relationship with measures of handwriting speed and accuracy, as well as fine motor skills. Lastly, the potential of lognormal parameters to enhance the accuracy and specificity of ADHD diagnosis was examined, with the expectation that these parameters would successfully distinguish between children with and without ADHD.

Methodology.   24 children aged 8 to 11 years took part in this study: 12 with ADHD and 12 without. The children took several psychometric tests: the Wechsler Intelligence Scale for Children – 4th Edition (WISC-IV¼ [156], the Pen Stroke Test (PST) on digitizer, the BHK (Échelle d'évaluation rapide de l'écriture chez l'enfant) [28], the Purdue Pegboard [150], the Finger Tapping Test (FTT) [143] and the TWISC-IV- Coding subtest [156]. The questionnaires completed by the parents assessed the presence of inattention and hyperactivity/impulsivity behaviours and of developmental coordi-nation disorders.

An optimal algorithm was used to extract the Sigma-Lognormal model parameters from the PST. For each child, the mean value of the following parameters for the 30 strokes was used in the analyses [79]. The signal-to-noise ratio (SNR) between the original velocity profile and the reconstructed velocity profile measures the quality of the Sigma-Lognormal reconstruction. The number of lognormal functions required to reconstruct the original velocity profile (nbLog) represents the fluidity of movement of the participant. Other parameters can be obtained from analyzing the participants' graphomotor behavior. Two parameters represent the neuromotor action plan. Time required (t_0; in seconds) for the brain to produce a motor command. The amplitude of the movement (D) associated with each motor command, in millimeters, is the distance planned to be covered by the pen for each lognormal.

Results.   Independent measures t-tests carried out on the PST parameters revealed a significant inter-group difference on SNR/nbLog. This parameter indicated significantly poorer quality of motor control in the ADHD group. Mean nbLog was significantly higher for the ADHD group (t = 3.475; p = 0.002), which indicated that more lognormals were required to reconstruct the pen stroke signal and that the children's movements were less fluid. A significant inter-group difference was found also in terms of t_0. This parameter was greater for the ADHD group, indicating a longer delay for command preparation (t = -3.607; p = 0.002). In addition, the D parameter was significantly smaller the ADHD

group (t = 2.306; p = 0.031), which reflects a smaller movement amplitude in their action plans. No significant difference was observed on the other parameters. Variability in values obtained for the PST parameters was calculated for each child and the groups were then compared. Mean SNR/nbLog variability was greater for the ADHD group (t = -2.975; p = .007) This suggests that the ADHD group had greater variability in motor control across pen strokes. Intra-individual variability was not significantly different between groups on any other parameter. The area under the ROC curve (AUC) was calculated to assess the capacity of PST parameters to discriminate between the ADHD and control groups [68]. An AUC of 1 indicates a perfect diagnostic test, whereas an AUC of .5 indicates a test performing at chance level, that is, unable to discriminate between two groups (Hajian- Tilaki, 2013). The AUC was 0.87 for SNR/nbLog; 0.84 for t_0; 0.91 for nbLog; and 0.79 for D. In other words, these four parameters discriminated between the two groups. The AUC for nbLog was significantly greater than the AUC for D (p = 0.032), indicating that nbLog can better discriminate between the two groups than D. A correlation was found between t_0 and writing speed as measured by the BHK in the ADHD group (r = -0.67; p = 0.018) indicating that the faster a child wrote, the shorter the motor command production delay (t_0). In the ADHD group, nbLog was negatively associated with performance on the WISC-IV Coding subtest (r = -0.64; p = 0.024) and the FTT (r = -0.64; p = 0.026). As such, the number of lognormals needed to reconstruct the strokes was associated with lower scores on these two tests. In the ADHD group, a correlation was observed between D and the FTT (r = 0.81; p = 0.002) indicating that greater amplitude of movement on the PST was associated with faster motor speed. In the control group, a significant correlation was found between SNR and the total score on the BHK (r = -0.80; p = 0.002) indicating that higher SNR is associated with more controlled handwriting. Finally, a significant correlation was found for the control group between SNR and the scores on the Purdue Pegboard task (r = 0.59; p = 0.043) indicating that higher SNR is associated with better manual dexterity.

Outcomes. This study explored the usefulness of the PST in evaluating fine motor skill impairment in children with ADHD. The Kinematic Theory of rapid human movements and the Sigma-Lognormal analysis allowed the use of objective parameters obtained from reconstructing fast pen stroke movements as indicators of child motor control capacity [79, 116]. A significant difference emerged between children with and without ADHD on four PST parameters: SNR/ nbLog, nbLog, t_0 and D. Moreover, children with ADHD demonstrated greater intra-individual variability in quality of motor control (SNR/nbLog). This suggests that children with ADHD are less able than peers without ADHD to control a single stroke. The results indicate that children with ADHD may have a graphomotor skill impairment at the level of motor planning, as reflected by longer t_0 and smaller D, as well as, at the execution level. The PST, based on the Sigma-Lognormal analysis, shows promise as it may offer a fast and effective way of detecting motor skills problems in children with ADHD and may contribute to refining ADHD diagnosis. Together, the findings suggest that it may be important to include assessment of motor and graphomotor skills in the clinical evaluation of children with ADHD.

6.5 Screening for Developmental Problems in Preterm Born Children: Utility of the Pen Stroke Test During the Preschool Period

Context. Yearly in Canada, about 8% of all live births occur prematurely, i.e. before 37 weeks of gestational age (GA). Among these preterm births, approximately 90% occur between 29 and 36 weeks' GA. Children born at 29–36 weeks' GA display physiological immaturity and instability making their developing brain vulnerable to various insults related to preterm birth complications and treatments [21, 134], thus increasing the risk for developmental problems, including attention deficit and hyperactivity disorders, developmental coordination disorders or difficulties with writing skills [7, 33, 76, 145, 154]. In kindergarten, 34–40% of children born between 29–36 weeks' GA have ≥1 area of vulnerability for school readiness (i.e., the developmental abilities and behavior necessary to meet school demand) due to NDD, a red flag for future learning challenges [94]. Rehabilitation services, if timely implemented, can optimize academic achievement by addressing educational needs prior to school [31, 74]. In this perspective, early identification of children born between 29–36 weeks' GA at highest risk of learning challenges is crucial.

We previously recruited 241 children born between 29–36 weeks' GA to test a developmental screening protocol combining biological and clinical markers assessed from birth to 4 months CA to identify those at higher risk of NDD at 2 years CA. Now that this cohort is growing beyond the toddler years, longitudinal follow-up is necessary as 2-year outcomes may not be sufficient to predict long-term neurodevelopment [45]. Moreover, the dynamic process of brain development may uncover emerging signs of dysfunctions that could be identified. To this end, a non-invasive, rapid, unexpensive and easily available screening instrument is necessary. The Pen Stroke Test [45, 79] (PenStroke), developed by R. Plamondon, responds to these criteria, but needs to be validated first.

The PenStroke consists in producing handwriting strokes on a computerized interface which are then analyzed using the sigma-lognormal model [105, 117]. This model provides 2 parameters describing the general state of the neuromotor system and the quality of the modeling: the number of lognormals (nbLog) and the measure of the quality of the sigma-lognormal reconstruction, or the Signal-to-Noise Ratio (SNR). A stroke that approaches the 'perfect' model is made up of 2 lognormals; the higher the nbLog, the lesser is the motor control. In contrary, the higher is the SNR, the better is the fitting and the motor control. Evidence supports the utility of the PenStroke parameters to discriminate levels of graphomotor performance achieved by children aged 3 to 5 years [19]. Performing the PenStroke as screening measures prior to school entry could improve the clinical discrimination of preterm children born at 29–36 weeks' GA at risk of NDD. However, the screening accuracy of this tool in preterm preschool children needs to be determined.

The overarching aim of our research program is to improve early identification of NDD in preterm children born at 29–36 weeks' GA. The current study specifically aims to examine the concurrent accuracy of the PenStroke in identifying NDD at age 4.5 years.

We hypothesize that the PenStroke parameters will correlate with neurodevelopmental skills at 4.5 years of age and will enhance our developmental screening protocol in predicting neurodevelopment prior to school entry.

Methods. We are currently conducting a prospective longitudinal follow-up study of an established cohort. All participants from the initial cohort (n = 241) were recruited at the Centre Hospitalier Universitaire Sainte-Justine (CHUSJ). Children aged 4.5 yrs old (±3 mo) still enrolled in the initial study (217/241 children), in which inclusion criteria were birth between 29–36 6/7 wks' GA and admission for ≥48 h in the NICU, are eligible, but those under child protection services (for consent issues) are excluded. Recruitment will run between March 2023 and May 2027 as children of our cohort reach 4.5 years old. Data are collected at a 4.5-year-old visit at CHUSJ research center. The PenStroke is first administered and then followed by a neurodevelopmental assessment. Both are conducted by trained research assistants blinded to participant's history. For the PenStroke, three simple movements on a digitizer (Wacom Cintiq 13HD, digitized at 200 Hz) are completed. The first 2 movements consist of making 30 rapid pen strokes, each time, between a starting zone, identified by a black point, and an arrival zone displayed in gray. The first time, a sound cue (at 1 kHz for 500 ms) emitted by the computer is the go signal for the child to execute the movement and the second time, a green visual stimulus is used as the cue. The third movement consists in drawing a triangle 30 times by connecting three points displayed on the screen. These tests generate optimal parameters to express the quality of the neuromotor control of the upper limb. A global optimal algorithm is used to extract the sigma-lognormal model parameters (nbLog and SNR). The whole process is synchronized with Sign@medic, an in-house program. The neurodevelopmental assessment includes 9 standardized tests covering intellectual functioning, attention, language, motor skills, behavior, and adaptive functioning. For this study, NDD will be defined as 2 or more test scores (out of 9) that fall 1 standard deviation below the mean. Concurrent accuracy of the PenStroke will be determined by Receiver Operating Characteristic curves.

Preliminary Results. To date, from the 23 families contacted, 19 accepted to participate (83%) and 12 visits have been completed at a mean age of 4.4 years old (+/− 0.2). Data collection was complete for all assessed participants. For all 12 participants, we were able to reconstruct the recorded movements produced by the children using the Sigma-Lognormal model and to extract the parameters. Figures 9A and 9B show 2 examples of the reconstruction with the auditory stimulus: 9A from a child with a better motor control than the one pictured in 9B. Overall, the ratios between the SNR and the nbLog seem to vary between participants as shown in Fig. 10, supporting the use of the Sigma-Lognormal model to characterize preschool children motricity. Next steps will involve further data collection and the analysis of the associations between the Penstroke parameters and the neurodevelopmental profile of the participants.
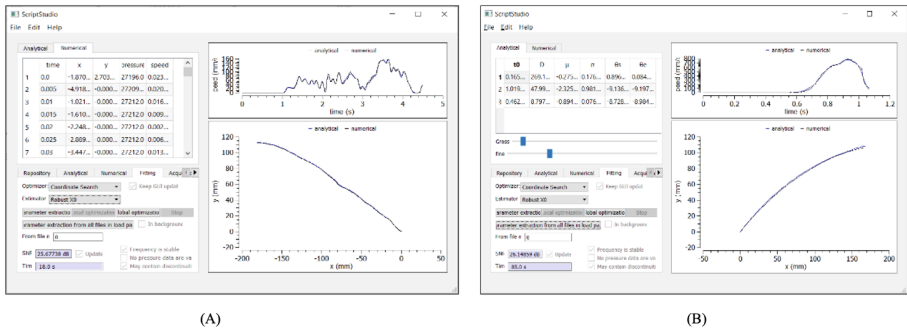
(A)   (B)

Fig. 9. (A) Reconstruction with auditory stimulus, (B) Reconstruction with auditory stimulus
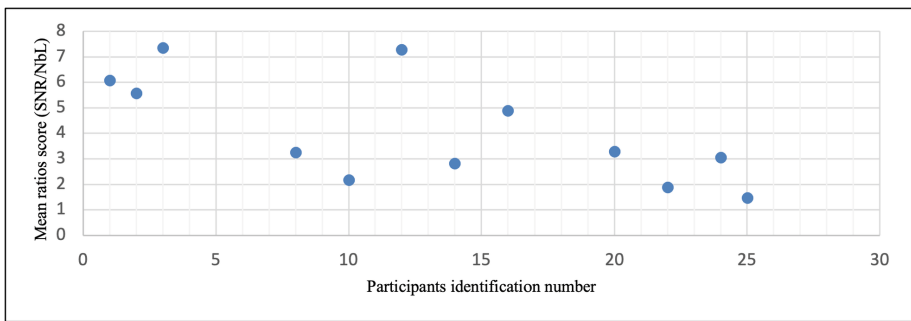

Fig. 10. Mean performance of the participants with the auditory stimulus

Expected Outcomes. Integrating the PenStroke at 4.5 years old to our developmental screening protocol could enhance the clinical discrimination of preterm children born at 29–36 weeks' GA at risk of NDD before school entry and optimize support prior to school entry.

6.6 Exploring the Benefits of Virtual Reality Lognormality Analysis for Diagnosing ADHD in Children

Context. At least 15% of the children with learning problems experience anxiety, dependency and depression, leading to loss of motivation and, in the worst cases, dropping out of school. Treatment of these cases is often costly and there is a shortage of professionals to provide follow-up care. It is estimated that it costs a minimum of $60,000 per child in Canada to carry out these diagnoses and their follow-ups.

A New Tool. To tackle these problems, the AeoVR team (https:///aleovr.com) is developing an educational tool in the form of virtual reality (VR) experiences that aim to support the development of school-aged children with learning disabilities. The spin-off company proposes a set of adapted challenges, a series of virtual reality games based on exercises used by Ortho pedagogy clinics. For example, to increase participation and

motivation, interactive and immersive exercises to stimulate fun and motivate learning are used. The system offers a personalized and adapted environment to foster academic development in an appropriate virtual world based on monitored stimuli. Detailed reports are provided to track progress and facilitate communication between the various parties involved. Figure 11 shows the basic VR data capture.

Fig. 11. VR data capture system

A Proof of Concept for VR Lognormality Study. Among the various improvements that AleoVr is exploring, stands the integration of a 3D lognormality analysis package to evaluate, characterize and monitor the child performances. A preliminary study has been run that check this hypothesis. The right-hand $x(t)$, $y(t)$, $z(t)$ coordinates of 13 participants aged between 19 and 63 were recorded. Each of the volunteers had to touch: 1. A cup on a table at about 1m from the floor, 2. The corner of table located approximately at 1m from floor, 3. A chair at approximately 30 cm from floor, 4. A tree house visible in the distance which was requiring an extension of the arm of approximately 30 cm above the eye level, 5. Their left shoulder whose distance was depending on the size of the participant. Each movements had to be repeated twice.

The ten gestures per participant were reproduced using the 3D Sigma-Lognormal extractor [59, 145] and the quality of the reconstruction was computed as a feasibility measure. The SNR were always above 15dB, (the accepted threshold for considering a movement as made up of lognormals) with a mean value of 23.4 ± 3.2 dB.

Expected Outcomes. These very preliminary results confirm that the 3D Sigma-Lognormal model can be used to extract neuromotor parameter from complex 3D movements collected with a VR system and that it could be exploited for developing objective numerical metrics to study these gestures.

7   Conclusion

Looking back and ahead at the numerous applications that involve lognormality, it becomes more and more implicit that this emergent property stands among the universal behaviour that has emerged through the evolution of species, the central limit

theorem slowly but surely acting as a growth force. Lognormals have been found and used as hidden primitives in numerous applications dealing with healthy and non-healthy subjects, ranging handwriting analysis and recognition [20, 51], signature verification [37, 38, 58], signal processing [43, 67], human-machine interfaces [87, 90, 96, 152] and biomarker definition [84] as well as for speech processing [25, 27]) and for Turing tests [88, 89]. We have summarized in this special session a subset of these applications, focusing on those presented in French at ACFAS 2023. Many other studies are going on in e-Security, e-Learning and e-Health [126, 129], and new fields are also expected to develop providing for examples a new set of functions for 2D and 3D smoothest curve modelling, anthropomorphic arm design, exoskeletons and prosthetics control, human-like movements modelling of virtual reality objects. Moreover, applications for fish [55, 141], farming [40] and robots [38, 39, 97, 132] are under investigations. By extending the range and significance of the Lognormality Principle these applications may, in the long run, cement lognormality as a fundamental law of nature.

Section 6.1: Denis Alamargot (Université Poitiers, France), Marie-France Morin (UdeS - Université de Sherbrooke), Nadir Faci (PM), Réjean Plamondon (PM).

Section 6.2: Céline Rémi (Université des Antilles, Guadeloupe, UA), Jimmy Na-geau (UA), Jean Vaillant (UA), Réjean Plamondon (PM), Emma-nuel Biabiany (UA).

Section 6.3 : Nadir Faci (PM), Naddley Désiré (SickKids Hospital Toronto), Miriam Beauchamp (UdeM - Université de Montréal), Isabelle Gagnon (Université McGill), Réjean Plamondon (PM).

Section 6.4: Raphaëlle Fortin (UdeM - Université de Montréal), Patricia Laniel (UdeM), Nadir Faci (PM), Miriam Beauchamp (UdeM), Réjean Plamondon (PM) Bruno Gauthier (UdeM).

Section 6.5: Marie-Noëlle Simard (UdeM - Université de Montréal), Thuy Mai Luu (CHU Ste-Justine, UdeM), Mathieu Dehaes (UdeM), Anne Gallagher (Université de Montréal), Anik Cloutier (CHU Sainte-Justine), Réjean Plamondon (PM).

Section 6.6: Caroline Bazinet (AleoVr), Alexis Maher (AleoVr), Charles Tétreault (AleoVr), Catherine Bazinet (AleoVr) , Andreas Fischer (University of Applied Sciences and Arts Western Switzerland, HES-SO) , Réjean Plamondon (PM).

References

1. Adler, C.H., et al.: Low clinical diagnostic accuracy of early vs advanced Parkinson disease: clinicopathologic study. Neurology **83**, 406–412 (2014)
2. Alamargot, D., Morin, M.-F.: Does handwriting on a tablet screen affect students' grapho-motor execution? A comparison between grades two and Nine. Hum. Mov. Sci. **44**, 32–41 (2015). https://doi.org/10.1016/j.humov.2015.08.011
3. Alamargot, D., Morin, M.-F.: Approche cognitive de la production écrite: Principaux résultats et apports pour l'apprentissage et l'enseignement à l'école. Approche Neuropsychologique des Apprentissages chez l'Enfant (ANAE) **163**, 713721 (2019)
4. Alamargot, D., Chesnet, D., Dansac, C., Ros, C.: Eye and pen: a new device for studying reading during writing. Behav. Res. Methods **38**(2), 287–299 (2006). https://doi.org/10.3758/BF03192780
5. Alamargot, D., Morin, M.-F., Simard-Dupuis, É.: Handwriting in signing deaf middle-school students and relationship with text composition and spelling. Read. Writ. **31**, 10171038 (2018). https://doi.org/10.1007/s11145-018-9824-y
6. Alamargot, D., Morin, M.-F., Simard-Dupuis, É.: Handwriting delay in dyslexia: children at the end of primary school still make numerous short pauses when producing letters. J. Lean. Disabil. **53**(3), 163–175 (2020). https://doi.org/10.1177/0022219420903705
7. Arpi, E., Ferrari, F.: Preterm birth and behaviour problems in infants and preschool-age children: a review of the recent literature. Dev. Med. Child Neurol. **55**, 788–796 (2013). https://doi.org/10.1111/dmcn.12142
8. Asif, H., et al.: Objective structured assessment of technical skill (OSATS) in the surgical skills and technology elective program (SSTEP): comparison of peer and expert raters. Am. J. Surg. **223**(2), 276–279 (2022). https://doi.org/10.1016/j.amjsurg.2021.03.064
9. Awal, M., et al.: Design and optimization of ECG modeling for generating different cardiac dysrhythmias. Sensors **21**(5), 1638 (2021). https://doi.org/10.3390/s21051638

10. Barlow, K.M.: Postconcussion syndrome: a review. J. Child Neurol. **31**, 57–67 (2016)
11. Beery, K.E., Buktenica, N.A., Beery, N.A.: The Beery-Buktenica Developmental Test of Visual-Motor Integration: Administration, Scoring, and Teaching Manual, 6th edn. NCS Pearson, Minneapolis (2010)
12. Bensalah, A., Chen, J., Fornés, A., Carmona-Duarte, C., Lladós, J., Ferrer, M.Á.: Towards stroke patients' upper-limb automatic motor assessment using smartwatches. In: Del Bimbo, A., et al. Pattern Recognition. ICPR International Workshops and Challenges. ICPR 2021, LNCS, vol. 12661, pp. 476–489. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-68763-2_36
13. Berio, D., Fol Leymarie, F., Asente, P., Echevarria, J.: StrokeStyles: stroke-based segmentation and stylization of fonts. ACM Trans. Graph. **41**(3), 1–21 (2022). Article #28, https://doi.org/10.1145/3505246
14. Berio, D., Akten, M., Fol Leymarie, F., Grierson, M., Plamondon, R.: Calligraphic stylisation learning with a physiologically plausible model of movement and recurrent neural networks. In: ACM Proceedings of 4th International Conference on Movement Computing (MOCO), London, UK (2017). https://doi.org/10.1145/3077981.3078049
15. Berio, D., Calinon, S., Fol Leymarie, F.: Learning dynamic graffiti strokes with a compliant robot. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Daejeon, South Korea, pp. 3981–3986 (2016).
16. Berio, D., Fol Leymarie, F., Plamondon, R.: Expressive curve editing with the sigma lognormal model. In: Diamanti, O., Vaxman. A. (eds.), Proceedings of Eurographics, Delft, The Netherlands, pp. 33–36 (2018)
17. Berio, D., Fol Leymarie, F., Plamondon, R.: Kinematics reconstruction of static calligraphic traces from curvilinear shape features. In: Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.) The Lognormality Principle and its Applications in e-Security, e-Learning and e-Health. World Scientific Publishing, Series in Machine Perception and Artificial Intelligence (88), Ch. 11, pp. 237–268 (2021)
18. Bernard, C.O., Ponsford, J.A., McKinlay, A., McKenzie, D., Krieser, D.: Predictors of post-concussive symptoms in young children: injury versus non-injury related factors. J. Int. Neuropsychol. Soc. **22**(8), 793–803 (2016)
19. Bidias, S.P., David, J.-P., Savaria, Y., Plamondon, R.: On the use of interval arithmetic to bound delta-lognormal rapid human movements. In: Proceedings International Conference on pattern Recognition and Artificial Intelligence: Workshop on the Lognormality Principle and its Applications, Montréal, Canada, pp. 738–742 (2018)
20. Bhattacharya, U., Plamondon, R., Dutta Chowdhury, S., Goyal, P., Parui, S.K.: A sigma-lognormal model-based approach to generating large synthetic online handwriting samples databases. Int. J. Doc. Anal. Recogn. (IJDAR). **20**(71), 1–17 (2017)
21. Boyle, E.M., et al.: Neonatal outcomes and delivery of care for infants born late preterm or moderately preterm: a prospective population-based study. Arch Dis Child Fetal Neonatal Ed **100**, F479-485 (2015). https://doi.org/10.1136/archdischild-2014-307347
22. Boyoguéno-Bidias, S.P.: Extracteur de paramètres Delta-Lognormaux globalement optimaux par séparation et évaluation exploitant l'arithmétique par intervalles : thèse doctorat, Polytechnique Montréal (2022)
23. Cancer, A., Minoliti, R., Crepaldi, M., Antonietti, A.: Identifying developmental motor difficulties: a review of tests to assess motor coordination in children. J. Funct. Morphol. Kinesiol. **5**(1), 16 (2020). https://doi.org/10.3390/jfmk5010016
24. Cardarilli, G.C., Di Nunzio, L., Fazzolari, R., Re, M., Silvestri, F.: Improvement of the cardiac oscillator based model for the simulation of bundle branch blocks. Appl. Sci. **9**(18), 3653 (2019). Article 18. https://doi.org/10.3390/app9183653
25. Carmona-Duarte, C., Ferrer, M.A., Plamondon, R., Gómez-Rodellar, A., Gómez-Vilda, P.: Sigma-lognormal Modeling of Speech. Cogn. Comput. **13**(2), 488–503 (2021)

26. Carmona-Duarte, C., Ferrer, M.A., Gómez-Vilda, P., Van Gemmert, A.W.A., Plamondon, R.: Evaluating Parkinson's disease in voice and handwriting using the same methodology. In: The Lognormality Principle and its Applications in e-Security, e-Learning and e-Health, pp. 161–175 (2020). https://doi.org/10.1142/9789811226830_0007

27. Carmona-Duarte, C., Gomez, P., Ferrer, M.A., Plamondon, R., Londral, A.: Study of several parameters for the detection of amyotrophic lateral sclerosis from articulatory movement. Loquens. **4**(2), 038 (2017)

28. Charles, M., Soppelsa, R., Albaret, J.M.: BHK échelle d'évaluation rapide de l'écriture chez l'enfant [BHK Concise Evaluation Scale for Children's Handwriting]. ECPA (2004)

29. Chen, M., AlRegib, G., Juang, B.-H.: Air-writing recognition – part ii: detection and recognition of writing activity in continuous stream of motion data. IEEE Trans. Hum.-Mach. Syst. **46**(3), 436–444 (2016)

30. Cinnera, A.M., Morone, G.: Motor recovery in stroke rehabilitation supported by robot-assisted therapy. In: Assistive Technologies for Assessment and Recovery of Neurological Impairments, pp. 304–321 (2022). https://doi.org/10.4018/978-1-7998-7430-0.ch015

31. Cioni, G., Inguaggiato, E., Sgandurra, G.: Early intervention in neurodevelopmental disorders: underlying neural mechanisms. Dev. Med. Child Neurol. **58**(Suppl 4), 61–66 (2016). https://doi.org/10.1111/dmcn.13050

32. Côté, J.N.: Adaptations to neck/shoulder fatigue and injuries. Adv. Exp. Med. Biol. **826**, 205–228 (2014)

33. Cserjesi, R., et al.: Functioning of 7-year-old children born at 32 to 35 weeks' gestational age. Pediatrics Pediatrics **130**(4), e838–846 (2012). https://doi.org/10.1542/peds.2011-2079

34. Dankovičová, Z., Drotár, P., Gazda, J., Vokorokos, L.: Overview of the handwriting processing for clinical decision support system. In: IEEE 14th International Scientific Conference on Informatics, p. 6367 (2017). https://doi.org/10.1109/INFORMATICS.2017.8327223

35. Danna, J., Longcamp, M., Nalborczyk, L. Velay, J-L., Commengé, C., Jover, M.: Interaction between orthographic and graphomotor constraints in learning to write. Learn. Instr. **80**, 101622 (2022)

36. Delenclos, M., Jones, D.R., McLean, P.J., Uitti, R.J.: Biomarkers in Parkinson's disease: advances and strategies. Parkinsonism Rel. Disord **22**, S106–S110 (2016)

37. Diaz, M., Fischer*, A., Ferrer, M.A., Plamondon, R.: Dynamic signature verification system based on one real signature. IEEE Trans. Cybern. **48**(1), 228–239 (2018)

38. Diaz, M., Ferrer, M.A., Impedovo, D., Malik, M.I., Pirlo, G., Plamondon, R.: A perspective analysis of handwritten signature technology ACM Comput. Surv. **51**(6), 39 (2019)

39. Diaz, M., Quintana, J.J., Ferrer, M.A., Carmona-Duarte, C., Wolniakowski A., Miatliuk, K.: Lognormality of velocity profiles in rapid robotic arm movements. In: 19th International Graphonomics Conference (IGS2019), Cancun, México, June 9–12 (2019)

40. Diaz, M., et al.: Studying the principle of lognormality in livestocks. In: Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.) The Lognormality Principle and its applications in e-security, e-learning and e-health, World Scientific, Series in Machine Perception Artificial Intelligence, ISBN: 978-081-122-682-2, vol. 88, pp. 397–398 (2021). https://doi.org/10.1142/9789811226830

41. Djioua, M., Plamondon, R.: A new algorithm and system for the characterization of handwriting strokes with delta-lognormal parameters. IEEE Trans. Pattern Anal. Mach. Intell. **31**(11), 2060–2072 (2009)

42. Djioua, M., Plamondon, R.: A new algorithm and system for the extraction of delta-lognormal parameters. Technical Report, EPM-RT-2008-04, École Polytechnique de Montréal (2008)

43. Djioua, M., & Plamondon, R. (2008). A new methodology to improve myoelectric signal processing using handwriting. In: Proseedings International Conference on Frontiers in Handwriting Recognition, Montreal, ICFHR'2008, pp. 112–117

44. Douret, L., Auzias, M.: Le développement de l'organisation motrice et temporo-spatiale de l'écriture chez l'enfant. Approche Neuropsychologique des Apprentissages chez l'Enfant (ANAE) **5**, 2935 (1993)

45. Duval, T., Rémi, C., Plamondon, R., Vaillant, J., O'Reilly, C.: Combining sigma-lognormal modeling and classical features for analyzing graphomotor performances in kindergarten children. Hum. Mov. Sci. **43**, 183–200 (2015). https://doi.org/10.1016/j.humov.2015.04.005

46. Duval, T, Rémi, C., Plamondon, R., O'Reilly, C.: On the use of the sigma-lognormal model to study children handwriting. In: 16th Biennial Conference of the Graphonomics Society (IGS 2013), Japan, pp. 26–29 (2013)

47. Eliasson, A.-C., Ro€sblad, B., Forssberg, H.: Disturbances in programming goal-directed arm movements in children with ADHD. Dev. Med. Child Neurol. **46**(1), 19–27 (2004). https://doi.org/10.1111/j.1469-8749.2004.tb00429.x

48. Faci, N., Désiré, N., Beauchamp, M.H., Gagnon, I., Plamondon, R.: Lognormality in children with mild traumatic brain injury: a preliminary pilot study. In: Proceedings of ICPRAI 2018. ICPRAI 2018 First International Workshop on the Lognormality Principle and its Applications, Montréal, Canada, pp. 790–794 (2018)

49. Faci, N., Boyogueno Bidias, S.P., Plamondon, R., Bergeron, N.: An interactive tablet-based system to run neuromuscular tests. In: Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.) The Lognormality Principle and Its Applications in E-Security, e-Learning and e-Health, World Scientific Publishing, Series in Machine Perception and Artificial Intelligence, vol.88, pp 269–288 (2021). https://doi.org/10.1142/9789811226830_0012

50. Faci, N., Désiré, N., Beauchamp, M.H., Gagnon, I., Plamondon, R.: Lognormality in children with mild traumatic brain injury: a preliminary pilot study. In Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.) The Lognormality Principle and its Applications in e-Security, e-Learning and e-Health. World Scientific Publishing, Series in Machine Perception and Artificial Intelligence, vol. 88, pp. 143–160 (2021)

51. Faundez-Zanuy, M., Fierrez, J., Ferrer, M.A., Diaz, M., Tolosana, R., Plamondon, R.: Handwriting biometrics: applications and future trends in e-security and e-health. Cogn. Comput. **12**, 940–953 (2020)

52. Fawaz, H.I., Forestier, G., Weber, J., Idoumghar, L., Muller, P.-A.: Accurate and interpretable evaluation of surgical skills from kinematic data using fully convolutional neural networks. Int. J. Comput. Assist. Radiol. Surg. **14**(9), 1611–1617 (2019). https://doi.org/10.1007/s11548-019-02039-4

53. Feldman, J., Singh, M.: Information along contours and object boundaries. Psychol. Rev. **112**(1), 243–252 (2005)

54. Ferrer, M.A., Diaz, M., Carmona-Duarte, C., Plamondon, R.: iDelog: iterative dual spatial and kinematic extraction of sigma-lognormal parameters. IEEE Trans. Pattern Anal. Mach. Intell. (PAMI). 42(1): 114–125 (2020)

55. Ferrer, M., et al.: From operculum and body tail movements to different coupling of physical activity and respiratory frequency in farmed gilthead sea bream and European sea bass. Insights on aquaculture biosensing. Comput. Electron. Agric. **175**(105531), 1–9 (2020). ISSN 0168–1699

56. Ferrer, M., Diaz, M., Quintana-Hernandez, J., Carmona-Duarte, C., Plamondon, R.: A multilognormal analysis of 3D human movements. IEEE Trans. Pattern Anal. Mach. Intell. 13, March 2023

57. Fahr, A., Keller, J.W., Balzer, J., Lieber, J., van Hedel, H.J.A.: Quantifying age-related differences in selective voluntary motor control in children and adolescents with three assessments. Hum. Mov. Sci. **77**, 102790 (2021). https://doi.org/10.1016/j.humov.2021.102790

58. Fischer, A., Plamondon, R.: Signature verification based on the kinematic theory of rapid human movements. IEEE Trans. Hum. Mach. Syst. **47**(2), 169–180 (2017)

59. Fischer, A., Schindler, R., Bouillon, M., Plamondon, R.: Modeling 3d movements with the kinematic theory of rapid human movements. In: Plamondon, R., Marcelli, A., Ferrer, M. (eds.), The Lognormality Principle and its Applications in e-Security, e-Learning and e-Health, chapter 15, pp. 327–342. World Scientific (2021)

60. Fong, W.C.: Why Chinese painting is history. Art Bull. **85**(2), 258–280 (2003)

61. Freedberg, D., Gallese, V.: Motion, emotion and empathy in esthetic experience. Trends Cogn. Sci. **11**(5), 197–203 (2007)

62. Freyd, J.J.: Representing the dynamics of a static form. Mem. Cognit. **11**(4), 342–346 (1983)

63. Funke, I., Mees, S.T., Weitz, J., Speidel, S.: Video-based surgical skill assessment using 3D convolutional neural networks. Int. J. Comput. Assist. Radiol. Surg. **14**(7), 1217–1225 (2019). https://doi.org/10.1007/s11548-019-01995-1

64. Goldbraikh, A., D'Angelo, A.-L., Pugh, C.M., Laufer, S.: Video-based fully automatic assessment of open surgery suturing skills. Int. J. Comput. Assist. Radiol. Surg. **17**(3), 437–448 (2022). https://doi.org/10.1007/s11548-022-02559-6

65. Goubault, E., Martinez, R., Bouffard, J., Dowling-Medley, J., Begon, M., Dal Maso, F.: Shoulder electromyography-based indicators to assess manifestation of muscle fatigue during laboratory-simulated manual handling task. Ergonomics **65**(1), 118–133 (2022)

66. Goubault, E., Verdugo, F., Pelletier, J., Traube, C., Begon, M., Dal Maso, F.: Exhausting repetitive piano tasks lead to local forearm manifestation of muscle fatigue and negatively affect musical parameters. Sci. Rep. **11**(1), 8117 (2021)

67. Guerfali, W., Plamondon, R.: Signal processing for the parameter extraction of the delta lognormal model. In: Research in Computer and Robot Vision, pp. 217–232, World Scientific 1995

68. Hajian-Tilaki, K.: Receiver operating characteristic (ROC) curve analysis for medical diagnostic test evaluation. Caspian J. Intern. Med. **4**(2), 627–635 (2013)

69. Hamstra-Bletz, L., Blöte, A.W.: Development of handwriting in primary school: a longitudinal study. Percept. Mot. Skills **70**(3), 759770 (1990)

70. Hansen, E., Walster, G.W.: Global Optimization Using Interval Analysis: Revised and Expanded. CRC Press, Boca Raton (2003)

71. Heaton, J.: An empirical analysis of feature engineering for predictive modeling. Southeast-Con **2016**, 1–6 (2016). https://doi.org/10.1109/SECON.2016.7506650

72. Hira, S., et al.: Video-based assessment of intraoperative surgical skill. Int. J. Comput. Assist. Radiol. Surg. **17**(10), 1801–1811 (2022). https://doi.org/10.1007/s11548-022-02681-5

73. Hong, M., Perlmutter, J.S., Earhart, G.M.: A kinematic and electromyographic analysis of turning in people with parkinson disease. Neurorehabil. Neural Repair **23**(2), 166–176 (2009). https://doi.org/10.1177/1545968308320639

74. Hughes, A.J., Redsell, S.A., Glazebrook, C.: Motor development interventions for preterm infants: a systematic review and meta-analysis. Pediatrics **138**, (2016). https://doi.org/10.1542/peds.2016-0147

75. Kaiser, M.L., Schoemaker, M.M., Albaret, J.M., Geuze, R.H.: What is the evidence of impaired motor skills and motor control among children with attention deficit hyperactivity disorder (ADHD)? systematic review of the literature. Res. Dev. Disabil. **36C**, 338–357 (2015). https://doi.org/10.1016/j.ridd.2014.09.023

76. Kerstjens, J.M., et al.: Developmental delay in moderately preterm-born children at school entry. J. Pediatr. **159**, 92–98 (2011). https://doi.org/10.1016/j.jpeds.2010.12.041

77. Kieves, N.R.: Objective gait analysis: review and clinical applications. Vet. Clin. North Am. Small Anim. Pract. **52**(4), 857–867 (2022). https://doi.org/10.1016/j.cvsm.2022.03.009

78. Korkman, M., Kirk, U., Kemp, S.: NEPSY-II : Bilan neuropsychologique de l'enfant. Seconde édition. Pearson Canada Assessment, Toronto (2012)

79. Laniel, P., Faci, N., Plamondon, R., Beauchamp, M.H., Gauthier, B.: Kinematic analysis of fast pen strokes in children with ADHD. Appl. Neuropsychol. Child **9**, 125–140 (2020). https://doi.org/10.1080/21622965.2018.1550402

80. Laurent, A., Plamondon, R., Begon, M.: Central and peripheral shoulder fatigue pre-screening using the sigma-lognormal model: a proof of concept. Front. Hum. Neurosci. **14**, 171 (2020)

81. Laurent, A., Plamondon, R., Begon, M.: Reliability of the kinematic theory parameters during handwriting tasks on a vertical setup. Biomed. Sig. Process. Control **71**, 103157 (2022)

82. Lawhern, V.J., Solon, A.J., Waytowich, N.R., Gordon, S.M., Hung, C.P., Lance, B.J.: Eegnet: A compact convolutional network for EEG-based brain-computer interfaces (2016). CoRR, abs/1611.08024. http://arxiv.org/abs/1611.08024. arXiv:1611.08024

83. Lebel, K., Nguyen, H., Duval, C., Plamondon, R., Boissy, P.: Capturing the cranio-caudal signature of a turn with inertial measurement systems: methods, parameters robustness and reliability. Front. Bioeng. Biotech. **5**, 51 (2017). https://doi.org/10.3389/fbioe.2017.00051

84. Lebel, K., Duval, C., Nguyen, H., Plamondon, R., Boissy, P.: Turn cranio-caudal signature assessment from inertial systems for mobility deficit identification in Parkinson's disease patients. Parkinsonism Relat. Disord. **46**, E24–E25 (2018)

85. Lebel, K., Duval, C., Nguyen, H.P., Plamondon, R., Boissy, P.: Cranio-caudal kinematic turn signature assessed with inertial systems as a marker of mobility deficits in Parkinson's disease. Front. Neurol. **9**, 22 (2018). https://doi.org/10.3389/fneur.2018.00022

86. Lebel, K., Duval, C., Plamondon, R., Faci, N., Boissy, P.: Sigma-Lognormal modelling to assess mobility deficits: the case of turn signature assessed with inertial measurement units (IMUs). In: Plamondon, R., Marcelli, A., Ferrer, M.Á., (eds.) The Lognormality Principle and its Applications in e-Security, e-Learning and e-Health. World Scientific Publishing Series in Machine Perception and Artificial Intelligence, vol. 88, pp. 177–193 (2020). https://doi.org/10.1142/9789811226830_0008

87. Leiva, L.A., Martín-Albo, D., Plamondon, R., Vatavu, R.-D.: KeyTime: super-accurate prediction of stroke gesture production times. In: Proceedings of the 2018 CHI Conference on Human Factors in Computing Systems, vol. 2018 (2018)

88. Leiva, L., Diaz, M., Ferrer, M.A., Plamondon, R.: Human or machine? it is not what you write, but how you write It. In: Proceedings of ICPR 2020. 25th International Conference on Pattern Recognition, ICPR 2020, Milan, Italy, pp. 2612–2619 (2021)

89. Leiva, L., Vatavu, R.D., Martin Albo, D., Plamondon, R.: Omnis praedictio: estimating the full spectrum of human performance with stroke gestures. Int. J. Hum. Comput. Stud. **142**, 102466 (2020)

90. Leiva, L., Martin Albo, D, Plamondon, R., Vidal, E.: Gestures à go go: authoring synthetic human-like gestures using the kinematic theory of rapid movements. In: Proceedings of CHI 2015. CHI 2015, Seoul, Korea, pp.1–10 (2015)

91. Levien, R.: From spiral to spline: optimal techniques in interactive curve design, Ph.D. thesis, EECS Department, University of California, Berkeley (2009)

92. Leyton, M.: Symmetry-curvature duality. Comput. Vis. Graph. Image Process. **38**(3), 327–341 (1987)

93. Longcamp, M., Anton, J.L., Roth, M., Velay, J.L.: Visual presentation of single letters activates a pre-motor area involved in writing. Neuroimage **19**(4), 1492–1500 (2003)

94. Louis, D., et al.: School readiness among children born preterm in Manitoba. Canada. JAMA Pediatr **176**, 1010–1019 (2022). https://doi.org/10.1001/jamapediatrics.2022.2758

95. Mancini, M., et al.: Continuous monitoring of turning in Parkinson's disease: rehabilitation potential. NeuroRehabilitation **37**(1), 3 (2015). https://doi.org/10.3233/NRE-151236

96. Martin-Albo, D., Leiva, L.A., Huang, J., Plamondon, R.: Strokes of insight: user intent detection and kinematic compression of mouse cursor trails. Inf. Process. Manage. **52**(6), 989–1003 (2016)

97. Miatliuk, K., Wolniakowski, A., Diaz, M., Ferrer, M.A.: Universal robot employment to mimic human writing. In: 20th International Carpathian Control Conference, ISBN: 978-1-7281-0702-8, pp. 1–5, 26–29 May 2019. https://doi.org/10.1109/CarpathianCC.2019.8766027, Kraków-Wieliczka, Poland

98. Nadeau, A., et al.: A 12-week cycling training regimen improves upper limb functions in people with Parkinson's disease. Front. Hum. Neurosci. **12**(351), 1–10 (2018). https://doi.org/10.3389/fnhum.2018.00351

99. Mediavilla, C.: Calligraphy: From Calligraphy to Abstract Painting. Scirpus Publications, Wommelgem (1996)

100. Michaud, B., Bailly, F., Charbonneau, E., Ceglia, A., Sanchez, L., Begon, M.: Bioptim, a python framework for musculoskeletal optimal control in biomechanics. IEEE Trans. Syst. Man Cybern. **53**(1), 321–332 (2023)

101. Moore, R.E.: Interval analysis. Prentice-Hall, Englewood Cliffs (1966)

102. Moran, L.M., et al.: Quality of life in pediatric mild traumatic brain injury and its relationship to postconcussive symptoms. J. Pediatr. Psychol. **37**(7), 736–744 (2011)

103. Morin, M.-F., Bara, F., Alamargot, D.: Apprentissage de la graphomotricité à l'école : Quelles acquisitions? Quelles pratiques? Quels outils? Scientia Paedagogica Experimentalis, LIV **1–2**, 4782 (2017)

104. Müller, M., Röder, T., Clausen, M., Eberhardt, B., Krüger, B., Weber, A.: Documentation mocap database HDM05, Technical Report, CG-2007-2, Universität Bonn (2007)

105. O'Reilly, C., Plamondon, R.: Development of a sigma-lognormal representation for on-line signatures. Pattern Recogn. Spec. Issue Front. Handwriting Recogn. **42**(12), 3324–3337 (2009)

106. O'Reilly, C., Plamondon, R.: Prototype-based methodology for the statistical analysis of local features in stereotypical handwriting tasks. In: International Conference on Pattern Recognition, IEEE, Istanbul, Turkey, pp.1864–1867 (2010)

107. O'Reilly, C., Plamondon, R., Landou, M.K., Stemmer, B.: Using kinematic analysis of movement to predict the time occurrence of an evoked potential associated with a motor command. Eur. J. Neurosci. **37**(2), 173–180 (2013). https://doi.org/10.1111/ejn.12039

108. O'Reilly, C., Plamondon, R.: A globally optimal estimator for the delta-lognormal modeling of fast reaching movements. IEEE Trans. Syst. Man Cybern. Part B (Cybernetics) **42**, 1428–1442 (2012). https://doi.org/10.1109/TSMCB.2012.2192109

109. O'Reilly, C., Oruganti, S.D.R., Tilwani, D., Bradshaw, J.: Model-driven analysis of ECG using reinforcement learning. Bioengineering **10**(6), 696 (2023)

110. Oğul, B.B., Gilgien, M., Özdemir, S.: Ranking surgical skills using an attention-enhanced siamese network with piecewise aggregated kinematic data. Int. J. Comput. Assist. Radiol. Surg. **17**(6), 1039–1048 (2022). https://doi.org/10.1007/s11548-022-02581-8

111. Pan, Z., Talwar, S., Plamondon, R., Van Gemmert, A.W.: Characteristics of bi-directional unimanual and bimanual drawing movements: The application of the delta-lognormal models and sigma-lognormal model. Pattern Recogn. Lett. **121**, 97–103 (2019)

112. Paz-Villagrán, V., Danna, J., Velay, J.-L.: Lifts and stops in proficient and dysgraphic handwriting. Hum. Movement Sci. **33**, 381–394 (2014). https://doi.org/10.1016/j.humov.2013.11.005

113. Pfeiffer, B., Moskowitz, B., Paoletti, A., Brusilovskiy, E., Zylstra, S.E., Murray, T.: Developmental test of visual-motor integration (VMI): an effective outcome measure for handwriting interventions for kindergarten, first-grade, and second-grade students? Am. J. Occup. Ther. **69**(4), 1–7 (2015). https://doi.org/10.5014/ajot.2015.015826

114. Pignocchi, A.: How the intentions of the draftsman shape perception of a drawing. Conscious. Cogn. **19**(4), 887–898 (2010)
115. Plamondon, R., Djioua, M., O'Reilly, C.: La Théorie cinématique des mouvements humains rapides : développements récents. Traitement du Signal, Numéro Spécial : Le Document Écrit. **26**(5), 377–394 (2009)
116. Plamondon, R., O'Reilly, C., Rémi, C., Duval, T.: The lognormal handwriter: learning, performing and declining. Front. Psychol. Cogn. Sci. **2013**, 1–14 (2013)
117. Plamondon, R., Djioua, M.: A multi-level representation paradigm for handwriting stroke generation. Hum. Mov. Sci. **25**(4–5), 586–607 (2006)
118. Plamondon, R.: A kinematic theory of rapid human movements. Part I. movement representation and generation. Biol. Cybern. **72**(4), 295–307 (1995a). https://doi.org/10.1007/BF00202785
119. Plamondon, R.: A kinematic theory of rapid human movements: part II: movement time and control. Biol. Cybern. **72**(4), 309–320 (1995). https://doi.org/10.1007/BF00202786
120. Plamondon, R.: A kinematic theory of rapid human movements: Part III: kinetic outcomes. Biol. Cybern. **78**, 133–145 (1998)
121. Plamondon, R.: The lognormality principle: a personalized survey. In: Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.), The Lognormality Principle and its applications in e-security, e-learning and e-health, World Scientific Publishing, Series in Machine Perception and Artificial Intelligence, vol. 88, pp 1–39 (2021)
122. Plamondon, R., Guerfali, W.: The generation of handwriting with delta-lognormal synergies. Biol. Cybern. **78**(2), 119–132 (1998). https://doi.org/10.1007/s004220050419
123. Plamondon, R., Feng, C., Djioua, M.: The convergence of a neuromuscular impulse response towards a lognormal, from theory to practice. (Technical Report EPM-RT-2008–08) (2008)
124. Plamondon, R.: A kinematic theory of rapid human movements part V. Movement representation and generation. Biol. Cybern. **72**, 295–307 (1995)
125. Plamondon, R., Feng, C., Woch, A.: A kinematic theory of rapid human movements: part IV: a formal mathematical proof and new insights. Biol. Cybern. **89**, 126–138 (2003)
126. Plamondon, R., Li, X., Djioua, M.: Extraction of delta-lognormal parameters from handwriting strokes. J. Front. Comput. Sci. China **1**(1), 106–113 (2007)
127. Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.) The lognormality principle and its applications in e-security, e-learning and e-health. World Sci. Publ. Ser. Mach. Percept. Artif. Intell. **88**, 415 (2021)
128. Plamondon, R., O'Reilly, C., Galbally, J., Almaksour, A., Anquetil, E.: Recent developments in the study of rapid human movements with the Kinematic theory: applications to handwriting and signature synthesis. Pattern Recogn. Lett. **35**(1), 225–235 (2014). https://doi.org/10.1016/j.patrec.2012.06.004
129. Plamondon, R., O'Reilly, C., Ouellet-Plamondon, C.: Strokes against stroke—strokes for strides. Pattern Recogn. **47**(3), 929–944 (2014). https://doi.org/10.1016/j.patcog.2013.05.004
130. Plamondon, R., Pirlo, G., Anquetil, E., Rémi, C., Teuling, H-L., Nakagawa, M.: Personal digital bodyguards for e-security, e-learning and e-health: a prospective survey. Pattern Recogn. **81**, 633–659 (2018). https://doi.org/10.1016/j.patcog.2018.04.012
131. Pontart, V., Bidet-Ildei, C., Lambert, E., Morisset, P., Flouret, L., Alamargot, D.: Influence of handwriting skills during spelling in primary and lower secondary grades. Front. Psychol. **4**, 19 (2013). https://doi.org/10.3389/fpsyg.2013.00818
132. Provincial council for maternal and child health. Final report of the neonatal follow-up clinics. vol. 47 (2015)
133. Quintana, J.J., Ferrer, M.A., Diaz, M., Feo, J.J., Wolniakowski, A., Miatliuk, K.: Uniform vs. lognormal kinematics in robots: perceptual preferences for robotic movements. Appl. Sci. **12**(23), 12045 (2022). ISSN: 2523–3963

134. Quiroz-Juárez, M.A., Jiménez-Ramírez, O., Vázquez-Medina, R., Breña-Medina, V., Aragón, J.L., Barrio, R.A.: Generation of ECG signals from a reaction-diffusion model spatially discretized. Sci. Rep. **9**(1), 19000 (2019). https://doi.org/10.1038/s41598-019-554 48-5

135. Raju, T.N.: Developmental physiology of late and moderate prematurity. Semin. Fetal Neonatal. Med. **17**, 126–131 (2012). https://doi.org/10.1016/j.siny.2012.01.010

136. Rémi, C., Nagau, J., Vaillant, J., Dorville, A., Plamondon, R.: Multimodal acquisition and analysis of children handwriting for the study of the efficiency of their handwriting movements: the @MaGma challenge. In: 16th International Conference on Frontiers in Handwriting Recognition, pp.459–464. Niagara Falls, United States (2018)

137. Rémi, C., Vaillant, J., Plamondon, R., Prévost, L., Duval, T.: Exploring the kinematic dimensions of kindergarten children's scribbles. In: 17th Biennial Conference of the International Graphonomics Society, 79–82. Guadeloupe (2015)

138. Rémi, C., Nagau, J., Vaillant, J., Plamondon, R.: Preliminary study of t0, a sigma-lognormal parameter extracted from young childrens controlled scribbles. In: Proceedings of the 18th International Graphonomics Society Conference, Italy, pp.93–97 (2017)

139. Rémi, C., Nagau, J., Vaillant, J., Plamondon, R.: Could sigma-lognormal modeling help teachers to characterize the kinematic efficiency of pupils' cursive procedures of handwriting? In: Plamondon, R., Marcelli, A., Ferrer, M.A. (eds.) The Lognormality Principle and its Applications in e-Security, e-Learning, and e-Health, pp. 87–116. World Scientific Publishing Company, Series in Machine Perception and Artificial Intelligence, vol. 88 (2021). https://doi.org/10.1142/9789811226830_0004

140. Rémi, C., Nagau, J., Copilotrace: a platform to process graphomotor tasks for education and graphonomics research. In: Carmona-Duarte, C., Diaz, M., Angel Ferrer, M., Morales, A. (eds.) 20th International Conference of the International Graphonomics Society 2022, LNCS, vol. 13424, pp. 129–143. Springer, Heidelberg (2022), https://doi.org/10.1007/978-3-031-19745-1_10

141. Rocca, W.A.: The burden of Parkinson's disease: a worldwide perspective [Comment]. Lancet Neurol. **17**, 928–929 (2018)

142. Rosell-Moll, E., et al.: Use of accelerometer technology for individual tracking of activity patterns, metabolic rates and welfare in farmed gilthead sea bream (Sparus aurata) facing a wide range of stressors. In: Aquaculture, ISSN 0044–8486, vol. 539, pp. 1–10, 30 (2021)

143. Rosenblum, S., Epsztein, L., Josman, N.: Handwriting performance of children with attention deficit hyperactive disorders: a pilot study. Phys. Occup. Ther. Pediatr. **28**(3), 219–234 (2008). https://doi.org/10.1080/01942630802224934

144. Schatz P. (2011) Finger Tapping Test. In: Kreutzer J.S., DeLuca J., Caplan B. (eds) Encyclopedia of Clinical Neuropsychology. Springer, New York, NY, 1050–1051

145. Schindler, R., Bouillon, M., Plamondon, R., Fischer*, A.: Extending the sigma-lognormal model of the kinematic theory to three dimensions. In: Proceedings of ICPRAI 2018. ICPRAI 2018 First International Workshop on the Lognormality Principle and its Applications, Montréal, Canada, 748–752 (2018)

146. Shah, P., Kaciroti, N., Richards, B., Oh, W., Lumeng, J.C.: Developmental outcomes of late preterm infants from infancy to kindergarten. Pediatrics **138** (2016). https://doi.org/10.1542/peds.2015-3496

147. Sica, M., et al.: Continuous home monitoring of Parkinson's disease using inertial sensors: a systematic review. PLoS ONE **16**(2), e0246528 (2021). https://doi.org/10.1371/journal.pone.0246528

148. Smits, E.J., et al.: Standardized handwriting to assess bradykinesia, micrographia and tremor in Parkinson's disease. PLoS ONE **9**(5), e97614 (2014)

149. Soangra, R., Sivakumar, R., Anirudh, E.R., Reddy Y.S.V., John, E.B.: Evaluation of surgical skill using machine learning with optimal wearable sensor locations. PLOS ONE **17**(6) (2022). https://doi.org/10.1371/journal.pone.0267936

150. Steinhart, S., Weiss, P.L., Friedman, J.: Proximal and distal movement patterns during a graphomotor task in typically developing children and children with handwriting problems. J. Neuroeng. Rehabil. **18**, 178 (2021). https://doi.org/10.1186/s12984-021-00970-9

151. Tiffin, J.: Purdue Pegboard Examiner Manual. Science Research Associates, Chicago (1968)

152. Todorov, E.: Optimality principles in sensorimotor control. Nat. Neurosci. **7**(9), 907–915 (2004)

153. Ungurean O.-C., Vatavu R.-D., Leiva L.A., Plamondon, R.: Gesture input for users with motor impairments on touchscreens: empirical results based on the kinematic theory. In: Proceedings of the 2018 CHI Conference on Human Factors in Computing Systems (2018)

154. Vaivre-Douret, L., Paquet, A.: L'évaluation standardisée développementale des fonctions neuro-psychomotrices: présentation de la batterie normée française NP-MOT. Neurophysiol. Clin. **49**(6), 421 (2019)

155. Vohr, B.: Long-term outcomes of moderately preterm, late preterm, and early term infants. Clin. Perinatol. **40**, 739–751 (2013). https://doi.org/10.1016/j.clp.2013.07.006

156. Walha, R., et al.: The accuracy and precision of gait spatio-temporal parameters extracted from an instrumented sock during treadmill and overground walking in healthy subjects and patients with a foot impairment secondary to psoriatic arthritis. Sensors **21**(18), 6179 (2021). https://doi.org/10.3390/s21186179

157. Wechsler, D.: WISC-IV: Échelle d'intelligence de Wechsler pour enfants [Wechsler Intelligence Scale for Children, 4th ed. (WISC-IV)]. Toronto, ON: PsychCorp (2004)

158. Woch, A., Plamondon, R., O'Reilly, C.: Kinematic characteristics of bidirectional delta-lognormal primitives in young and older subjects. Hum. Mov. Sci. **30**(1), 1–17 (2011). https://doi.org/10.1016/j.humov.2009.10.006

159. Wolniakowski, A., Quintana, J.J., Diaz, M., Miatliuk, K., Ferrer, M.A.: Towards human-like kinematics in industrial robotic arms: a case study on a UR3 robot. In: Procceding of the 54th International Carnahan Conference on Security Technology (ICCST), University of Hertfordshire, Hatfield, UK, pp. 1–5, 1–3, September 2022. https://doi.org/10.1109/ICCST49569.2021.9717393

160. World Health Organization. Neurological disorders: public health challenges: World Health Organization (2006)

161. Xia, L., Chen, C. Aggarwal, J.: View invariant human action recognition using histograms of 3d joints. In: Proceedings of IEEE Conference on Computer Vision and Pattern Recognition Workshops (CVPRW), pp. 20–27 (2012)

162. Yan, J.H., Thomas, J.R.: Arm movement control: differences between children with and without attention deficit hyperactivity disorder. Res. Q. Exerc. Sport **73**(1), 10–18 (2002). https://doi.org/10.1080/02701367.2002.10608987

163. Zemek, R., Osmond, M.H., Barrowman, N., on behalf of PERC Concussion Team.: In: Predicting and preventing postconcussive problems in paediatrics (5P) study: protocol for a prospective multicentre clinical prediction rule derivation study in children with concussion. BMJ Open **2013**(3), e003550 (2013). https://doi.org/10.1136/bmjopen-2013-003550

164. Zerveas, G., Jayaraman, S., Patel, D., Bhamidipaty, A., Eickhoff, C.: A transformer-based framework for multivariate time series representation learning. In: Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining, pp. 2114–2124 (2021). https://doi.org/10.1145/3447548.3467401

165. Zhou, H., Hu, H.: Human motion tracking for rehabilitation—a survey. Biomed. Sig. Proc. Control **3**(1), 1–18 (2008). Article 1. WorldCat.org. https://doi.org/10.1016/j.bspc.2007.09.001

166. Zia, A., Sharma, Y., Bettadapura, V., Sarin, E.L., Essa, I.: Video and accelerometer-based motion analysis for automated surgical skills assessment. Int. J. Comput. Assist. Radiol. Surg. **13**(3), 443–455 (2018). https://doi.org/10.1007/s11548-018-1704-z

167. Sady, M., Vaughan, C.G., Gioia, G.A.: Psychometric characteristics of the postconcussion symptom inventory in children and adolescents. Arch Clin. Neuropsychol. **29**(4), 348–63 (2014). https://doi.org/10.1093/arclin/acu014. Epub 2014 Apr 15